TMDs and Spin asymmetries in a light front quark-diquark model of the proton

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To probe the proton structure:

- Inclusive Process: Deep Inelastic Scattering(DIS)

$$l(k) + P(p) \to l(k') + X$$

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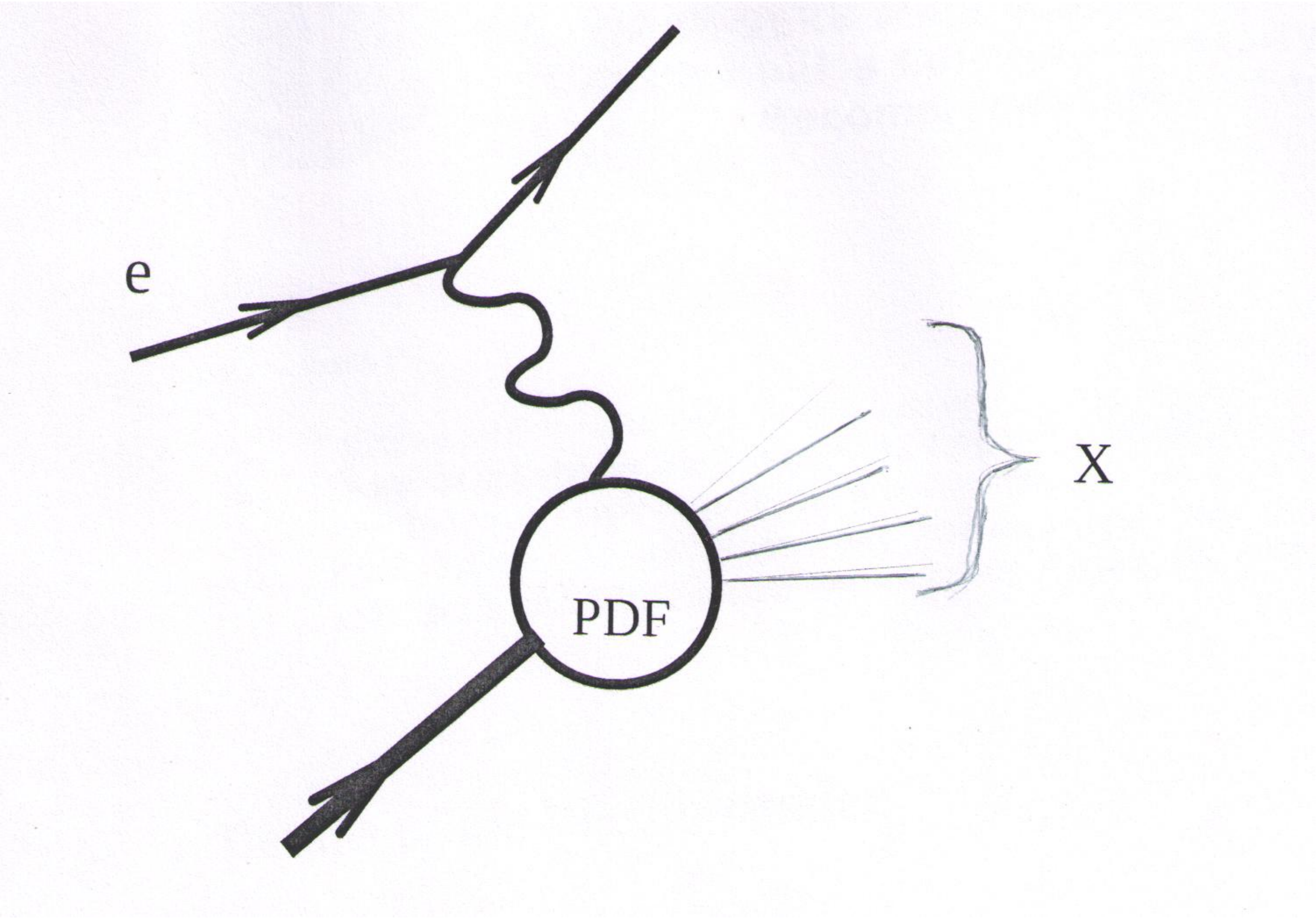

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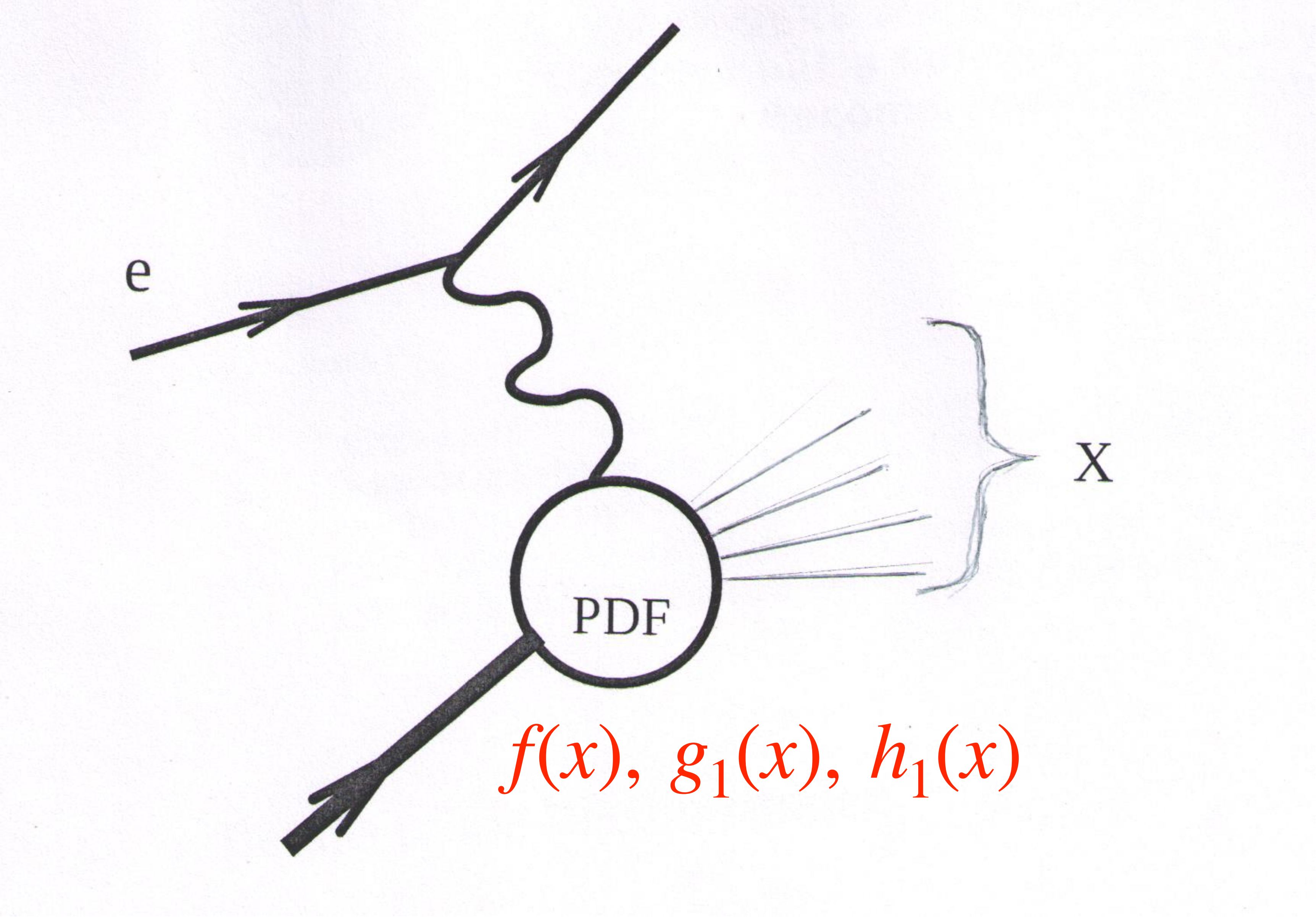
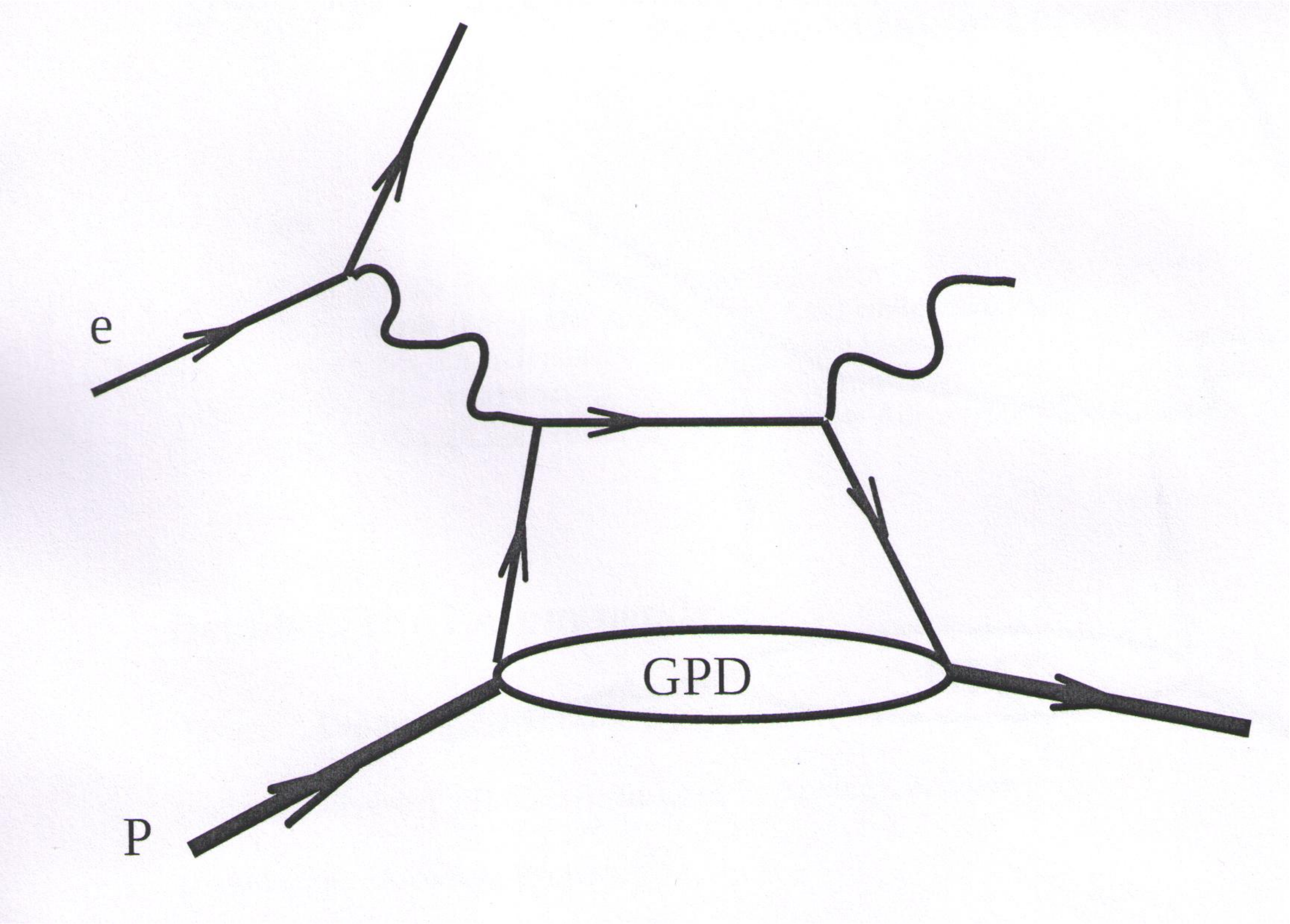


$f(x),\ g_1(x),\ h_1(x)$

- Exclusive Process, e.g. DVCS process:

$$\gamma^*(q) + P(p) \to \gamma(k) + P(p')$$



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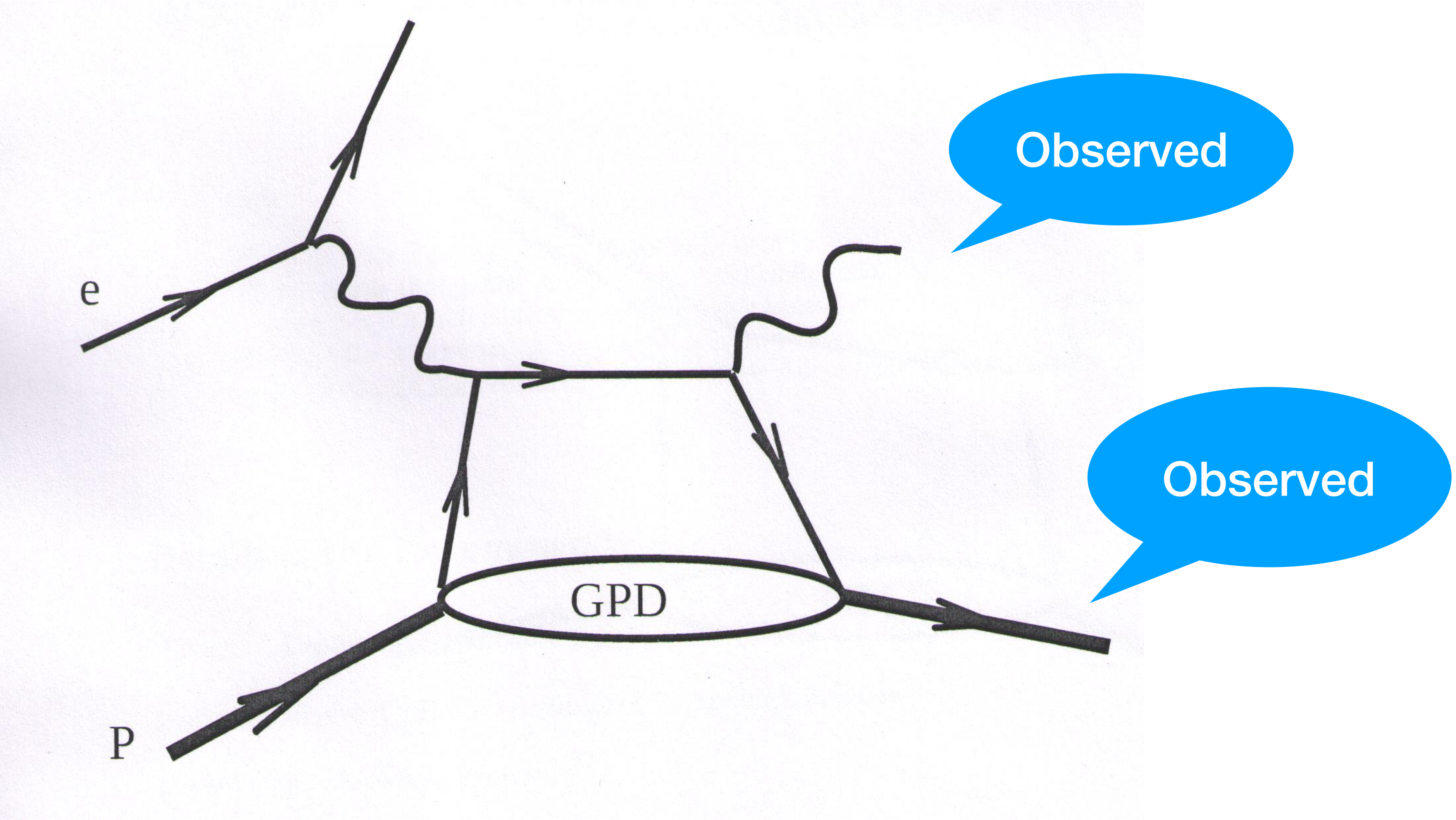

Final particles of a particular process are observed

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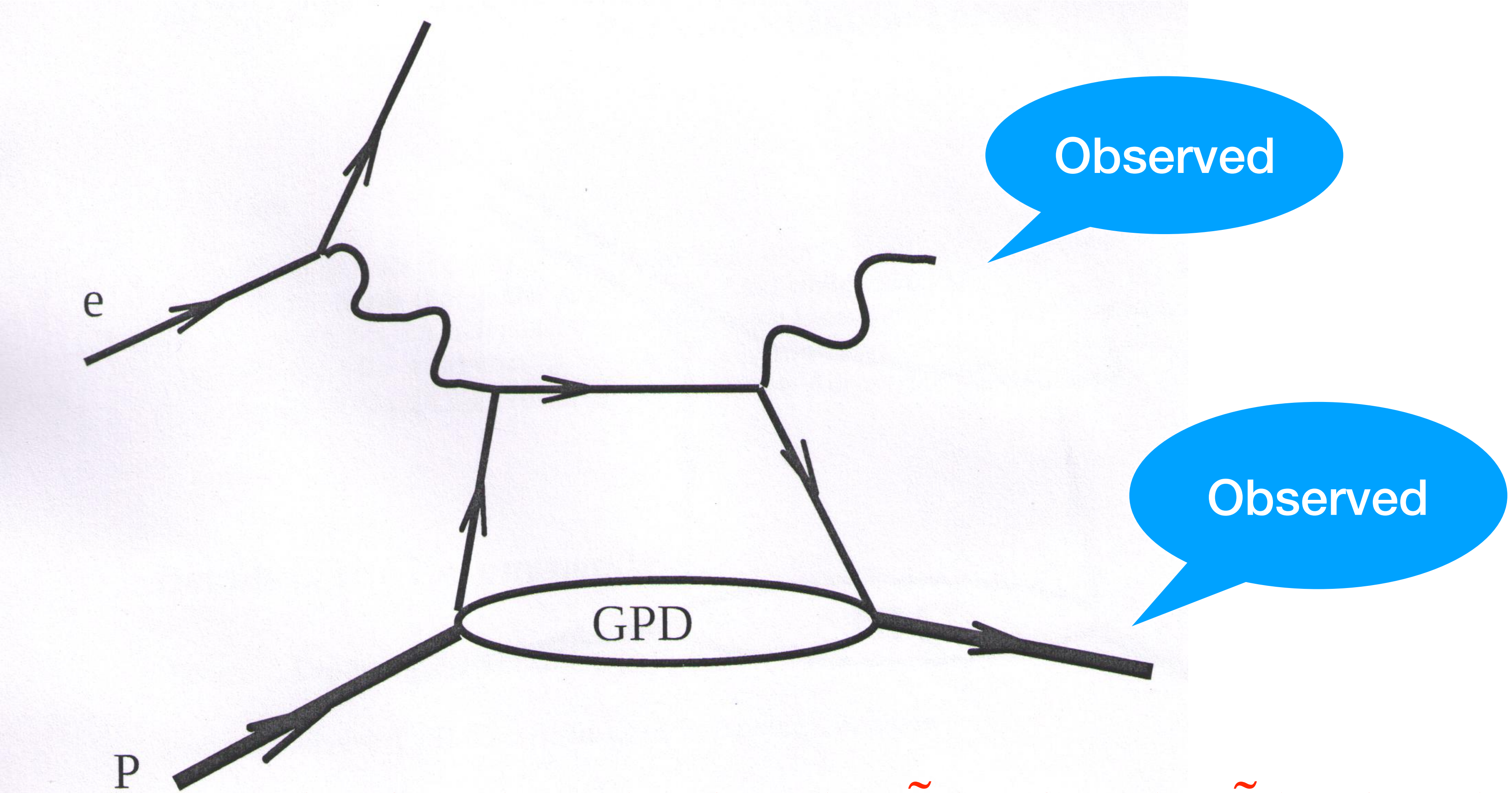
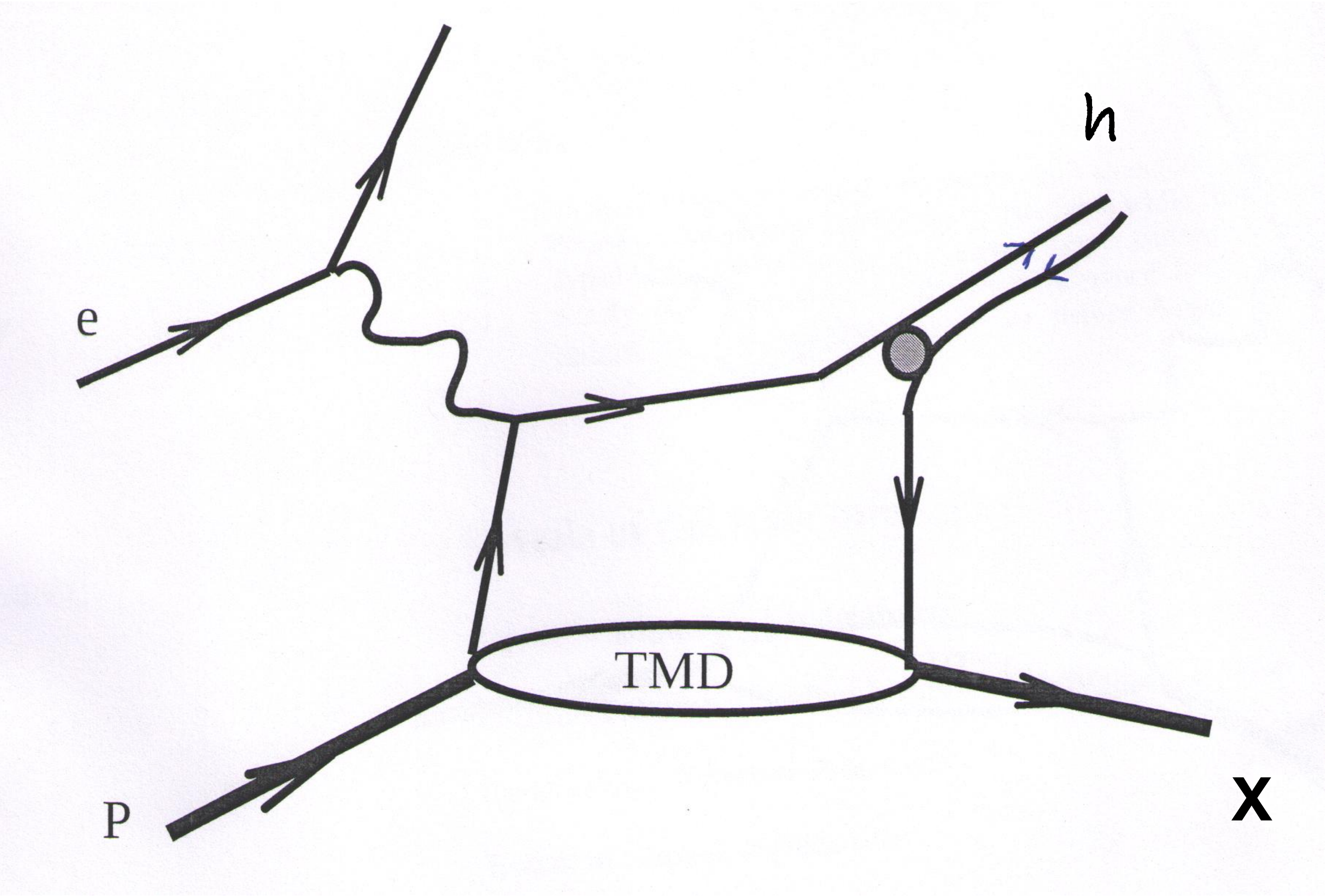


- $H(x, \Delta_\perp, t),\ E(x, \Delta_\perp, t),\ \tilde{H}(x, \Delta_\perp, t),\ \tilde{E}(x, \Delta_\perp, t)$

Final particles of a particular process are observed

- Semi Inclusive Process: Semi-Inclusive DIS

$$l(k) + P(p) \to l(k') + h(p_h) + X$$



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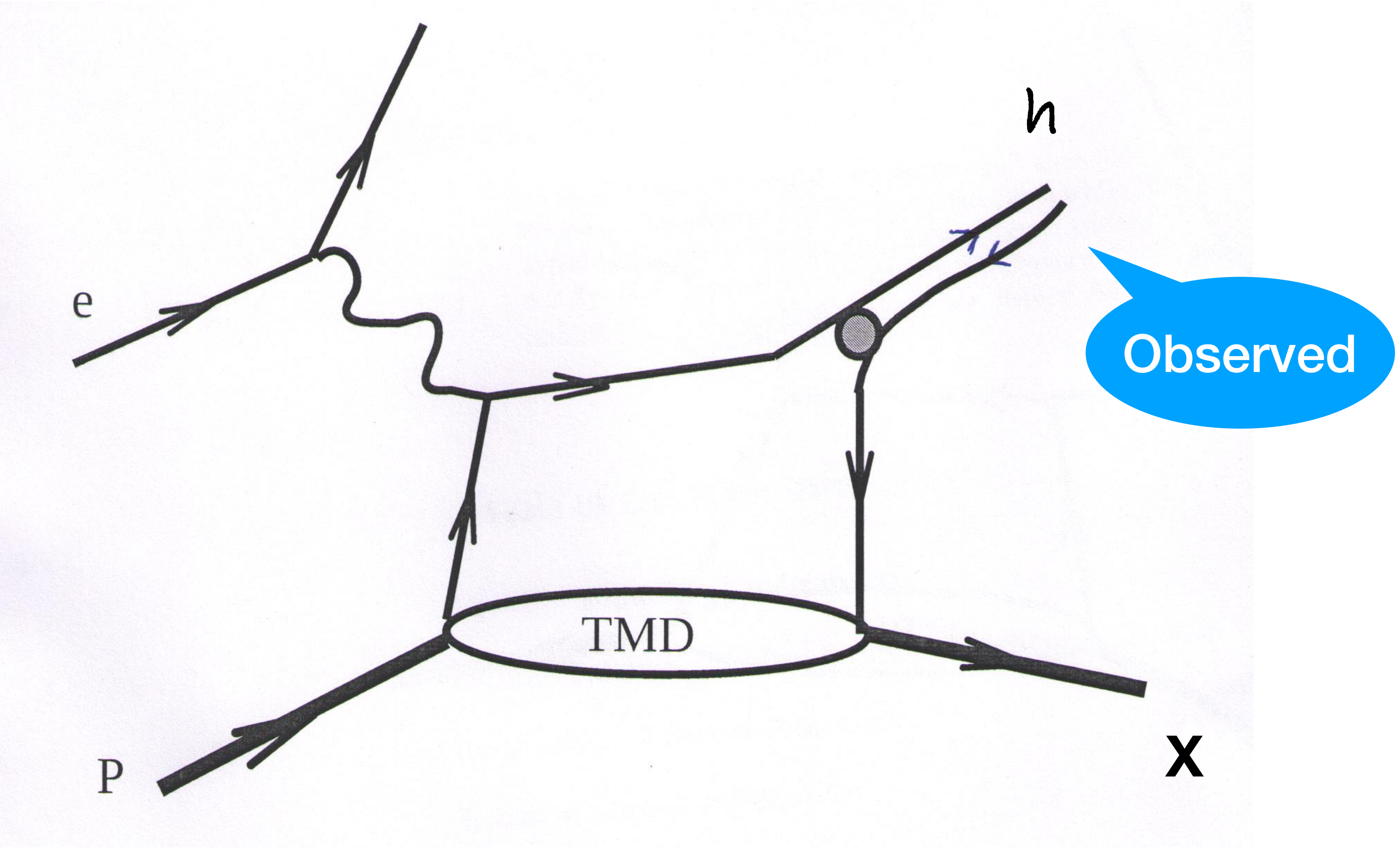
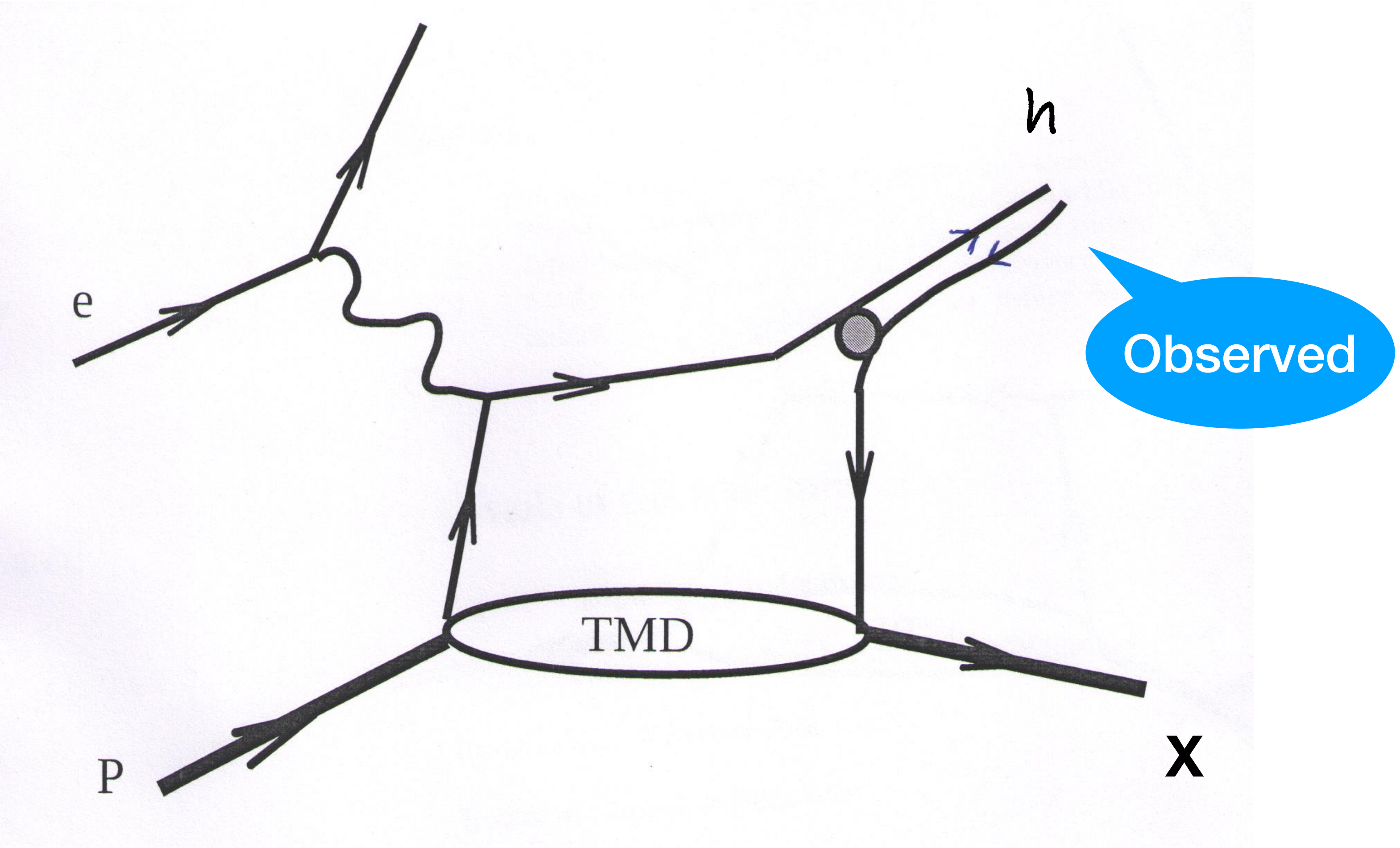


- **Only a particular particle in the final state is observed**

- Semi Inclusive Process: Semi-Inclusive DIS

$$l(k) + P(p) \to l(k') + h(p_h) + X$$



$$f_1(x, p_\perp^2),\ g_1(x, p_\perp^2),\ h_1(x, p_\perp^2), ..$$

- **Only a particular particle in the final state is observed**

Introduction

- The internal structure of proton is very complex and is not yet understood.

- To explain different experimental data, we require to understand not only 3D structure but also spin-orbital angular momentum distributions among the quark and gluons inside the proton.

- Solving nonperturbative QCD to understand the proton structure: horrendously complicated!

- To get knowledge about proton structure, people use models.

- Nonperturbative informations are encoded in GPDs, TMDs, Wigner distributions etc.

- GPDs and TMDs provide info on spatial structure of proton and spin-OAM distributions in the partonic level.

- Wigner distributions are the so-called mother distributions giving the most explicit information about the position and momentum space distributions of quarks and gluons, as well as spin-spin and spin-intrinsic transverse momentum correlations.

- On integrating the Wigner distributions over the transverse momentum one gets the impact-parameter dependent parton distribution functions (IPDPDFs) that are related to the generalized parton distributions (GPDs)

- on integrating over the transverse position one gets the transverse momentum dependent parton distributions (TMDs).

- Many experiments, e.g., COMPASS at CERN, HERMES at DESY, RHIC at BNL are providing important data towards the extraction of the GPDs and TMDs.

- Future planned experiments, like the EIC and AFTER@LHC, will provide results over wider and complementary kinematical regions.

- So, there are lot of activities in recent times to get a tomographic picture of the nucleon in terms of quarks and gluons, in three dimensional momentum space as well as in three dimensional position space.

- I'll present some results in a simplified light-front quark-diquark model of proton.

Our model

T. Maji, DC, PRD94, 094020 (2016)

- We proposed a light front quark-diquark model considering both scalar and axial-vector diquarks:

$$|p;\pm\rangle = C_s^2\,|u,S^0\rangle^{\pm} + C_V^2|u,A^0\rangle^{\pm} + C_{VV}^2|d,A^1\rangle^{\pm}$$

- S and A represent scalar and axial vector diquark with isospin at their superscript

- Two particle Fock state expansion for $J^z = \pm 1/2$ with spin-0 diquark:

$$|u,S\rangle^{\pm} = \int \frac{dx d^2p_\perp}{2(2\pi)^3\sqrt{x(1-x)}}\left[\psi_+^{\pm u}(x,p_\perp)|+\frac{1}{2},0;xP^+,p_\perp\rangle + \psi_-^{\pm u}(x,p_\perp)|-\frac{1}{2},0;xP^+,p_\perp\rangle\right]$$

- Where $|\lambda_q,\lambda_D;xP^+,p_\perp\rangle =$ two particle state with quark of helicity λ_q and diquark with helicity λ_D

$$\psi_{+}^{+u}(x,p_{\perp}) = N_s\phi_1^u(x,p_{\perp})$$

$$\psi_{-}^{+u}(x,p_{\perp}) = N_S(-\frac{p^1+ip^2}{xM})\phi_2^u(x,p_{\perp})$$

$$\psi_{+}^{-u}(x,p_{\perp}) = N_S(\frac{p^1-ip^2}{xM})\phi_2^u(x,p_{\perp})$$

$$\psi_{-}^{-u}(x,p_{\perp}) = N_S\phi_1^u(x,p_{\perp})$$

- Similarly the two particle Fock-state expansion for axial-vector diquark is given as:

$$|\nu\ A\rangle^{\pm} = \int \frac{dx\ d^2p_{\perp}}{2(2\pi)^3\sqrt{x(1-x)}}\Big[\psi_{++}^{\pm(\nu)}(x,p_{\perp})\,|+\frac{1}{2}\ \ +1;xP^+,p_{\perp}\rangle$$
$$+\psi_{-+}^{\pm(\nu)}(x,p_{\perp})\,|-\frac{1}{2}\ \ +1;xP^+,p_{\perp}\rangle+\psi_{+0}^{\pm(\nu)}(x,p_{\perp})\,|+\frac{1}{2}\ 0;xP^+,p_{\perp}\rangle$$
$$+\psi_{-0}^{\pm(\nu)}(x,p_{\perp})\,|-\frac{1}{2}\ 0;xP^+,p_{\perp}\rangle+\psi_{+-}^{\pm(\nu)}(x,p_{\perp})\,|+\frac{1}{2}\ \ -1;xP^+,p_{\perp}\rangle$$
$$+\psi_{--}^{\pm(\nu)}(x,p_{\perp})\,|-\frac{1}{2}\ \ -1;xP^+,p_{\perp}\rangle\Big]$$

- We adopt a generic ansatz of LFWF from the soft-wall AdS/QCD prediction:

$$\varphi_i^{(\nu)}(x, p_\perp) = \frac{4\pi}{\kappa}\sqrt{\frac{\log(1/x)}{1-x}}\, x^{a_i^\nu}(1-x)^{b_i^\nu} \exp\left[-\delta^\nu \frac{p_\perp^2}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right].$$

- The wave function reduces to the AdS/QCD prediction for the parameters $a_i^\nu = b_i^\nu = 0$ and $\delta^\nu = 1.0$

[Brodsky, Teramond, PRD 77, 056007(2008)]

- We use the AdS/QCD parameter $\kappa = 0.4\ GeV$

[DC, C. Mondal, PRD 88, 073006(2013)]

- The parameters in the model are fitted to nucleon and flavour form factors data at initial scale $\mu_0 = 0.8 \ GeV$

- In this model the parameters are assumed to be scale dependent

- The model reproduces PDFs to very large scale

$$\mu^2 = 1000 \ GeV^2$$

- Predicts axial and tensor charges with excellent agreement with data.

TMDs

- Collinear picture of DIS cannot explain the single or double spin asymmetries observed in SIDIS or Drell-Yan processes.

- Spin asymmetries require non-vanishing transverse momentum of the partons.

- So, Transverse Momentum dependent pdfs (or TMDs) are required.

- At leading twist there are 8 TMDs.

- Three of them reduces to three PDFs in collinear limit

Quark-quark correlator for SIDIS process:

$$\Phi^{\nu[\Gamma]}(x, p_\perp; S) = \frac{1}{2} \int \frac{dz^- d^2 z_T}{2(2\pi)^3} e^{ip.z} \langle P; S \,|\, \overline{\psi}^\nu(0) \Gamma \mathscr{W}_{[0,z]} \psi^\nu(z) \,|\, P; S \rangle \Bigg|_{z^+=0}$$

Different TMDs for different Γ **structure :**

$$\Phi^{\nu[\gamma^+]}(x, p_\perp; S) = f_1^\nu(x, p_\perp^2) - \frac{\epsilon_T^{ij} p_\perp^i S_T^j}{M} f_{1T}^{\perp\nu}(x, p_\perp^2)$$

$$\Phi^{\nu[\gamma^+\gamma^5]}(x, p_\perp; S) = \lambda g_{1L}^\nu(x, p_\perp^2) + \frac{p_\perp . S_T}{M} g_{1T}^\nu(x, p_\perp^2)$$

$$\Phi^{\nu[i\sigma^{j+}\gamma^5]}(x, p_\perp; S) = S_T^j h_1^\nu(x, p_\perp^2) + \lambda \frac{p_\perp^j}{M} h_{1L}^{\perp\nu}(x, p_\perp^2)$$

$$+ \frac{2p_\perp^j p_\perp . S_T - S_T^j p_\perp^2}{2M^2} h_{1T}^{\perp\nu}(x, p_\perp^2) - \frac{\epsilon_T^{ij} p_\perp^i}{M} h_1^{\perp\nu}(x, p_\perp^2)$$

λ **Is the nucleon helicity**

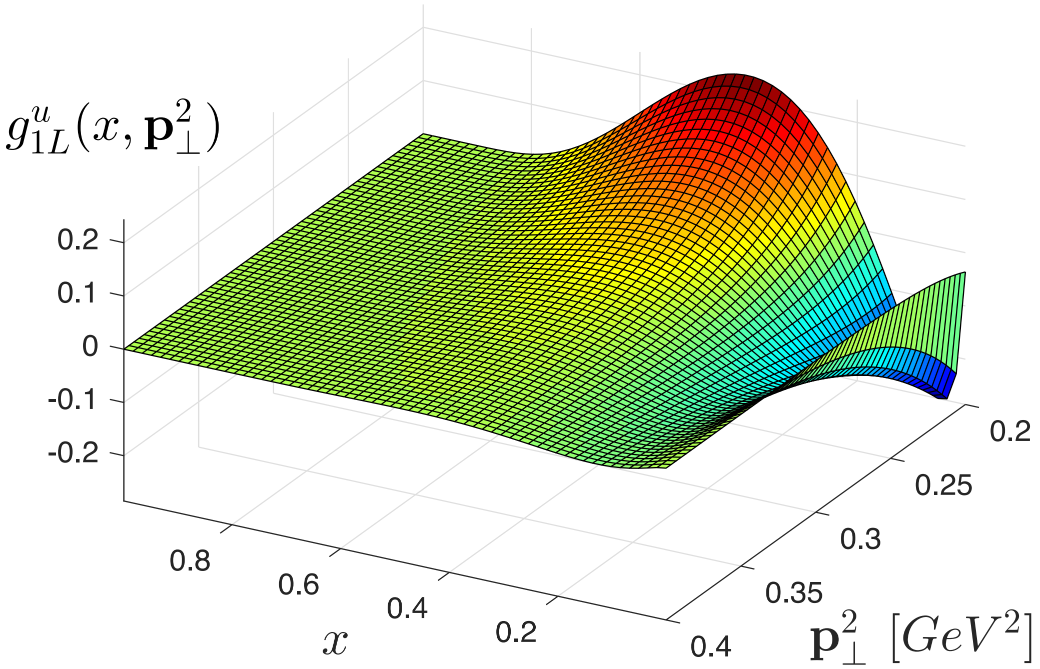
Quark polarization

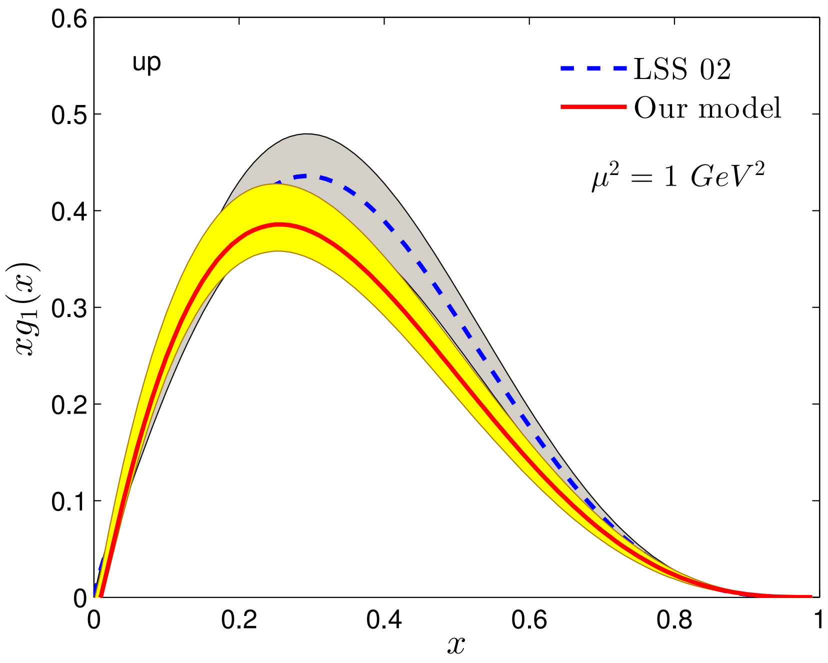
Proton Polarization

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1 \mid h_{1T}^\perp$

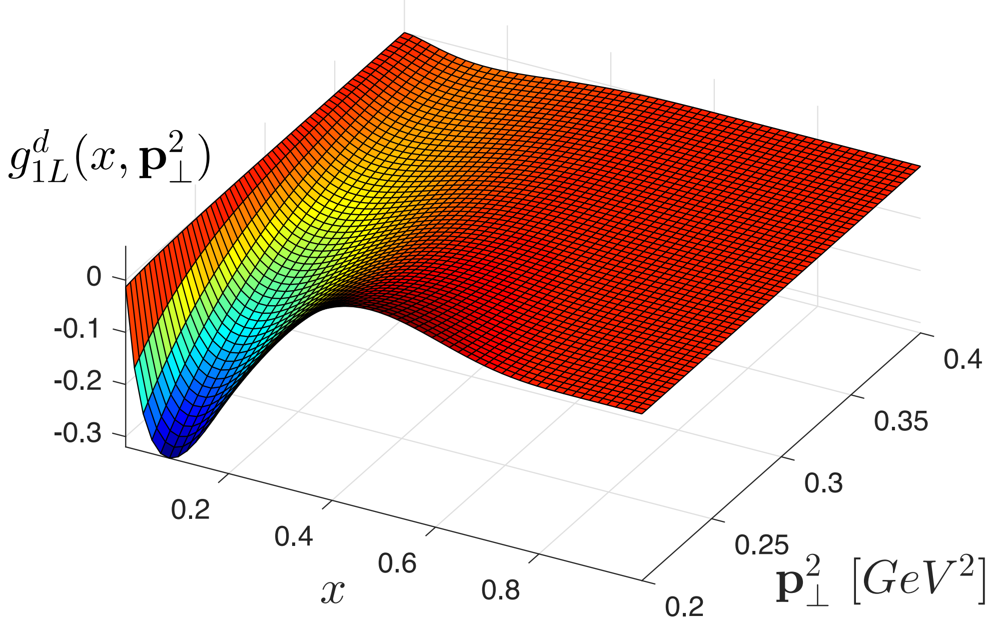
TMDs

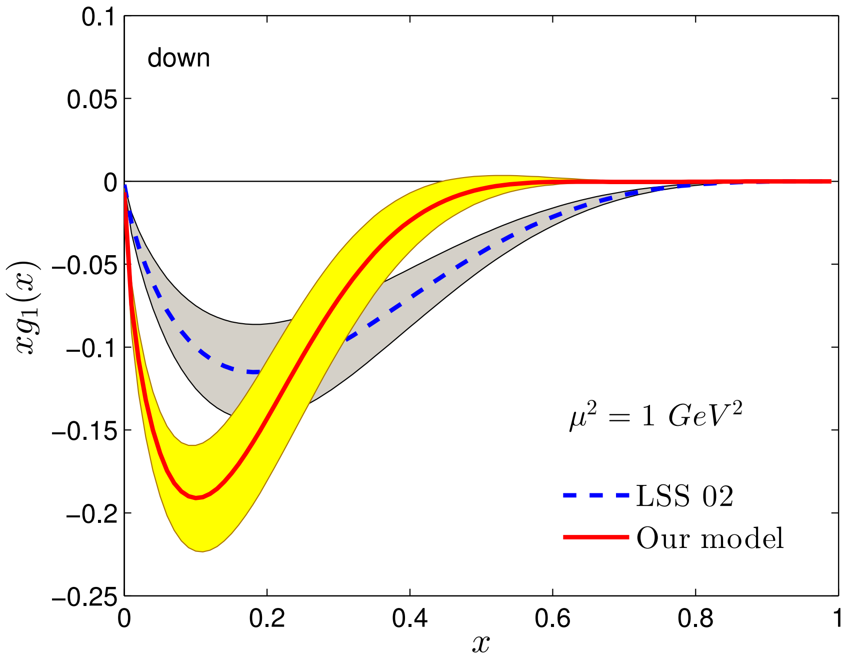
u-quark





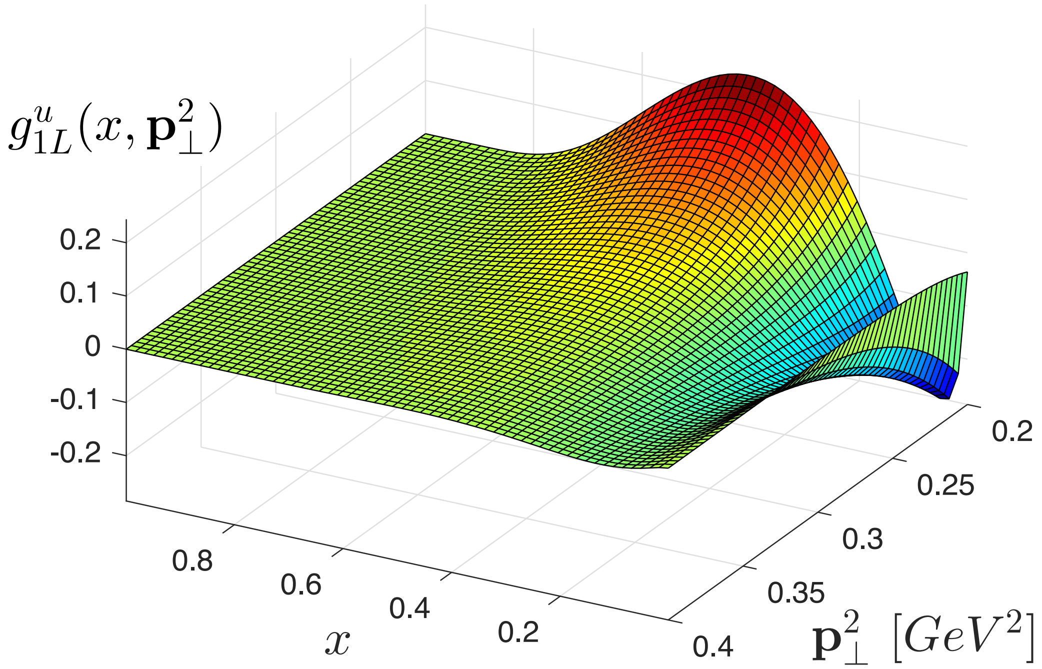
d-quark



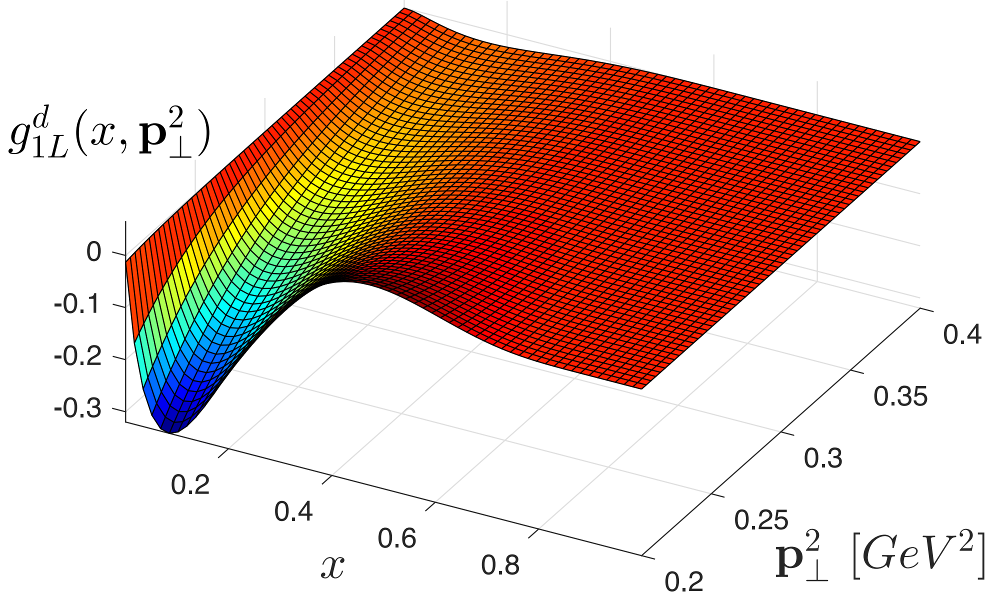


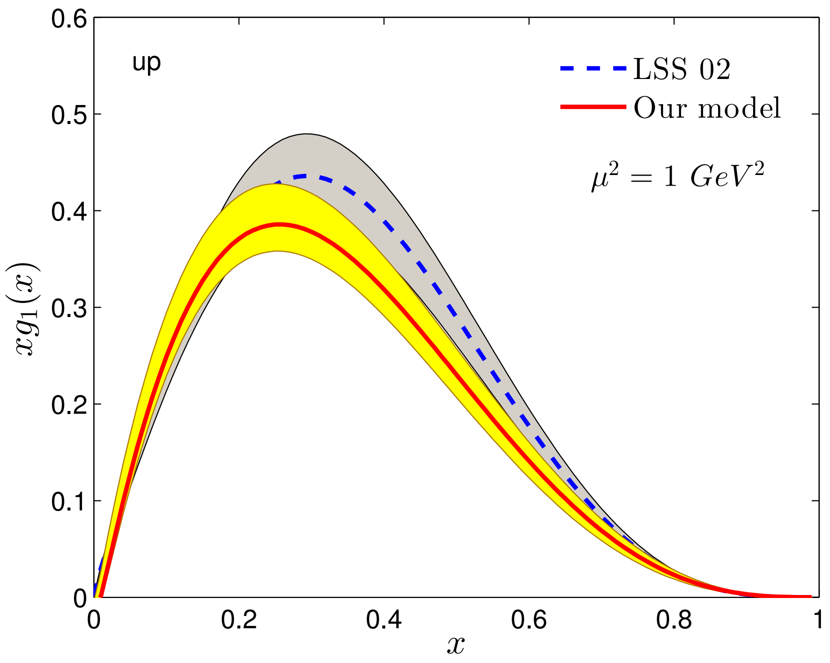
TMDs

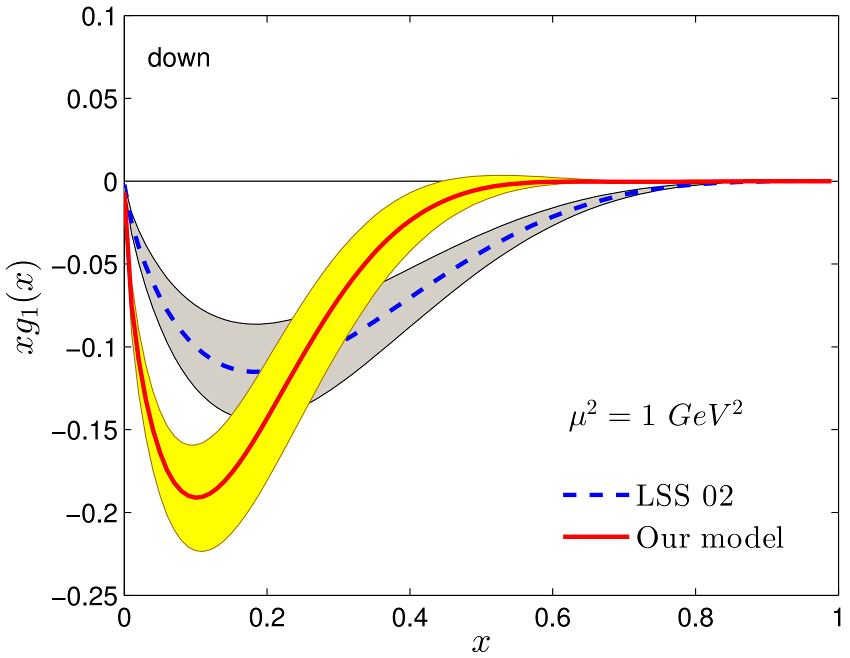
u-quark



d-quark





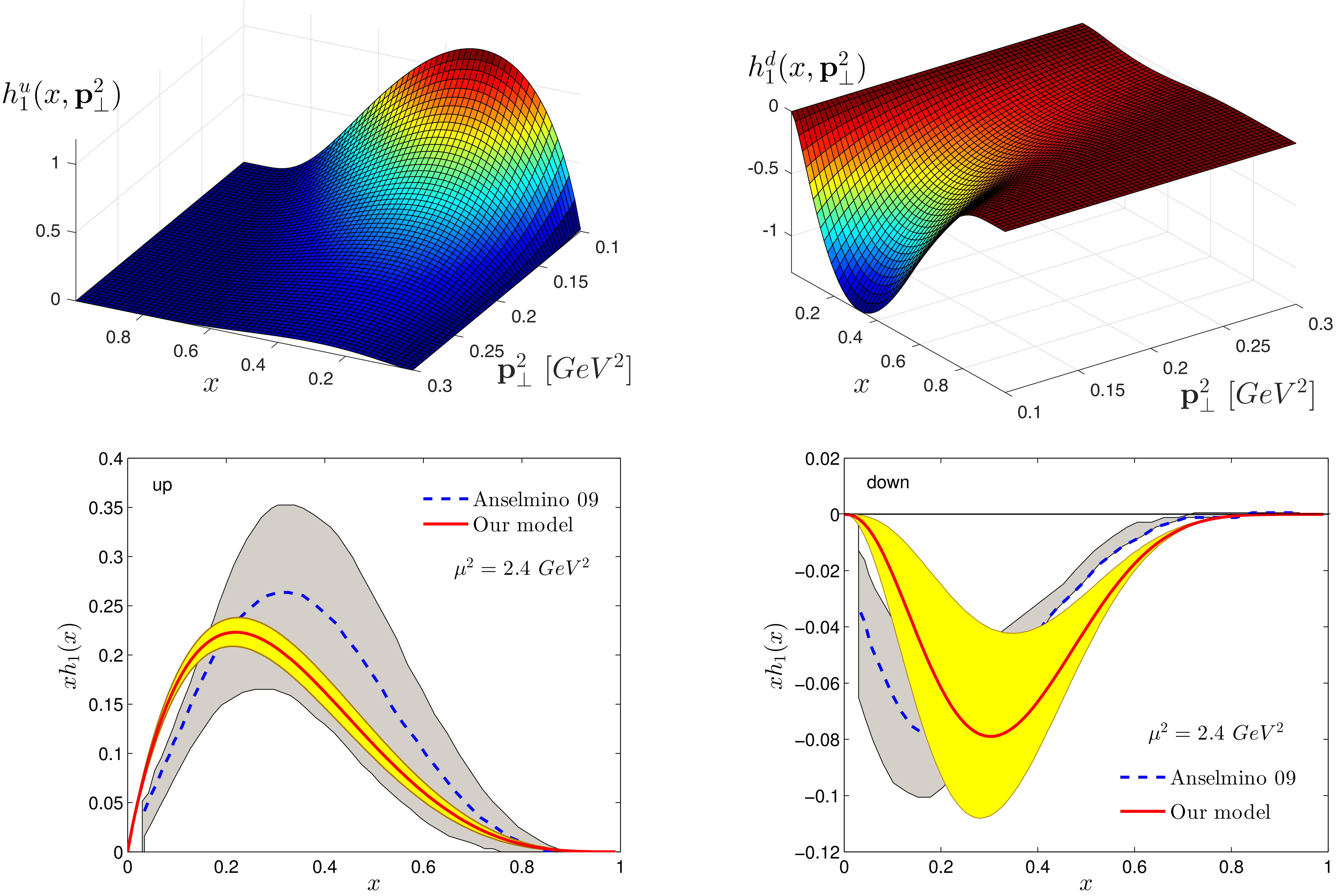


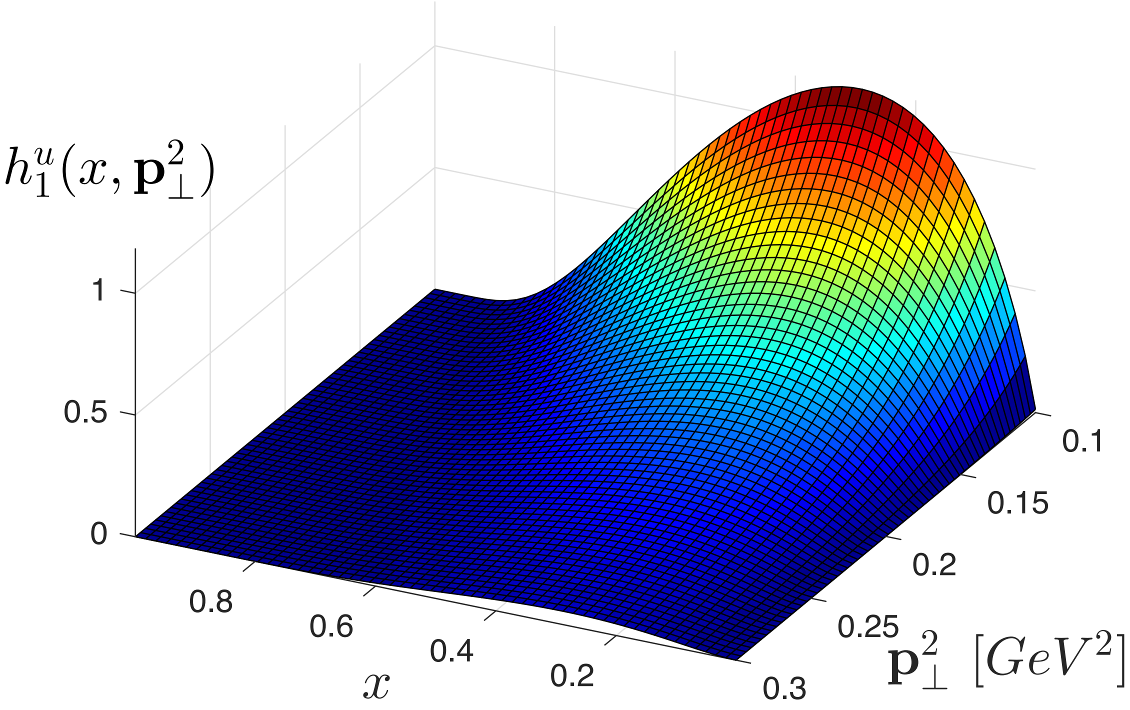
Proton axial charge: $$g_A = \int dx x g_1(x)$$

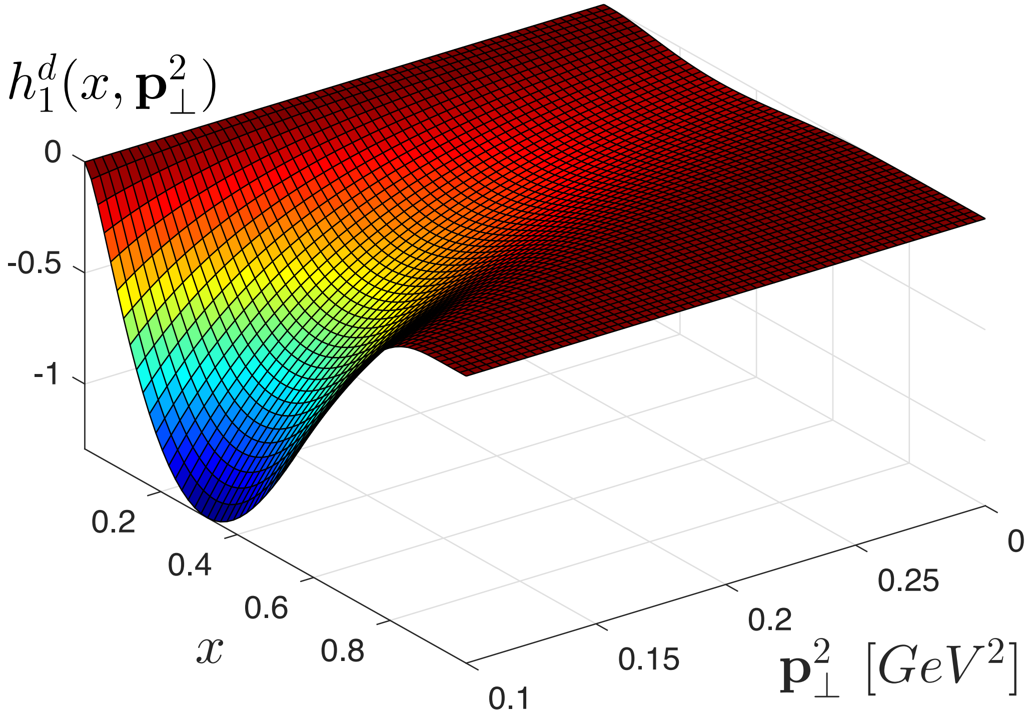
Our result: $g_A = 1.25^{+0.28}_{-0.22}$

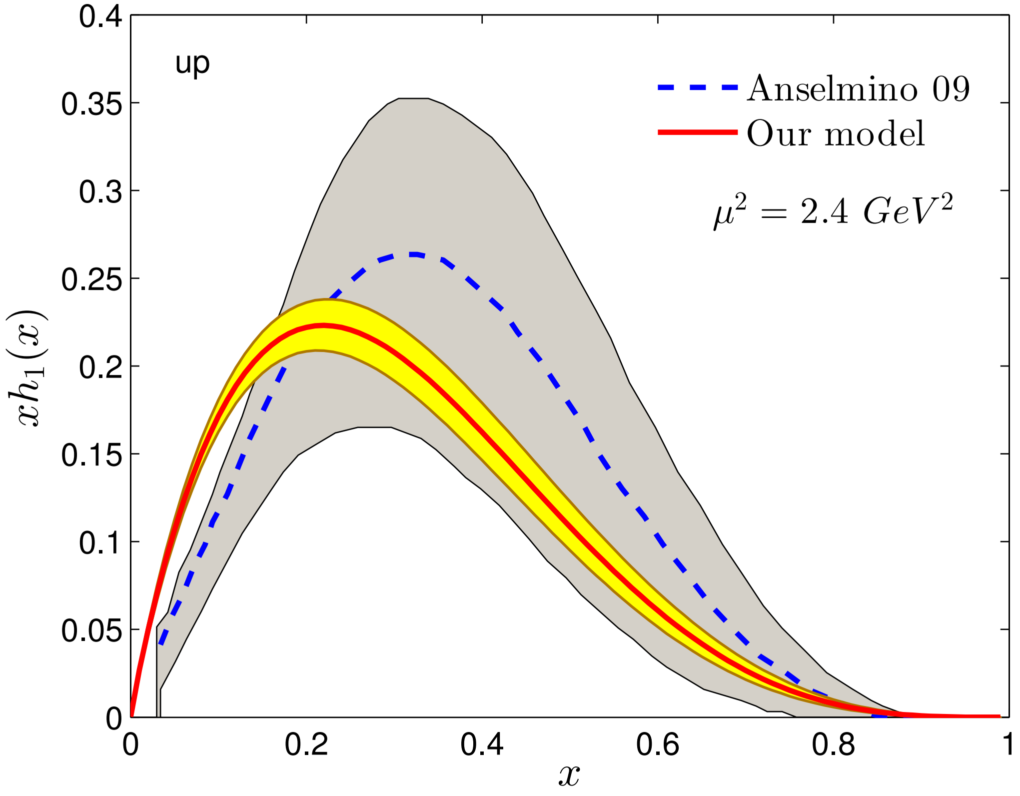
Measured value:

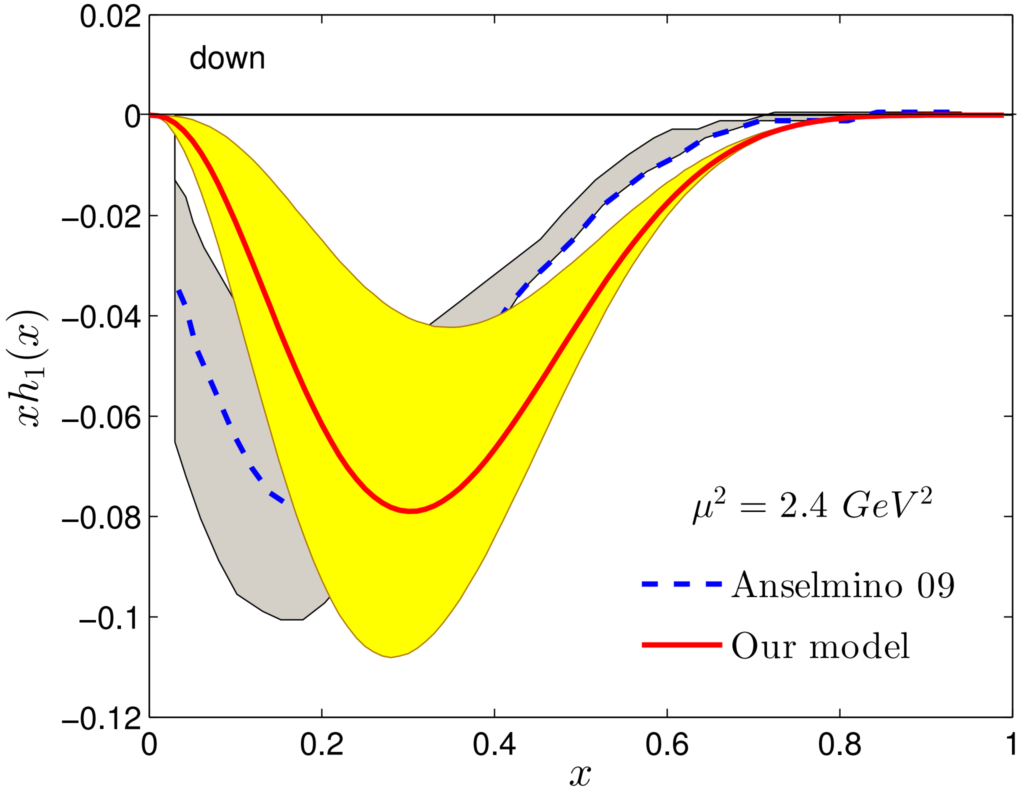
$g_A = 1.27 \pm 0.14$


$h_1^u(x,\mathbf{p}_\perp^2)$
1
0.5
0
0.8
0.6
0.4
0.2
x
0.1
0.15
0.2
0.25
0.3
$\mathbf{p}_\perp^2\ [GeV^2]$
$h_1^d(x,\mathbf{p}_\perp^2)$
0
-0.5
-1
0.2
0.4
0.6
0.8
x
0.1
0.15
0.2
0.25
0.3
$\mathbf{p}_\perp^2\ [GeV^2]$
up
Anselmino 09
Our model
$\mu^2 = 2.4\ GeV^2$
$xh_1(x)$
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1
x
down
0.02
0
−0.02
−0.04
−0.06
−0.08
−0.1
−0.12
$xh_1(x)$
$\mu^2 = 2.4\ GeV^2$
Anselmino 09
Our model
0
0.2
0.4
0.6
0.8
1
x









Tensor charge: $g_T = 0.51^{+0.12}_{-0.11}$

Measured value:

$g_T = 0.79^{+0.19}_{-0.20}$

TMD relations

[T. Maji, DC, PRD 95, 074009(2017)]

Soffer bound for TMDs:

$$|\, h_1^{\nu}(x, \mathbf{p}_\perp^2)\,| < \frac{1}{2}\left| f_1^{\nu}(x, \mathbf{p}_\perp^2) + g_{1L}^{\nu}(x, \mathbf{p}_\perp^2) \right|$$

Other inequalities:

$$|\, g_{1L}^{\nu}(x, \mathbf{p}_\perp^2)\,| < |\, f_1^{\nu}(x, \mathbf{p}_\perp^2)\,| \,.$$

$$\frac{\mathbf{p}_\perp}{2M^2}\,|\, h_{1T}^{\nu\perp}(x, \mathbf{p}_\perp^2)\,| < \frac{1}{2}\left| f_1^{\nu}(x, \mathbf{p}_\perp^2) - g_{1L}^{\nu}(x, \mathbf{p}_\perp^2) \right|$$

$$|\, f_1^{\nu}(x, \mathbf{p}_\perp^2)\,| > |\, h_1^{\nu}(x, \mathbf{p}_\perp^2)\,|$$

*** The relations are consistent with other models, e.g., Bag model, LCCQM, other quark-diquark models**

*** Relations are independent of the parameters in our model**

Quark densities

TMDs are related to quark densities inside a proton:

$$\rho^{\nu}_{UU}(\mathbf{p}_{\perp}) = f_1^{\nu(1)}(\mathbf{p}_{\perp}^2),$$

Here

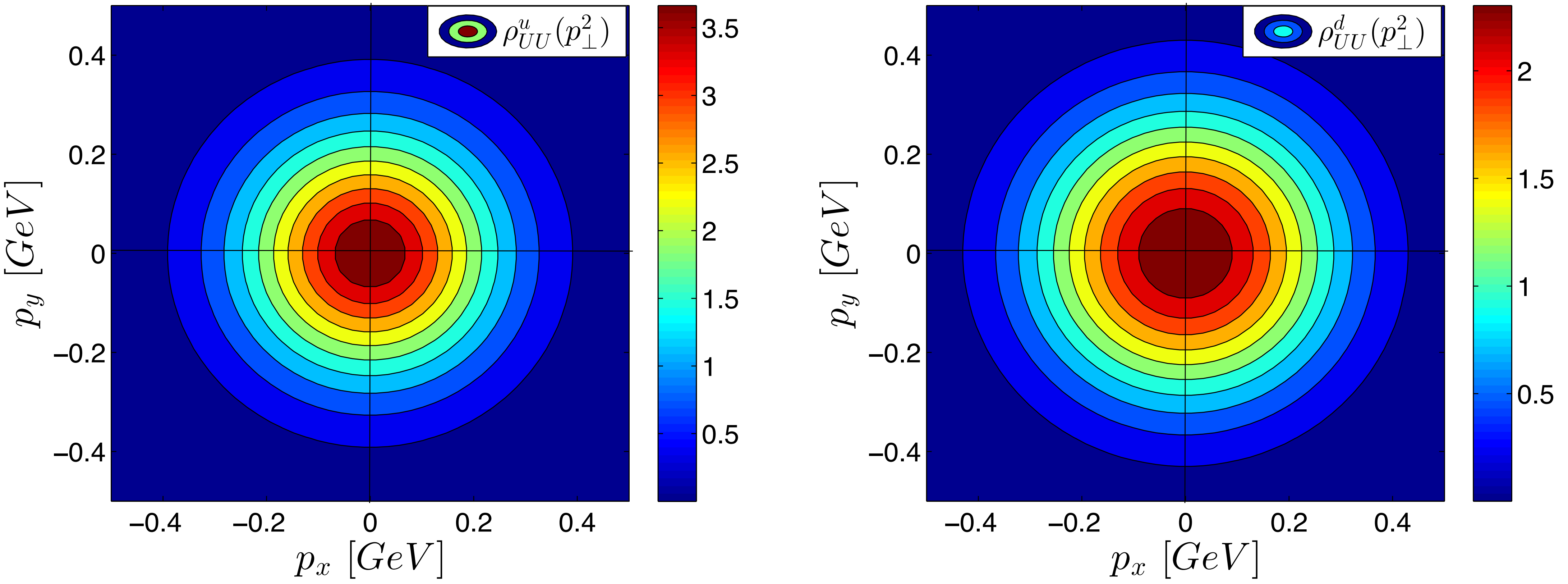
$$f_1^{\nu(1)}(\mathbf{p}_{\perp}^2) = \int dx f_1^{\nu}(x, \mathbf{p}^2)$$

$$\rho^{\nu}_{TL}(\mathbf{p}_{\perp}; \mathbf{S}_{\perp}, \lambda) = \frac{1}{2} f_1^{\nu(1)}(\mathbf{p}_{\perp}^2) + \frac{\lambda}{2} \frac{\mathbf{p}_{\perp} . \mathbf{S}_{\perp}}{M} g_{1T}^{\nu(1)}(\mathbf{p}_{\perp}^2)$$

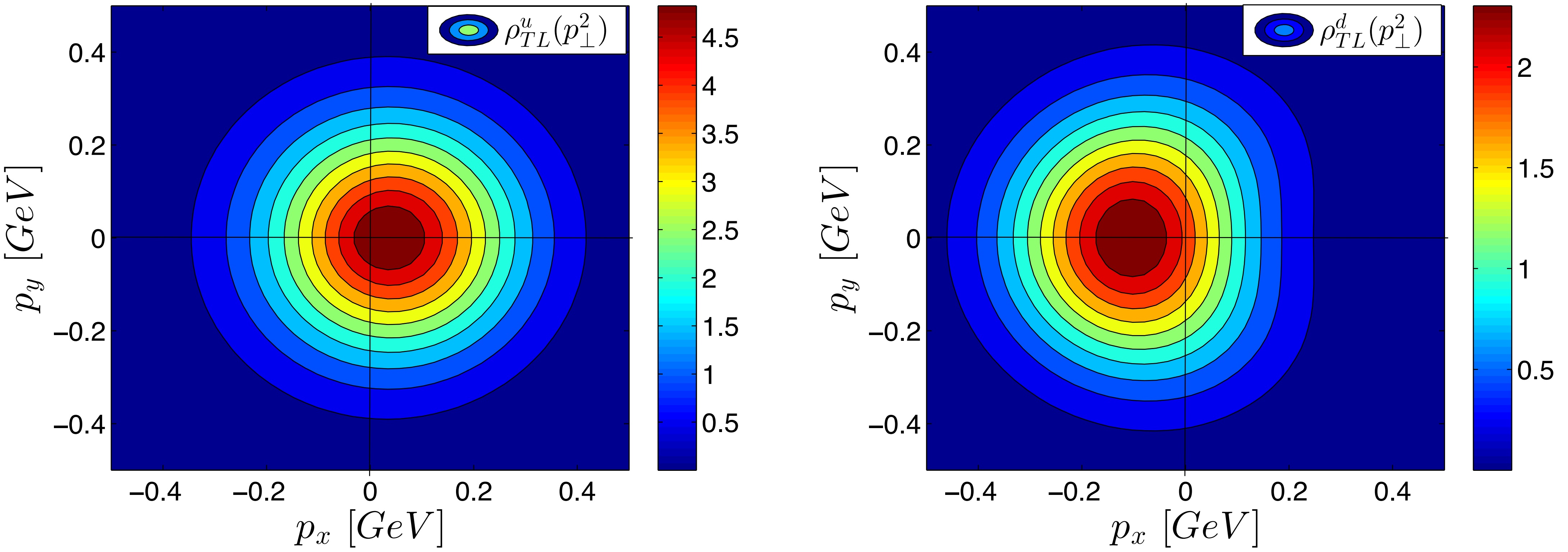
Proton polarization

Quark polarization

Unpolarised proton



Transversely polarised proton



Transverse shape of proton

Transverse shape of proton can be defined as

[G.A. Miller, PRC76, 065209 (2007)]

$$\frac{\hat{\rho}_{\text{REL}_T}(\mathbf{p},\mathbf{n})/M}{\tilde{f}_1(\mathbf{p}^2)} = 1 + \frac{\tilde{h}_1(\mathbf{p}^2)}{\tilde{f}_1(\mathbf{p}^2)}\cos\phi_n + \frac{\mathbf{p}^2}{2M^2}\cos(2\phi - \phi_n)\frac{\tilde{h}_{1T}^{\perp}(\mathbf{p}^2)}{\tilde{f}_1(\mathbf{p}^2)}$$

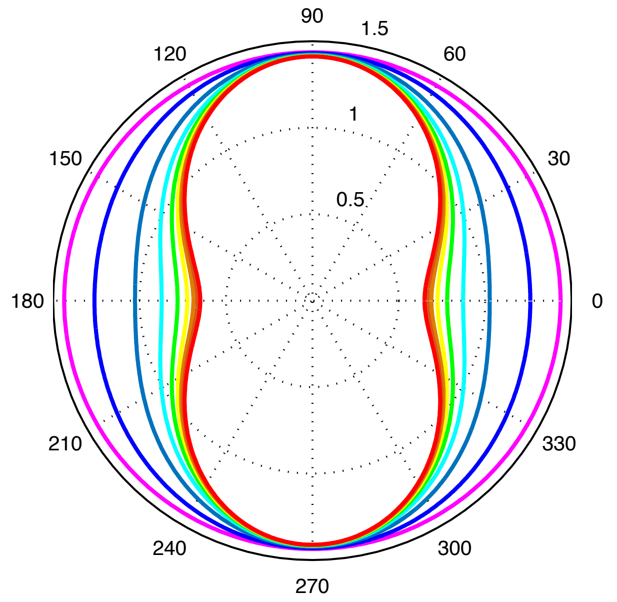
-

$\mathbf{n} =$ direction of the quark spin

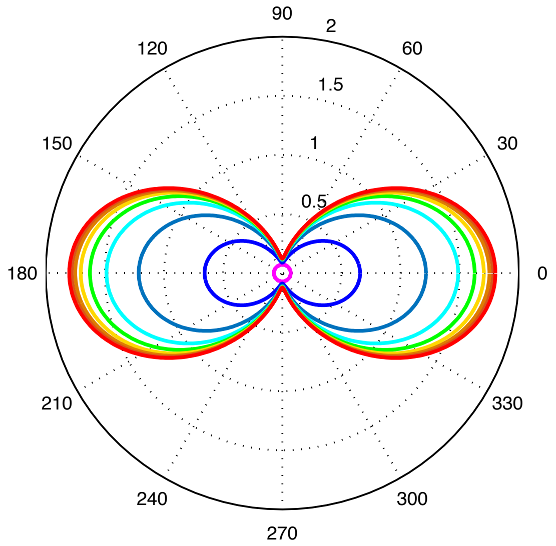
$\phi_n =$ angle between $\mathbf{n}$ and proton spin $\mathbf{S}_\perp$

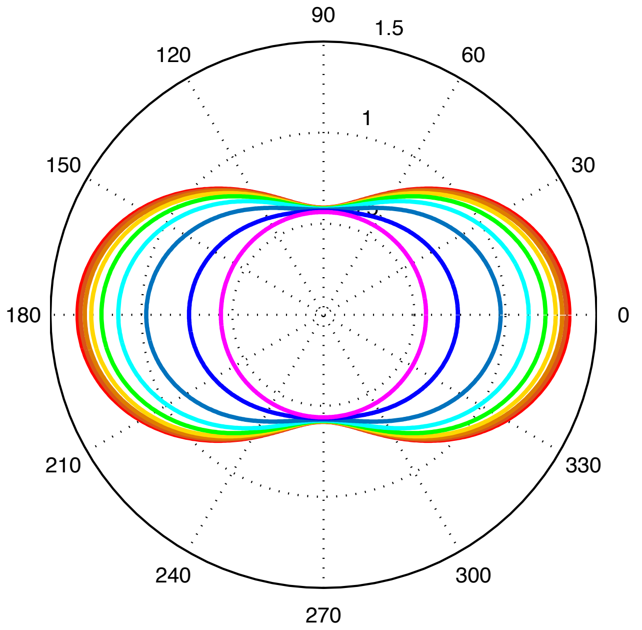
$\phi =$ angle between $\mathbf{p}$ and proton spin $\mathbf{S}_\perp$

u-quark **d-quark**

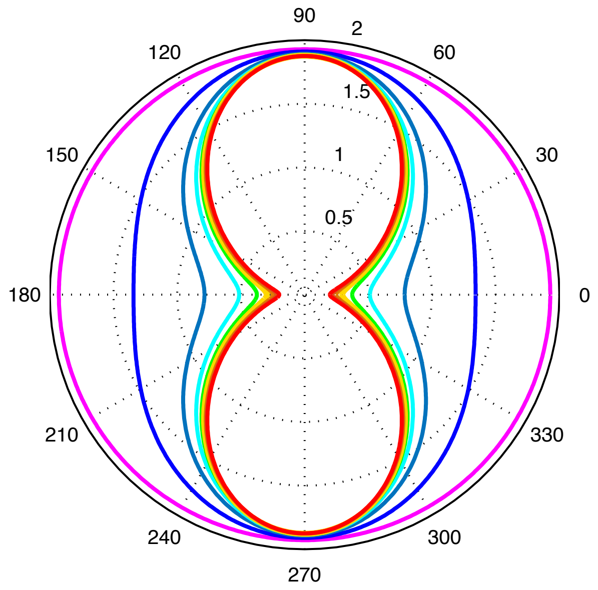


$\phi_n = 0$





$\phi_n = \pi$



color : red to blue $p_\perp = 0 \rightarrow 2\ GeV$

Spin asymmetries

- SIDIS shows asymmetries with target spins.
- SIDIS factorizes into TMDs and fragmentation functions.
- Spin asymmetries can be written as convolution of TMDs and fragmentation functions.
- 2 FFs for final unpolarised hadron: Chiral even $D_1(z, k_\perp^2)$

Fragmentation of an unpolarised quark

- Chiral odd FF: $H_1(z, k_\perp^2)$ (Collins function)

Fragmentation of a transversely polarised quark

- Chiral odd TMD $h_{1L}(x,p_\perp^2)$ couples with chiral odd $H_1(z,k_\perp^2)$ and give rise to the SSA measured with unpolarised lepton with longitudinally polarised proton.

$$A_{UL} \sim h_{1L}(x,p_\perp^2) \otimes H_1(z,k_\perp^2)$$

- Transversity TMD $h_1(x,p_\perp^2)$ produces the SSA with unpolarised lepton and transversely polarised proton:

$$A_{UT} \sim h_1(x,p_\perp^2) \otimes H_1(z,k_\perp^2)$$

- Chiral even TMD $g_{1T}(x,p_\perp^2)$ is accessed in double spin asymmetry:

$$A_{LT} \sim g_{1T}(x,p_\perp^2) \otimes D_1(z,k_\perp^2)$$

-

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-

Here we consider only π^+ and π^- channels

Collins asymmetry
comparison with HERMES data

- asymmetries are functions of $x, z, \mathbf{P}_{h\perp}, y$ and scale μ
  but exptl data are integrated asym for one variable at a time
- integrated asym are estimated by integrating over the variables in the corresponding kinematical limits

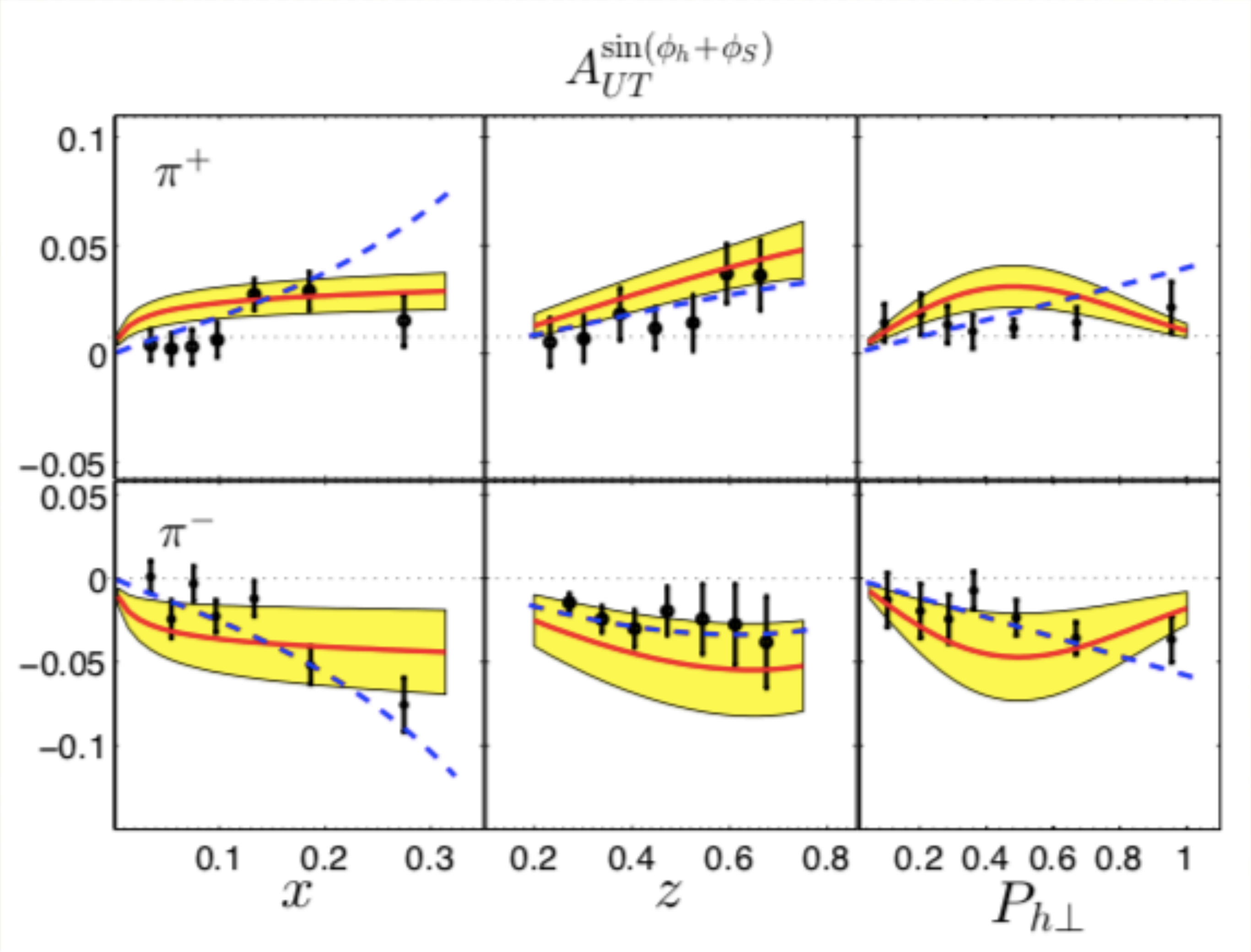
kinematical limits for HERMES

$0.023 \leq x \leq 0.4,$

$0.1 \leq y \leq 0.95$

$0.2 \leq z \leq 0.7$

red: QCD evolution
blue: parameter evol



h_1 at initial scale

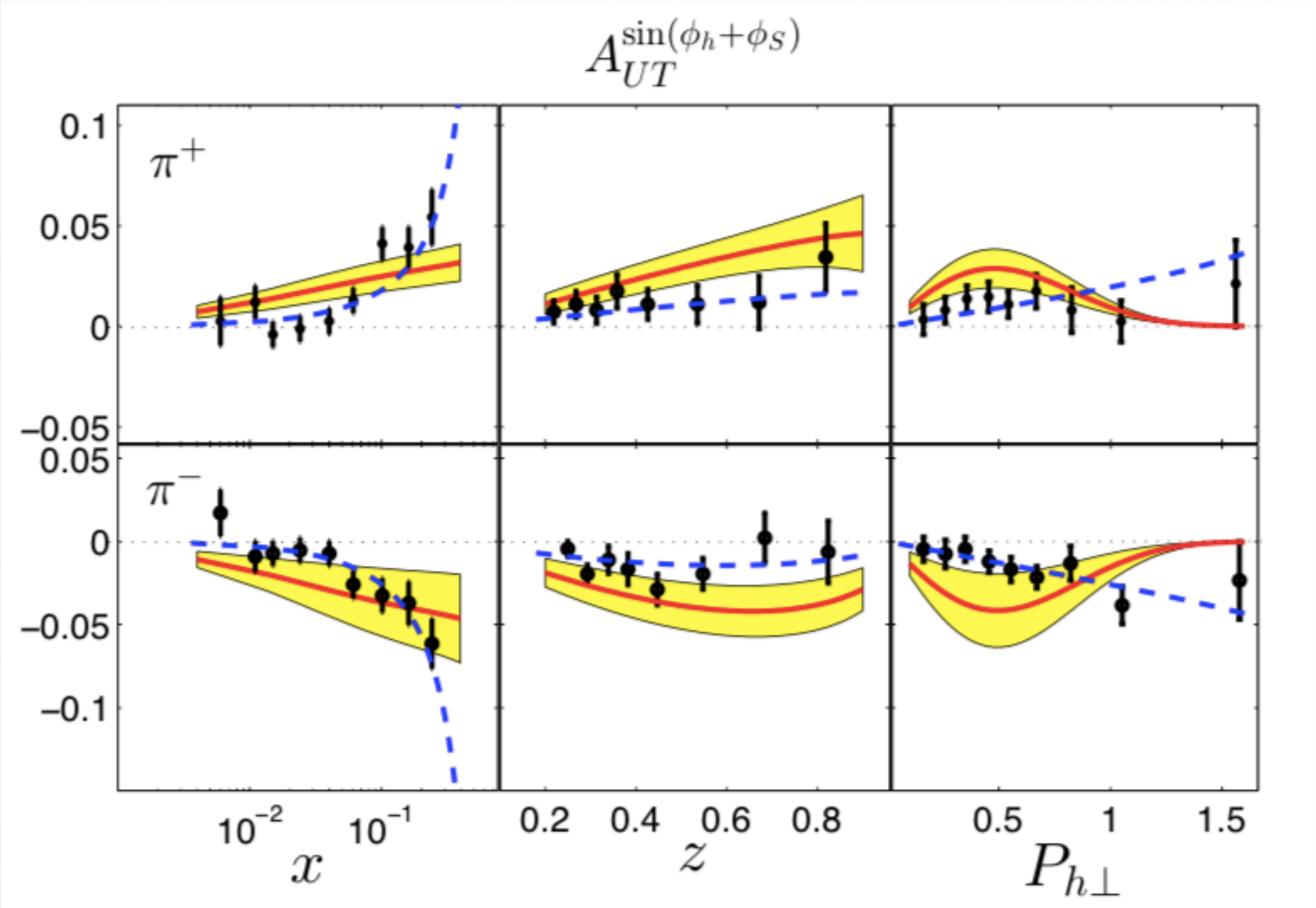
f_1^ν evolved to $\mu^2 = 2.5\ GeV^2$

comparison with COMPASS data

$0.003 \leq x \leq 0.7,$

$0.1 \leq y \leq 0.9$

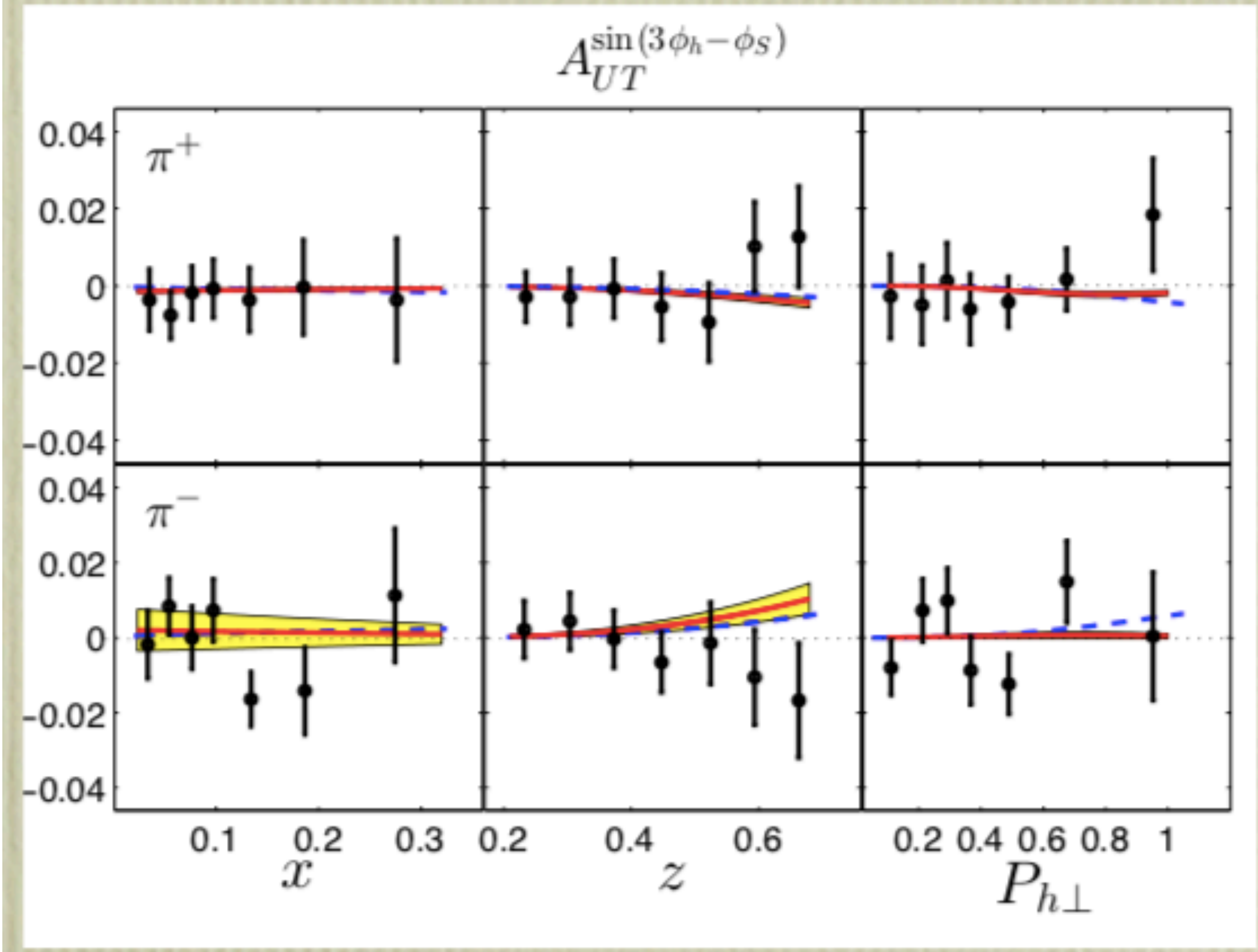
$0.2 \leq z \leq 1.0$

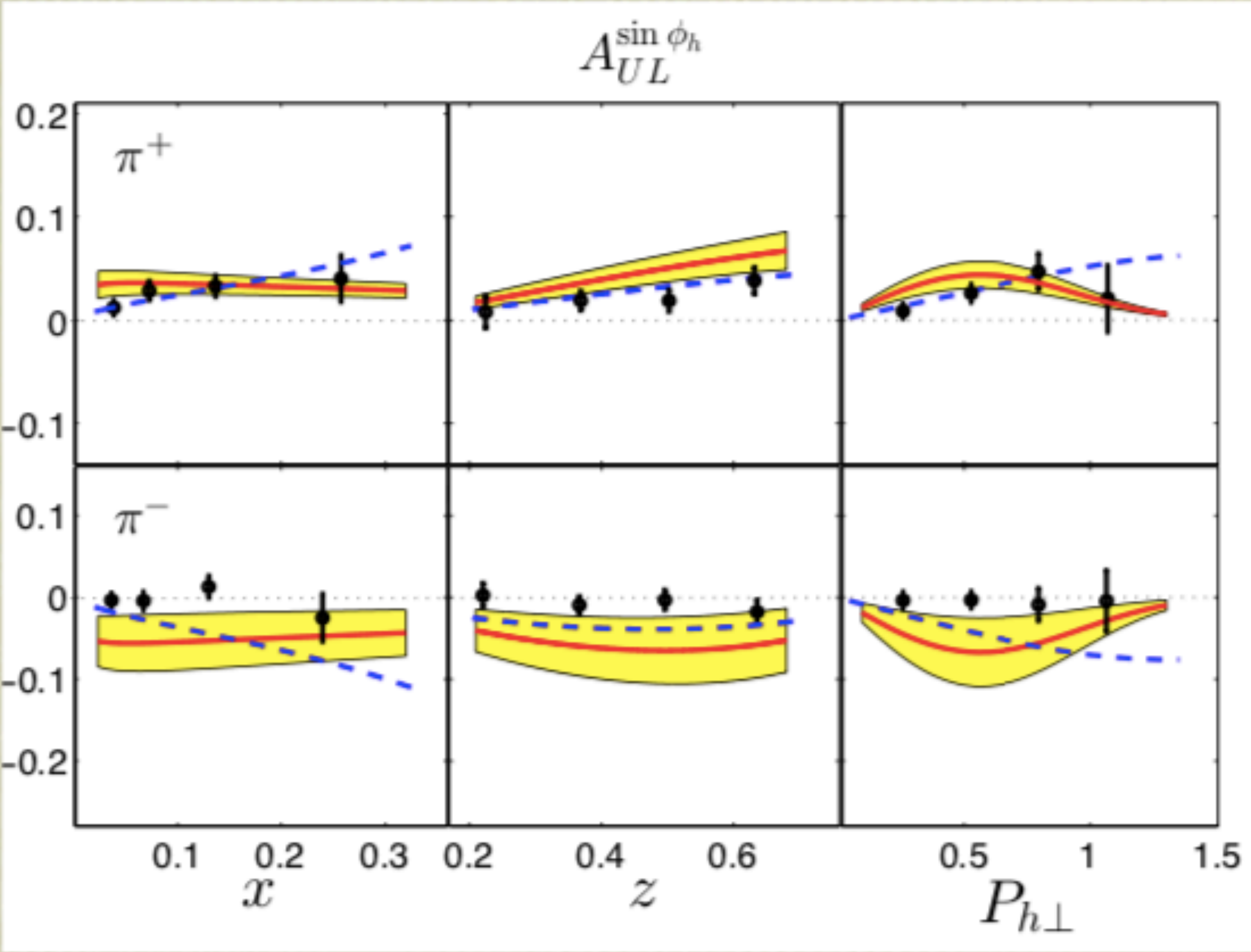


T. Maji, DC, O.V. Teryaev, PRD 96,114023

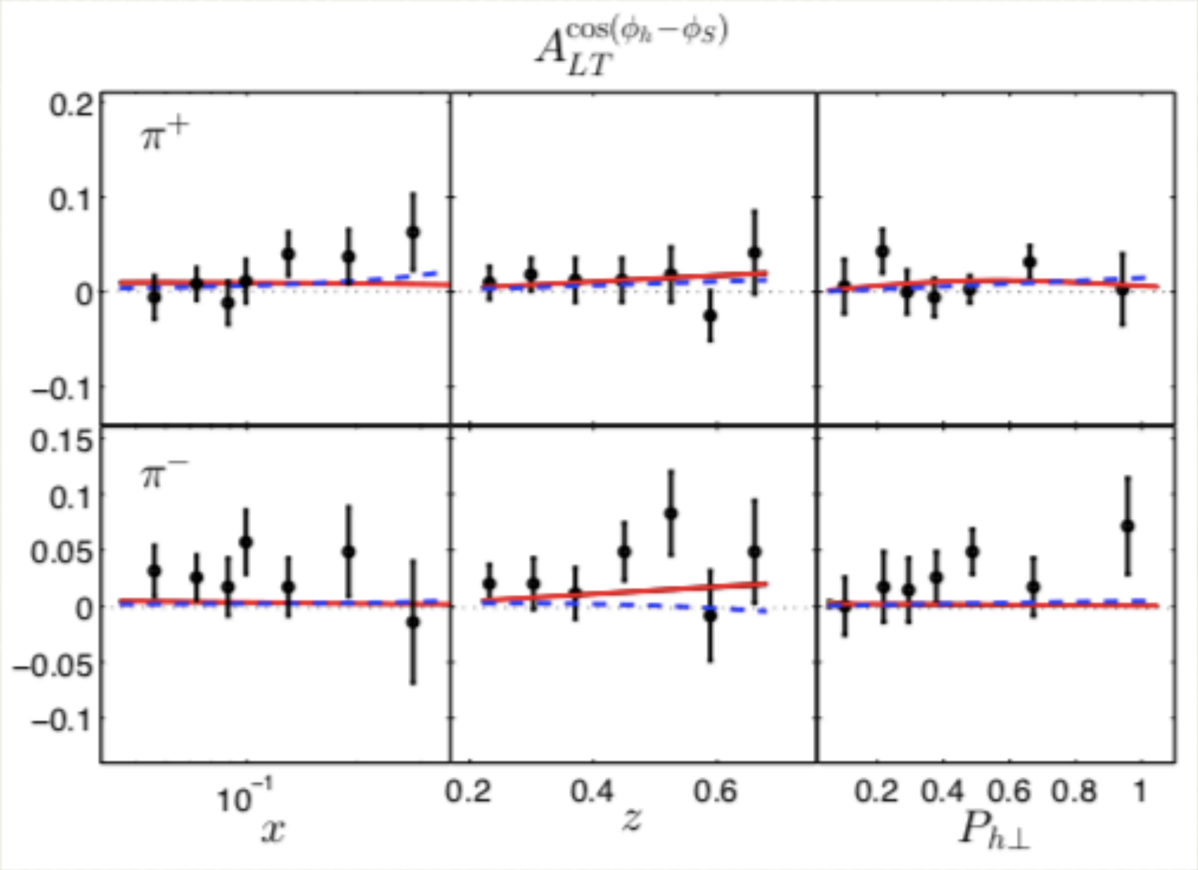
red: QCD evolution
blue: parameter evol

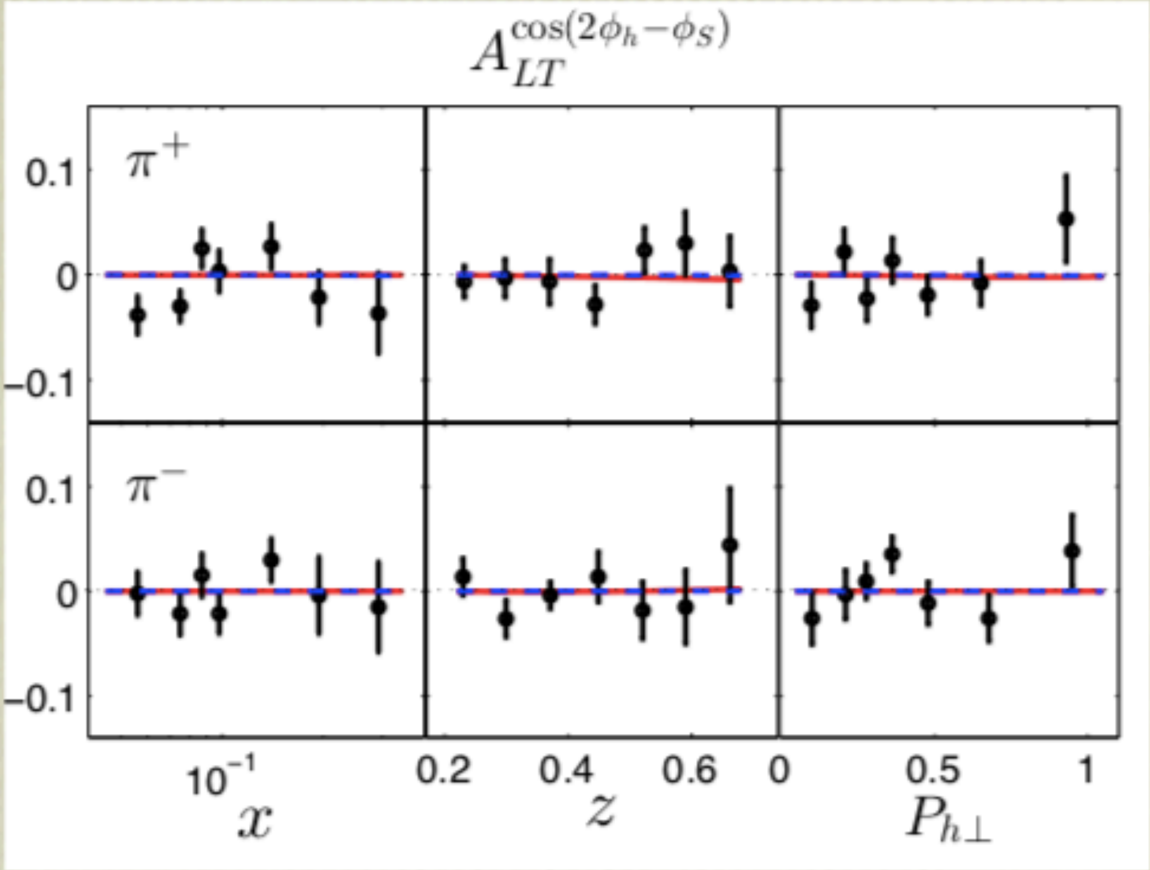
model predictions for other SSAs [HERMES data]





DSA: comparison with HERMES data:





Wigner Distributions

[X. Ji, PRL 91, 062001(2003)]

- In light-front framework, one defines the 5-dimensional quark Wigner distributions as

$$\rho^{\nu[\Gamma]}(\mathbf{b}_\perp, \mathbf{p}_\perp, x; S) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp . b_\perp} W^{\nu[\Gamma]}(\Delta_\perp, \mathbf{p}_\perp, x; S) \, .$$

- The correlator $W^{\nu[\Gamma]}$ relates the GTMDs in the Drell-Yan-West frame $(\Delta^+ = 0)$

$$W^{\nu[\Gamma]}(\Delta_\perp, \mathbf{p}_\perp, x; S) = \frac{1}{2} \int \frac{dz^-}{(2\pi)} \frac{d^2 z_T}{(2\pi)^2} e^{ip.z} \langle P''; S \,|\, \bar{\psi}_i^\nu(-z/2) \Gamma \mathscr{W}_{[-z/2, z/2]} \psi_j^\nu(z/2) \,|\, P'; S \rangle \Big|_{z^+=0}$$

The Wigner distributions for transversely polarised proton and different quark polarization:

$$\rho^{i\nu}_{TU}(\mathbf{b},\mathbf{p},x) = \frac{1}{2}[\rho^{\nu[\gamma^+]}(\mathbf{b},\mathbf{p},x;+\hat{S}_i) - \rho^{\nu[\gamma^+]}(\mathbf{b},\mathbf{p},x;-\hat{S}_i)]$$

$$\rho^{i\nu}_{TL}(\mathbf{b},\mathbf{p},x) = \frac{1}{2}[\rho^{\nu[\gamma^+\gamma^5]}(\mathbf{b},\mathbf{p},x;+\hat{S}_i) - \rho^{\nu[\gamma^+\gamma^5]}(\mathbf{b},\mathbf{p},x;-\hat{S}_i)]$$

$$\rho^{\nu}_{TT}(\mathbf{b},\mathbf{p},\mathbf{x}) = \frac{\mathbf{1}}{\mathbf{2}}\delta_{\mathbf{ij}}[\rho^{\nu[\mathbf{i}\sigma^{\mathbf{j}+}\gamma^{\mathbf{5}}]}(\mathbf{b},\mathbf{p},\mathbf{x};+\hat{\mathbf{S}}_{\mathbf{i}}) - \rho^{\nu[\mathbf{i}\sigma^{\mathbf{j}+}\gamma^{\mathbf{5}}]}(\mathbf{b},\mathbf{p},\mathbf{x};-\hat{\mathbf{S}}_{\mathbf{i}})]$$

Pretzelocity distribution

$$\rho^{\nu\perp}_{TT}(\mathbf{b},\mathbf{p},\mathbf{x}) = \frac{\mathbf{1}}{\mathbf{2}}\epsilon_{\mathbf{ij}}[\rho^{\nu[\mathbf{i}\sigma^{\mathbf{j}+}\gamma^{\mathbf{5}}]}(\mathbf{b},\mathbf{p},\mathbf{x};+\hat{\mathbf{S}}_{\mathbf{i}}) - \rho^{\nu[\mathbf{i}\sigma^{\mathbf{j}+}\gamma^{\mathbf{5}}]}(\mathbf{b},\mathbf{p},\mathbf{x};-\hat{\mathbf{S}}_{\mathbf{i}})]$$

 # Similar expressions for unpolarised and longitudinally polarised proton

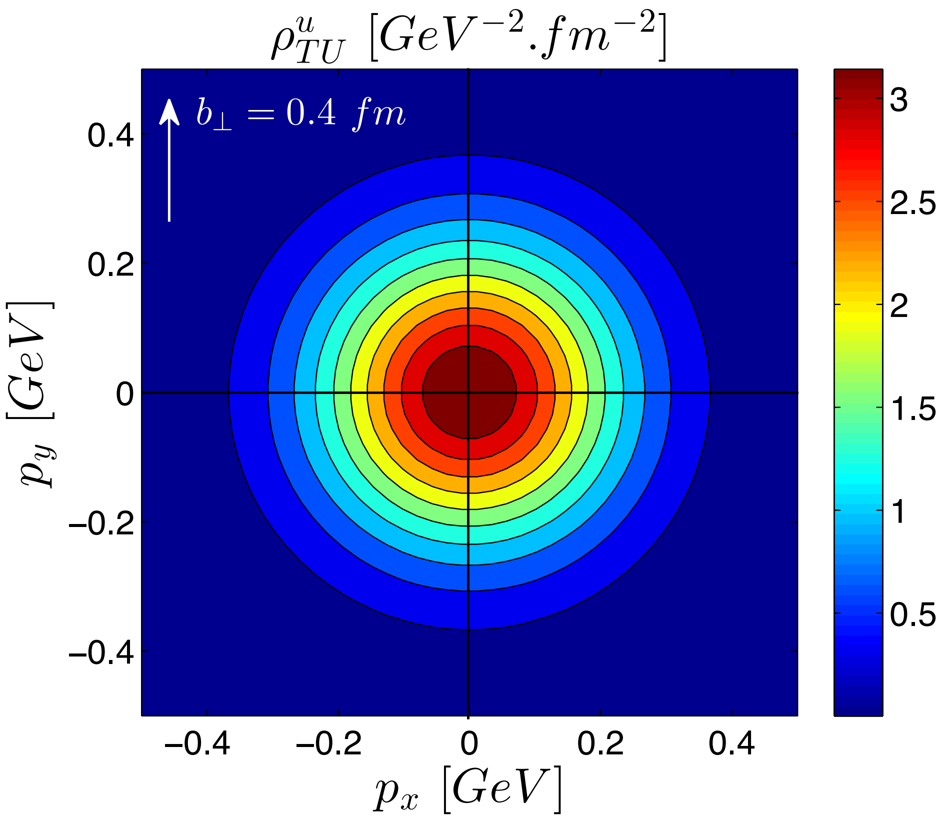
- At $z_{\perp} = 0$, Wigner distribution integrated over transverse momenta reduces to impact parameter dependent parton distribution.

- When integrated over $b_{\perp}$ Wigner distributions reduce to transverse momentum dependent distributions(TMDs).

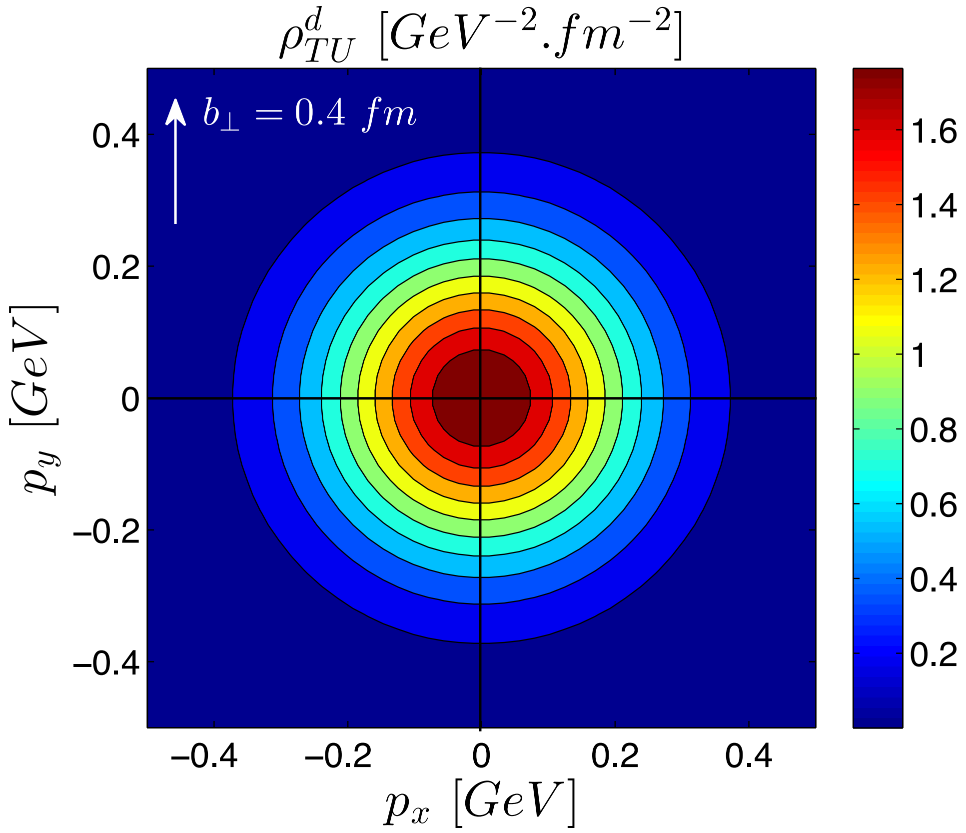
- One can also obtain the three dimensional quark densities by integrating over two mutually orthogonal components of transverse position and momentum.

- Wigner distributions are written in terms of GTMDs which are related to TMDs.

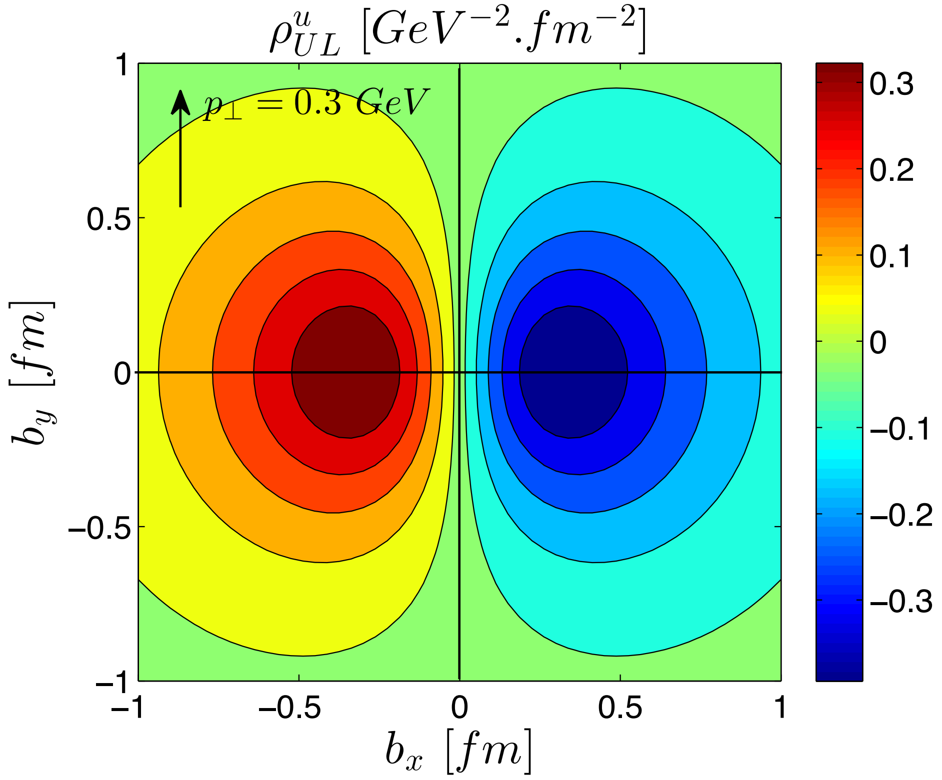
Wigner distributions



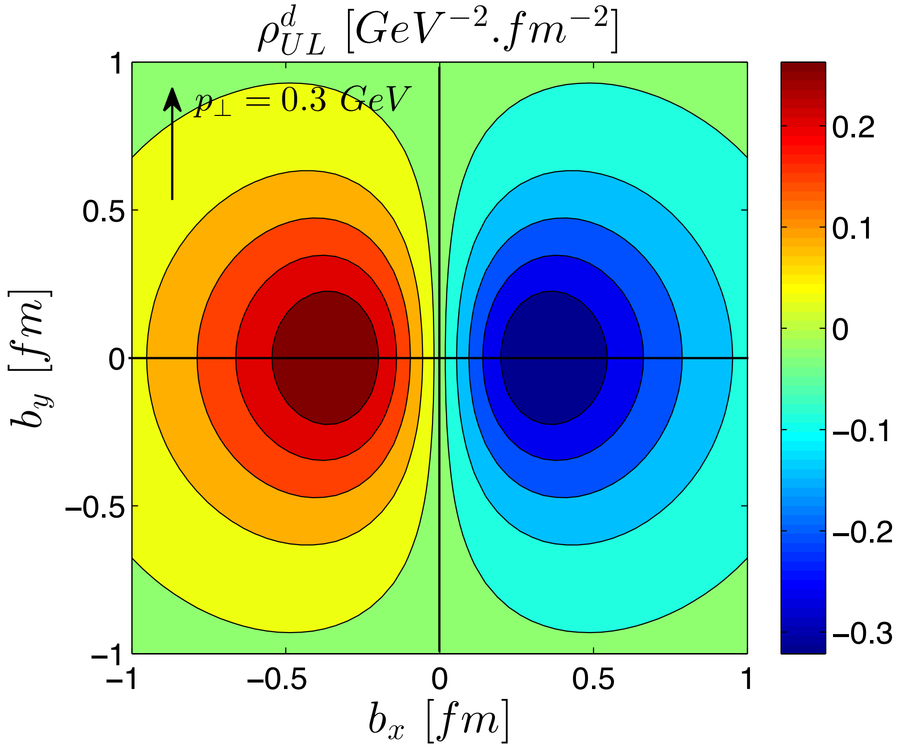
$\rho^u_{TU}\ [GeV^{-2}.fm^{-2}]$



$\rho^d_{TU}\ [GeV^{-2}.fm^{-2}]$

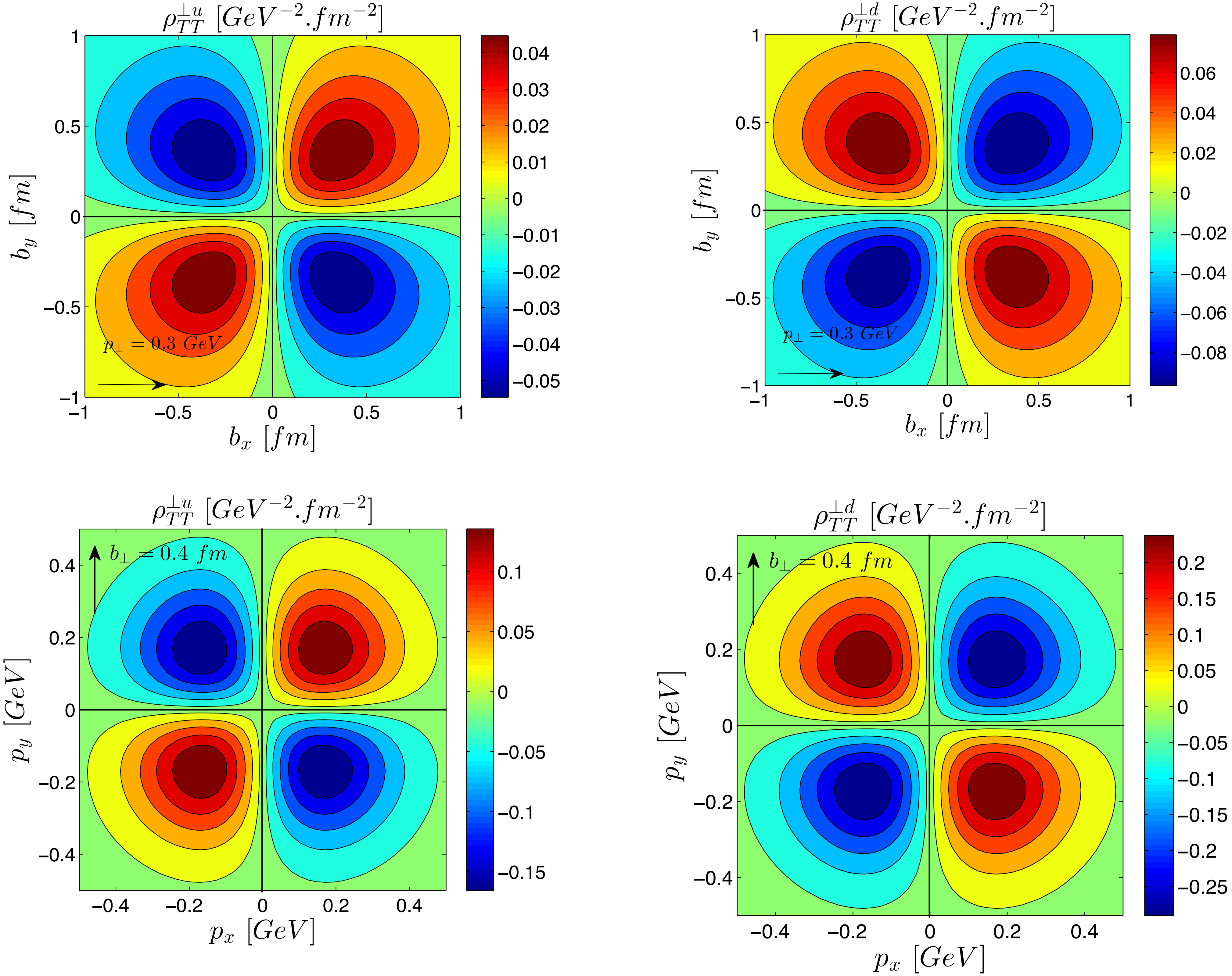


$\rho^u_{UL}\ [GeV^{-2}.fm^{-2}]$



$\rho^d_{UL}\ [GeV^{-2}.fm^{-2}]$

Pretzelous distribution



[DC, T. Maji, C. Mondal, A. Mukherjee, PRD 95, 074028]

Orbital angular momentum

The canonical OAM operator for quarks:

$$\hat{\ell}_z^\nu(b^-,\mathbf{b}_\perp,p^+,\mathbf{p}_\perp)=\frac{1}{4}\int\frac{dz^-d^2\mathbf{z}_\perp}{(2\pi)^3}e^{-ip.z}\bar{\psi}^\nu(b^-,\mathbf{b}_\perp)\gamma^+(\mathbf{b}_\perp\times(-i\partial_\perp))\psi^\nu(b^--z^-,\mathbf{b}_\perp)\,.$$

In terms of Wigner operator

$$\hat{\ell}_z^\nu=(\mathbf{b}_\perp\times\mathbf{p}_\perp)\hat{W}^{\nu[\gamma^+]}\,.$$

Spin-OAM correlation:

$$C_z^\nu(b^-,\mathbf{b}_\perp,p^+,\mathbf{p}_\perp)=\frac{1}{4}\int\frac{dz^-d^2\mathbf{z}_\perp}{(2\pi)^3}e^{-ip.z}\bar{\psi}^\nu(b^-,\mathbf{b}_\perp)\gamma^+\gamma^5(\mathbf{b}_\perp\times(-i\partial_\perp))\psi^\nu(b^--z^-,\mathbf{b}_\perp)\,.$$

$C_z^\nu > 0$ **Implies quark spin and quark OAM tend to align**

$C_z^\nu < 0$ **Implies quark spin and OAM tend to be anti parallel**

Some observations

[DC, T. Maji, C. Mondal, A. Mukherjee,
PRD 95, 074028]

- The quark OAM tends to be aligned with proton spin and anti-aligned to the quark spin for both u and d quarks.

- The difference in correlation strength between quark OAM-proton spin correlation and quark OAM-spin correlation is very small.

- Therefore, if the quark spin is parallel to the proton spin, the contributions of ρ_{UL} and ρ_{LU} interfere destructively resulting the circular symmetry for u and d quarks.

- If the quark spin is anti-parallel to the proton spin, the contributions of interfere constructively resulting a dipolar distribution for both quarks.

T-odd TMDs and Wigner distributions

- Final state interaction produces a non-trivial phase in the amplitude which produces the Sivers asymmetry.

[Brodsky et al, PLB 530, 99 (2002)]

- If the factorisation theorem holds for SIDIS and DY processes, Sivers and Boer Mulders asymmetries can be expressed as convolution of T-odd TMDs(Sivers and Boer-Mulder functions)+hard part+frag. function.

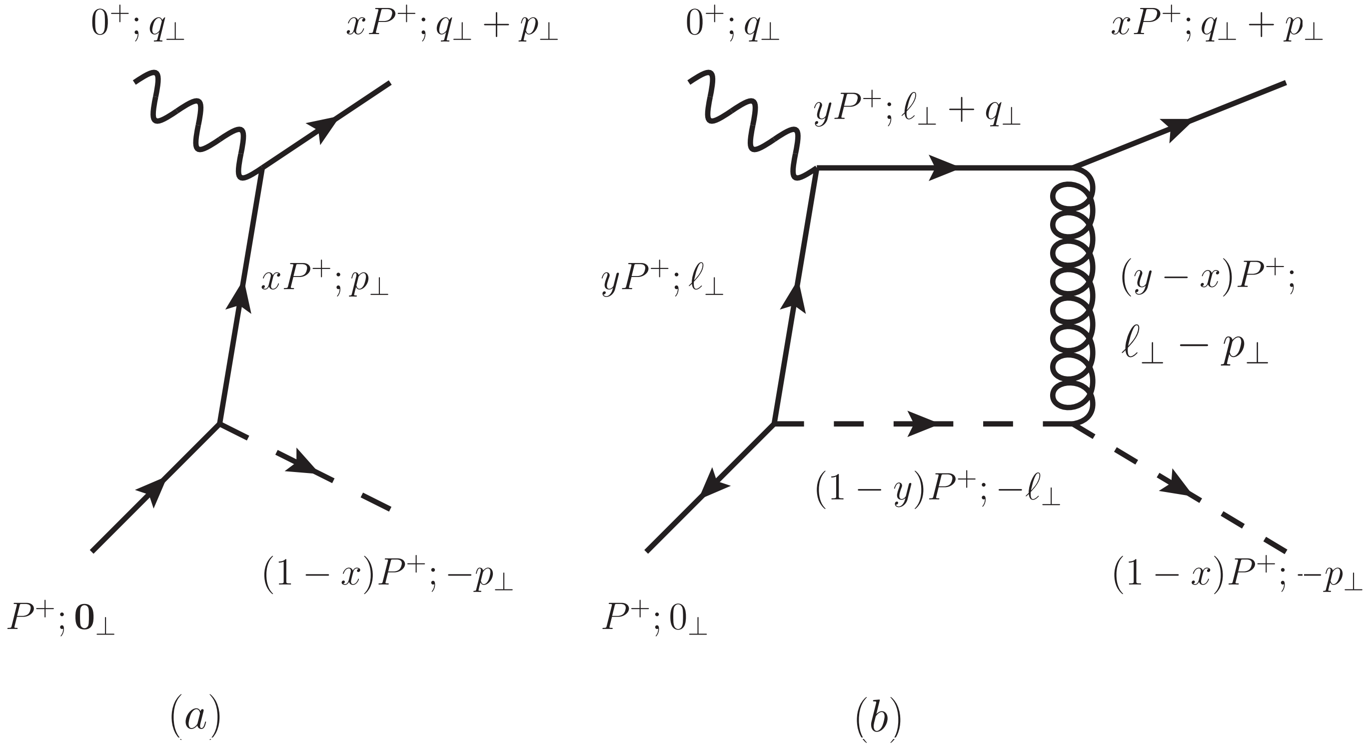
$$d\sigma = \sum_{\nu} f_{\nu/P}(x, p_\perp; Q^2) \otimes d\sigma^{lq \to lq} \otimes D_{h/\nu}(z, k_\perp; Q^2)$$

- Phenomenological extraction relies on factorisation of x, $k_\perp$ dependent parts.

Anselmino et al, JHEP 1704, 046, (2017)

- To have a wave function representation of T-odd TMDs and GTMDs, we incorporate the FSI in the LFWF with a complex phase.

Tree level and FSI diagram for $\gamma^{*}P \to qqq$



Modified wave functions

Wave function for scalar diquark:

[DC, T. Maji, A. Mukherjee, PRD97, 014016 (2018)]

$$\psi_{+}^{+(u)}(x,\mathbf{p}) = N_S \left[1 + i\frac{e_1 e_2}{8\pi}(\mathbf{p}^2 + B)g_1\right]\varphi_1^{(u)}(x,\mathbf{p})$$

$$\psi_{-}^{+(u)}(x,\mathbf{p}) = N_S\left(-\frac{p^1 + ip^2}{xM}\right)\left[1 + i\frac{e_1 e_2}{8\pi}(\mathbf{p}^2 + B)g_2\right]\varphi_2^{(u)}(x,\mathbf{p}),$$

$$\psi_{+}^{-(u)}(x,\mathbf{p}) = N_S\left(\frac{p^1 - ip^2}{xM}\right)\left[1 + i\frac{e_1 e_2}{8\pi}(\mathbf{p}^2 + B)g_2\right]\varphi_2^{(u)}(x,\mathbf{p})$$

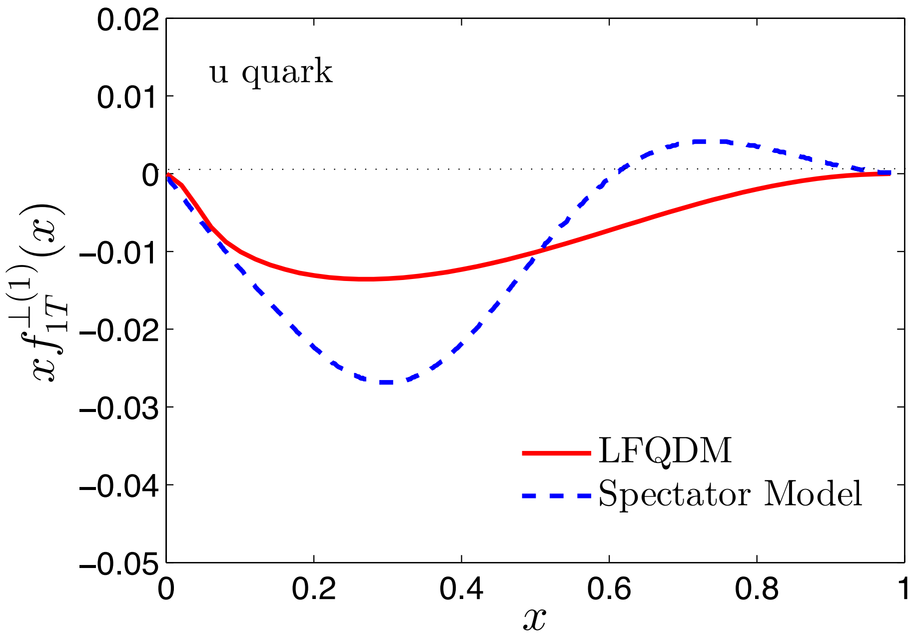
$$\psi_{-}^{-(u)}(x,\mathbf{p}) = N_S \left[1 + i\frac{e_1 e_2}{8\pi}(\mathbf{p}^2 + B)g_1\right]\varphi_1^{(u)}(x,\mathbf{p}),$$

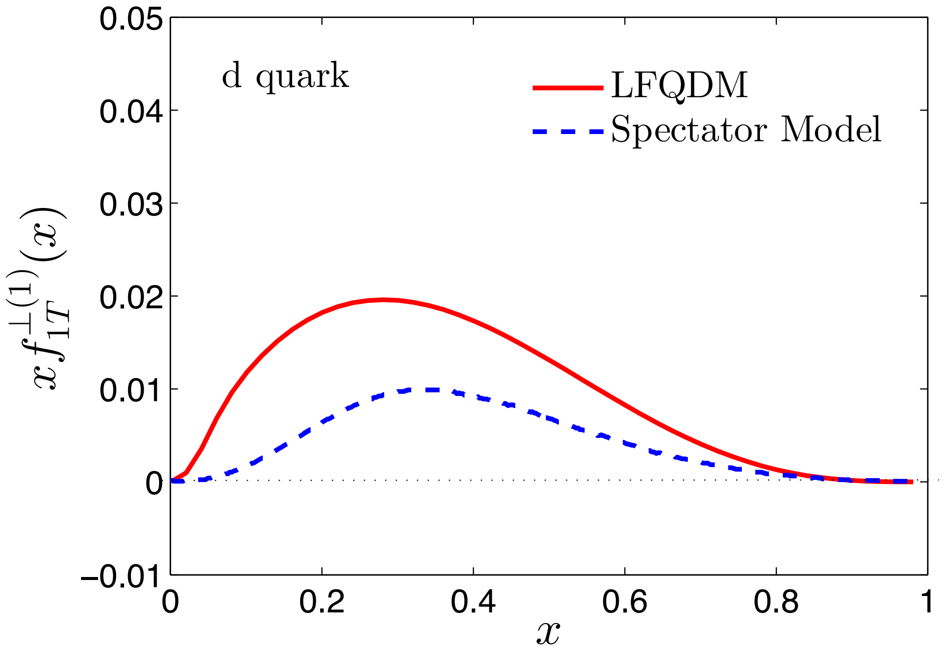
Similarly wave functions for axial vector diquark also acquire complex phases

$$g_1 = \int_0^1 d\alpha \frac{-1}{\alpha(1-\alpha)\mathbf{p}^2 + \alpha m_g^2 + (1-\alpha)B}$$

$$g_2 = \int_0^1 d\alpha \frac{-\alpha}{\alpha(1-\alpha)\mathbf{p}^2 + \alpha m_g^2 + (1-\alpha)B}$$

Sivers function

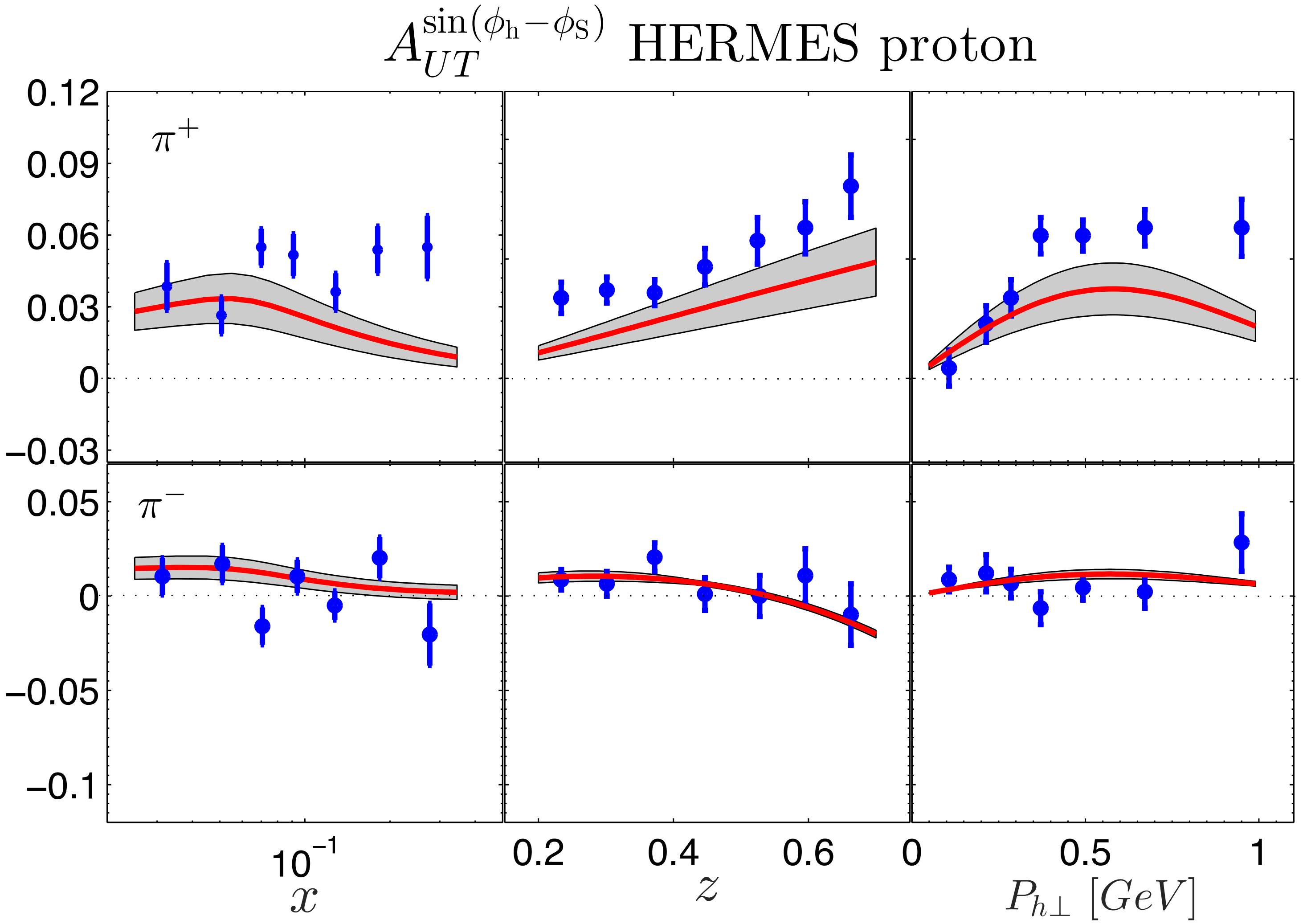




Here

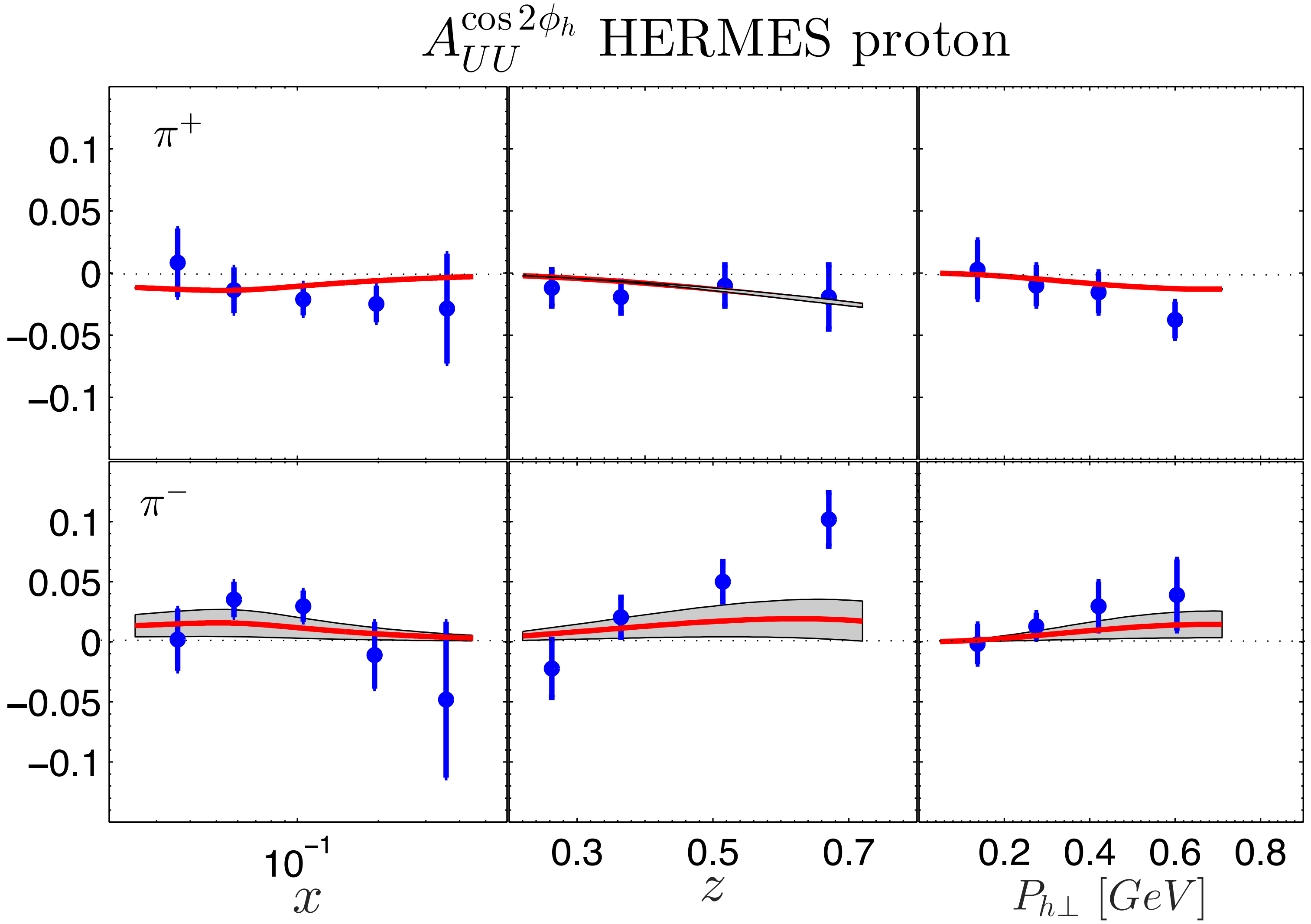
$$f_{1T}^{\perp(1)}(x) = \int d^2 p_\perp \frac{p_\perp^2}{2M^2} f_{1T}^{\perp}(x, \mathbf{p^2})$$

Sivers asymmetry



[DC, T. Maji, A. Mukherjee, PRD97, 014016 (2018)]

Boer-Mulders asymmetry



Spin density

- The spin density of unpolarised quarks in a transversely polarized proton is defined as

$$f_{\nu/P^{\uparrow}}(x, \mathbf{p}) = f_1^{\nu}(x, \mathbf{p}^2) - \frac{\mathbf{S}\,.\,(\hat{\mathbf{P}} \times \mathbf{p}_{\perp})}{M} f_{1T}^{\perp\nu}(x, \mathbf{p}^2)\,.$$

- Similarly, the spin density for transversely polarized quarks in an unpolarized proton is defined as

$$f_{\nu^{\uparrow}/P}(x, \mathbf{p}) = \frac{1}{2}[f_1^{\nu}(x, \mathbf{p}^2) - \frac{\mathbf{s}\,.\,(\hat{\mathbf{P}} \times \mathbf{p}_{\perp})}{M} h_1^{\perp\nu}(x, \mathbf{p}^2)]\,.$$

$\mathbf{S} =$ **Proton spin**

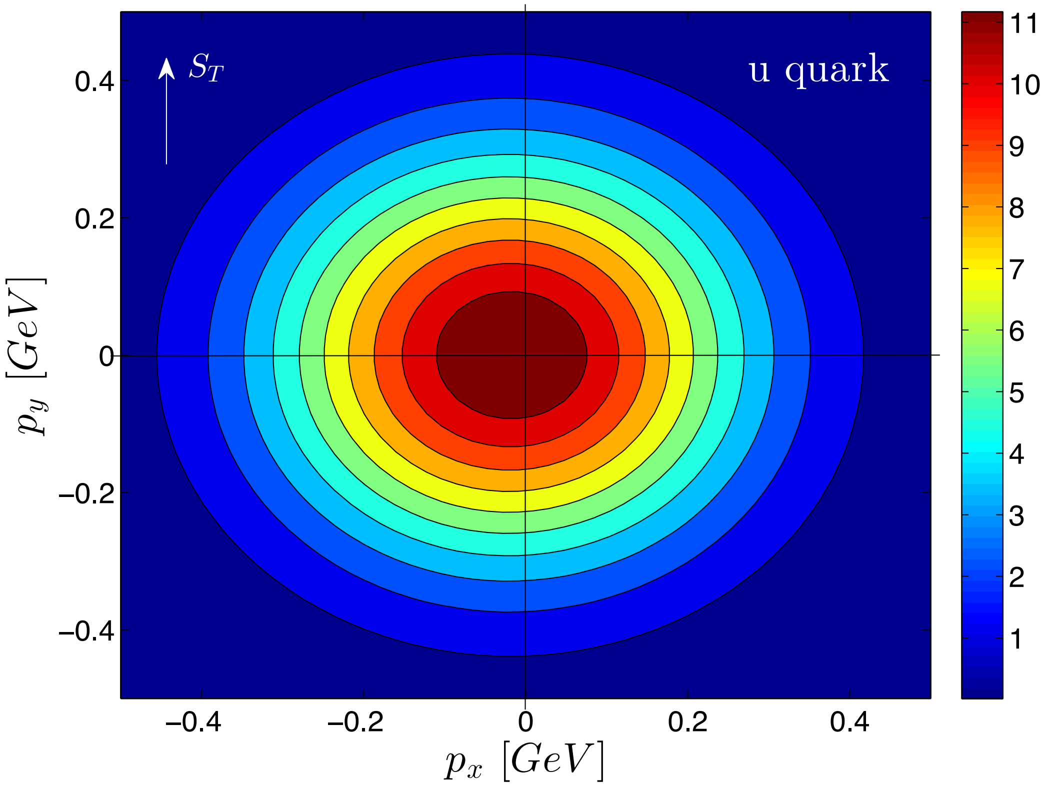
$\mathbf{s} =$ **Quark spin**

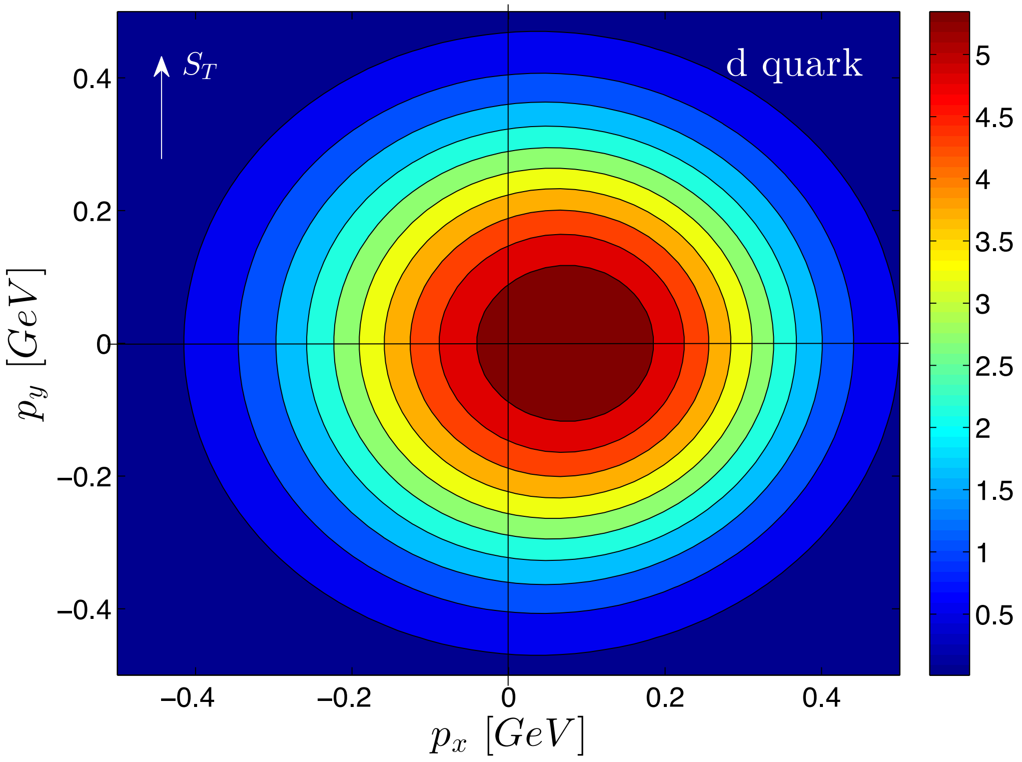
$\hat{\mathbf{P}} =$ **Proton momentum direction**

$\mathbf{p}_{\perp} =$ **Quark transverse momentum**

-

Spin densities in transversely polarised proton



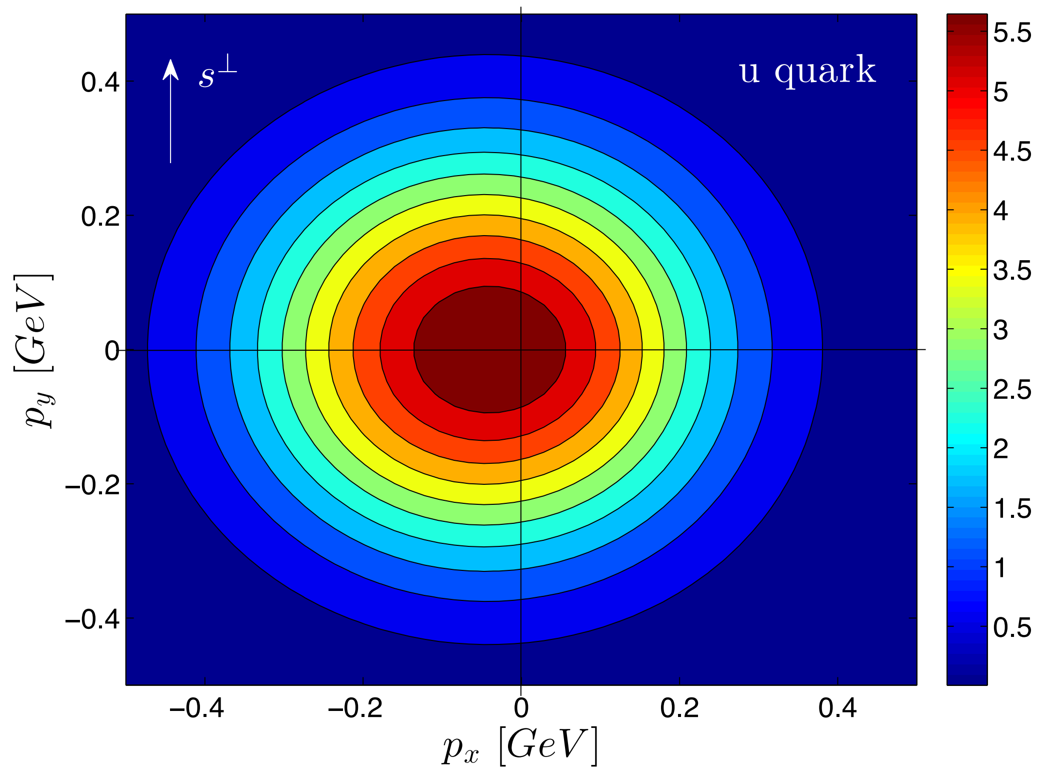


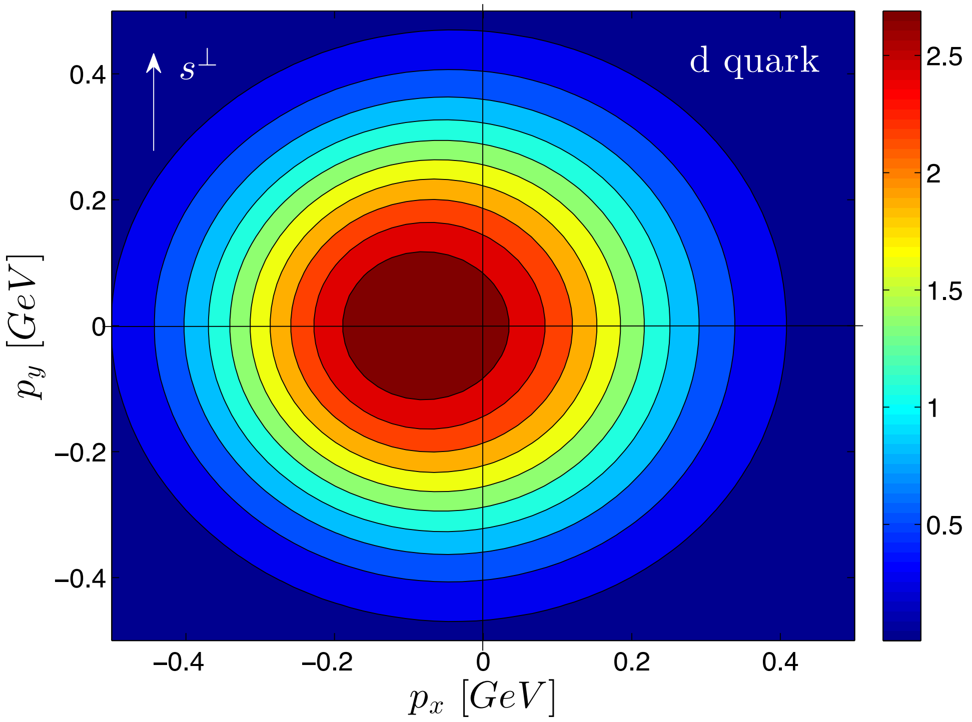
- Spin densities are not symmetric. Distorted due to Sivers function.

- Sivers function negative for u quark, positive for d-quark.

- Quarks in a transversely polarised proton have a transverse momentum asymmetry in the perpendicular direction of nucleon spin.

Spin densities in unpolarised proton





- Since Boer-Mulders functions are negative for both u and d quarks, we observed only a left-shift

FSI contribution to Wigner distributions

[DC, N. Kumar, T. Maji, A. Mukherjee, 1902.07051]

- Wigner distributions are parametrized in terms of GTMDs.

- Like TMDs, GTMDs also acquires T-odd components when FSI is incorporated in the wave functions, e.g.,

$$F^{e}_{1,2} + iF^{o}_{1,2}$$

- GTMD $F^{o}_{1,2}$ & $H^{o}_{1,1}$ reduce to Sivers and Boer-Mulder functions in the limit $\Delta_\perp = 0$

- The non-vanishing odd parts of GTMDs contribute to the Wigner distributions.

- Hermiticity property ensures that the Wigner distributions are real valued.

- Fourier transform of these leading twist T-odd GTMDs

$$\rho^{\nu}_{Siv}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp . \mathbf{b}_\perp} F^{o\nu}_{1,2}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x)$$

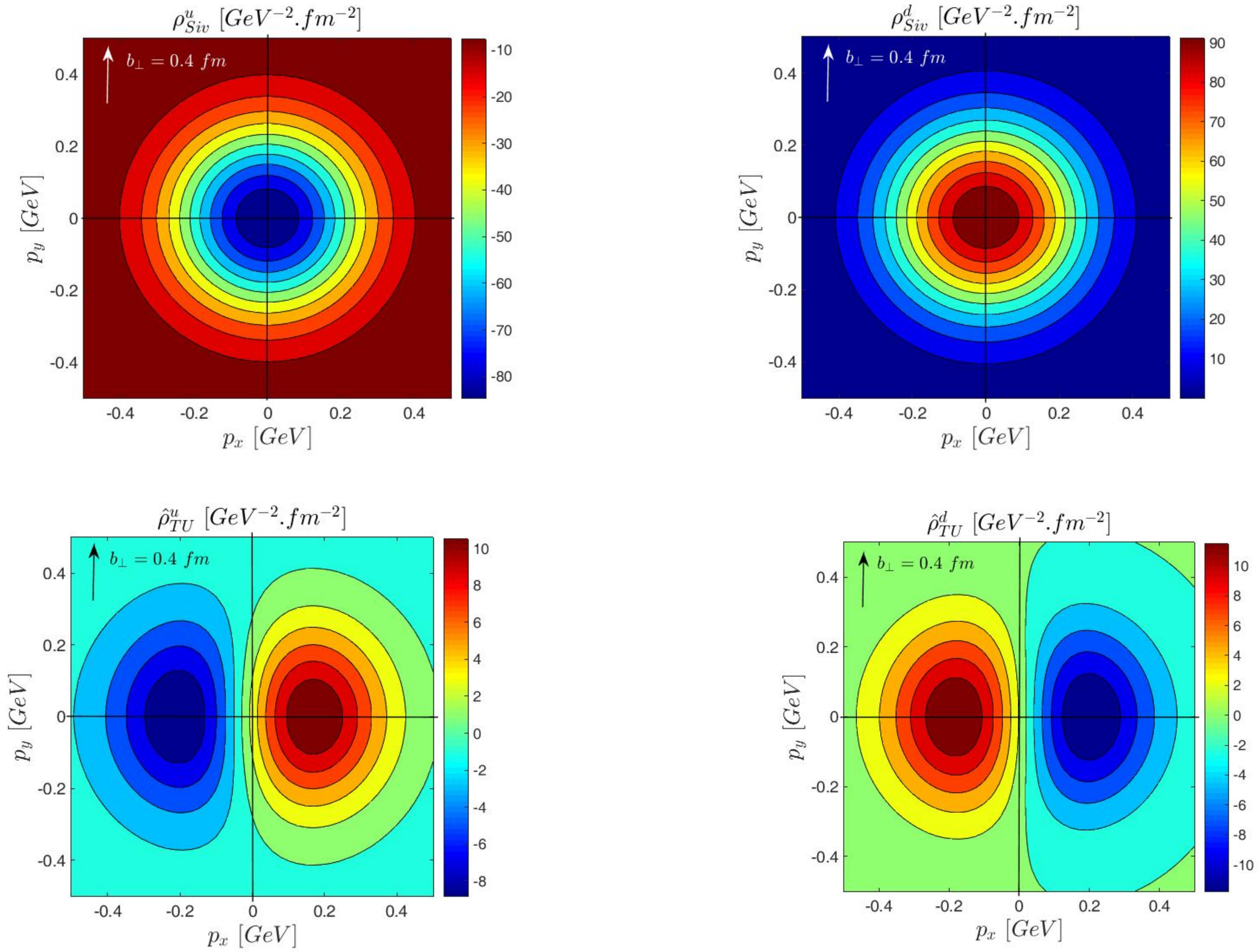
$$\rho^{\nu}_{BM}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp . \mathbf{b}_\perp} H^{o\nu}_{1,1}(\mathbf{\Delta}_\perp, \mathbf{p}_\perp, x)$$

- The modified Wigner distributions are

$$\hat{\rho}^{i\nu}_{TU}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) = \rho^{i\nu}_{TU}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) - \frac{1}{M} \epsilon^{ij}_\perp p^j_\perp \rho^{\nu}_{Siv}(\mathbf{b}_\perp, \mathbf{p}_\perp, x)$$

$$\hat{\rho}^{i\nu}_{UT}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) = \rho^{i\nu}_{UT}(\mathbf{b}_\perp, \mathbf{p}_\perp, x) + \frac{1}{M} \epsilon^{ij}_\perp p^j_\perp \rho^{\nu}_{BM}(\mathbf{b}_\perp, \mathbf{p}_\perp, x)$$

- Superscript i= transverse polarisation of proton, j=transverse polarisation of quark.



Note ρ^{ν}_{siv} is axially symmetric but total $\tilde{\rho}^{\nu}_{TU}$ is dipolar!

[DC, N. Kumar, T. Maji, A. Mukherjee, 1902.07051]

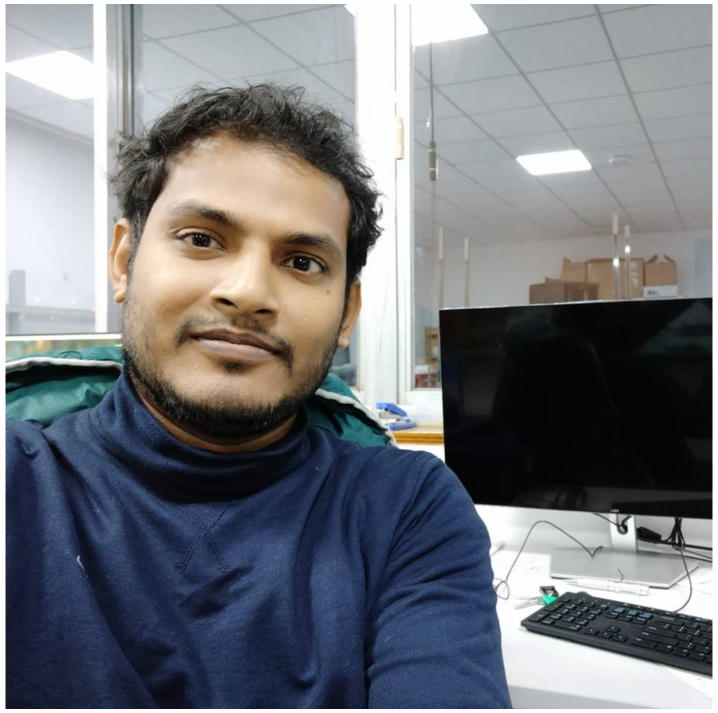
Conclusions

- Till now results are mostly model dependent.
- Our simple model predicts many data with good agreements.
- Found many relations among the different distribution functions which may help to constrain other models/ predictions.
- *Many interesting and unanswered questions to answer to understand the internal structure of proton…model independent/nonperturbative predictions…*

Collaborators

Chandan Mondal

Tanmay Maji



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THANKS