Infrared Structure of QCD, LHC and All That

V. Ravindran
The Institute of Mathematical Sciences,
Chennai, India

IISER-Pune, 23rd January 2017

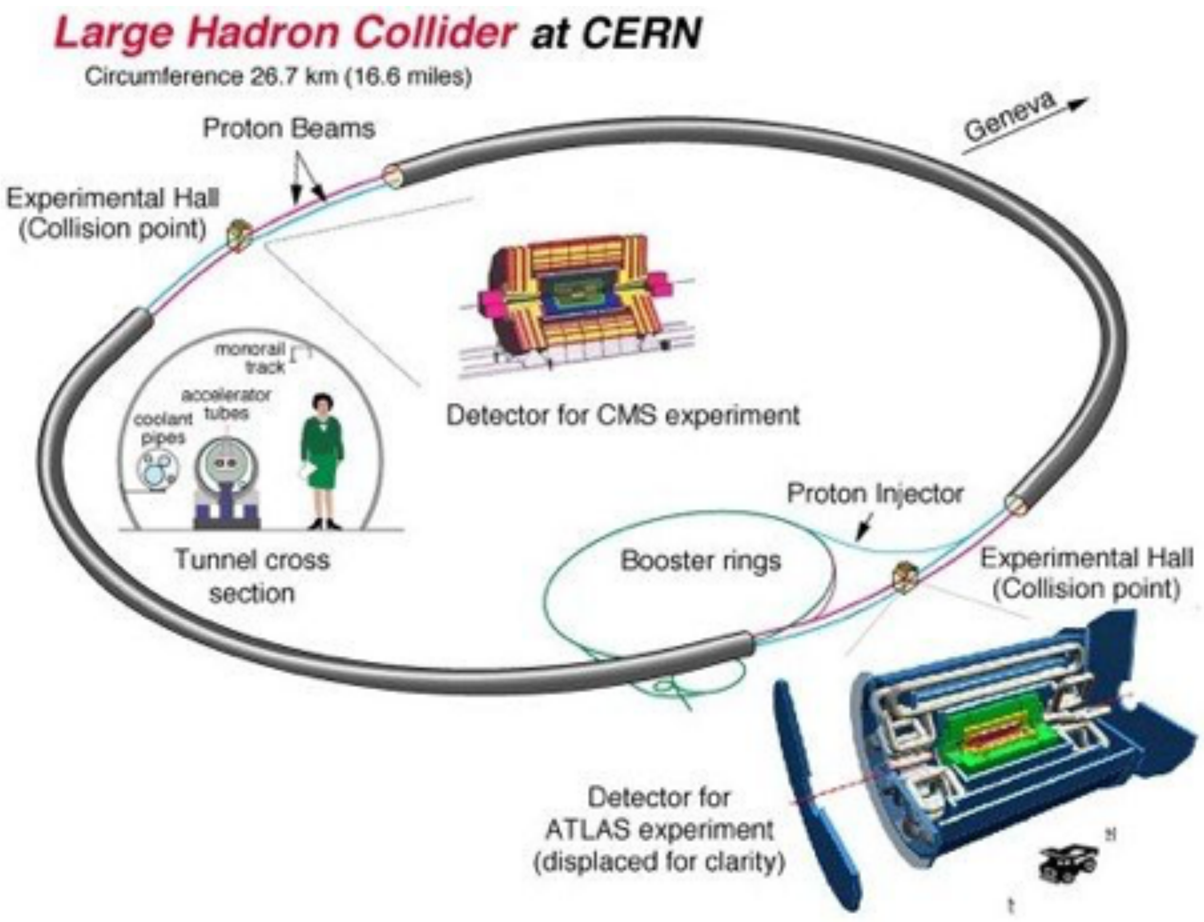
Plan

- LHC and Precision physics
- Infrared Structure
  - Soft
  - Collinear
- Multi-leg, Multi-loop amplitudes
  - K+G equation
  - Catani's proposal
- Factorisation and Resummation
- Casimir Duality
- UV from IR

Large Hadron Collider

- Excellent Discovery Reach

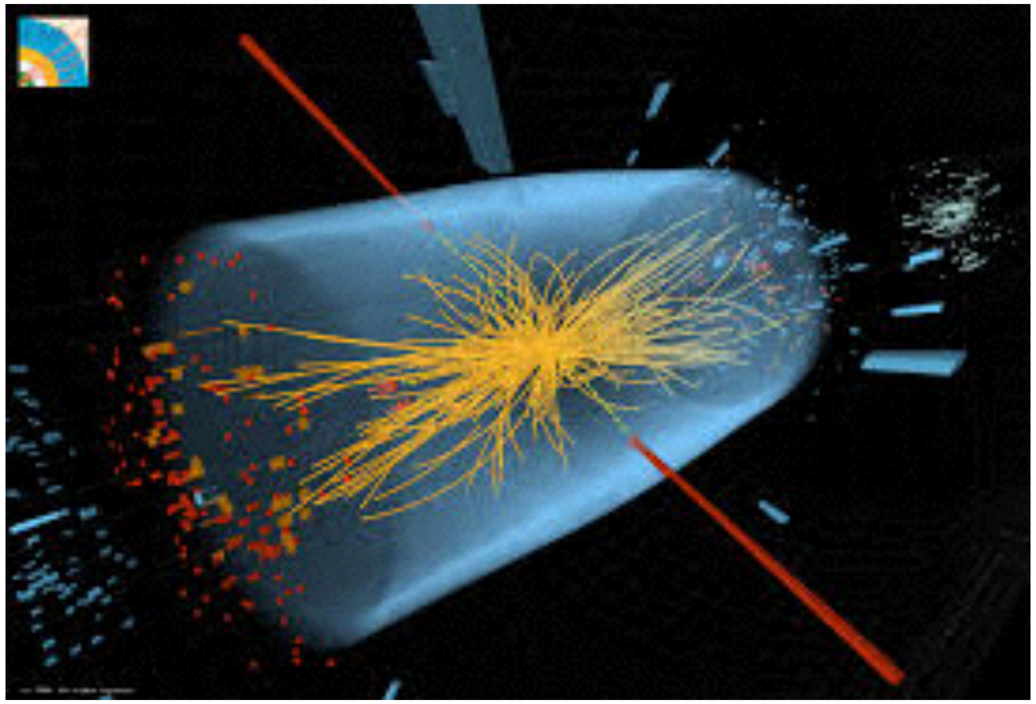
- Higgs
- Supersymmetry
- Extra-Dimensions
- Anything else



Large Hadron Collider

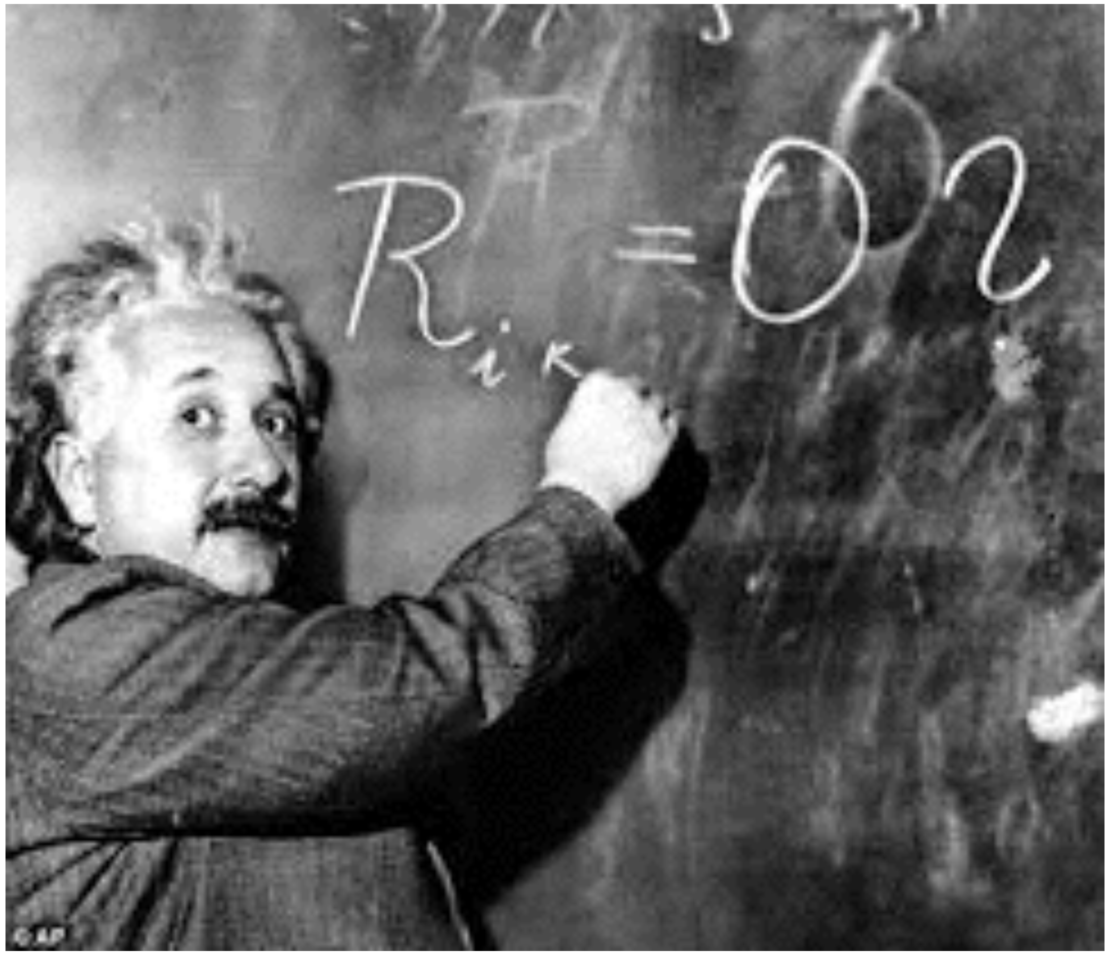
- Large amount of events

- $W \to e\nu$: 10^8 events
- $Z \to e^+e^-$: 10^7 events
- $t\bar{t}$ production 10^7 events
- Higgs production 10^5 events



Standard Model

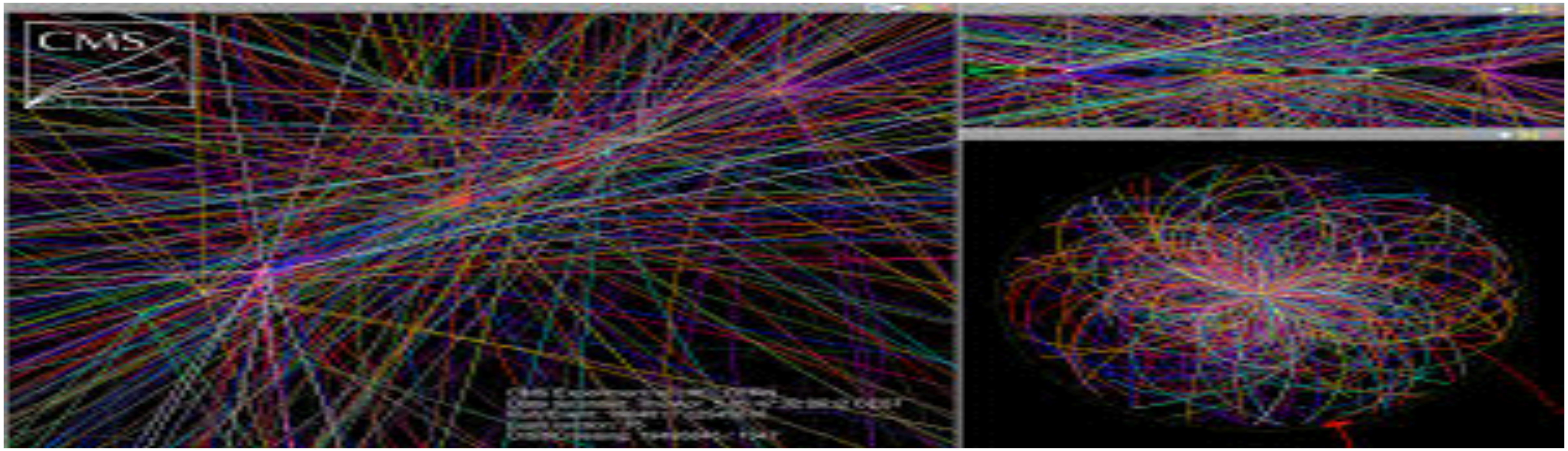
- Theories
  - Quantum Chromodynamics
  - Electroweak Theory (SM)
  - Theory of Gravity



Large Hadron Collider

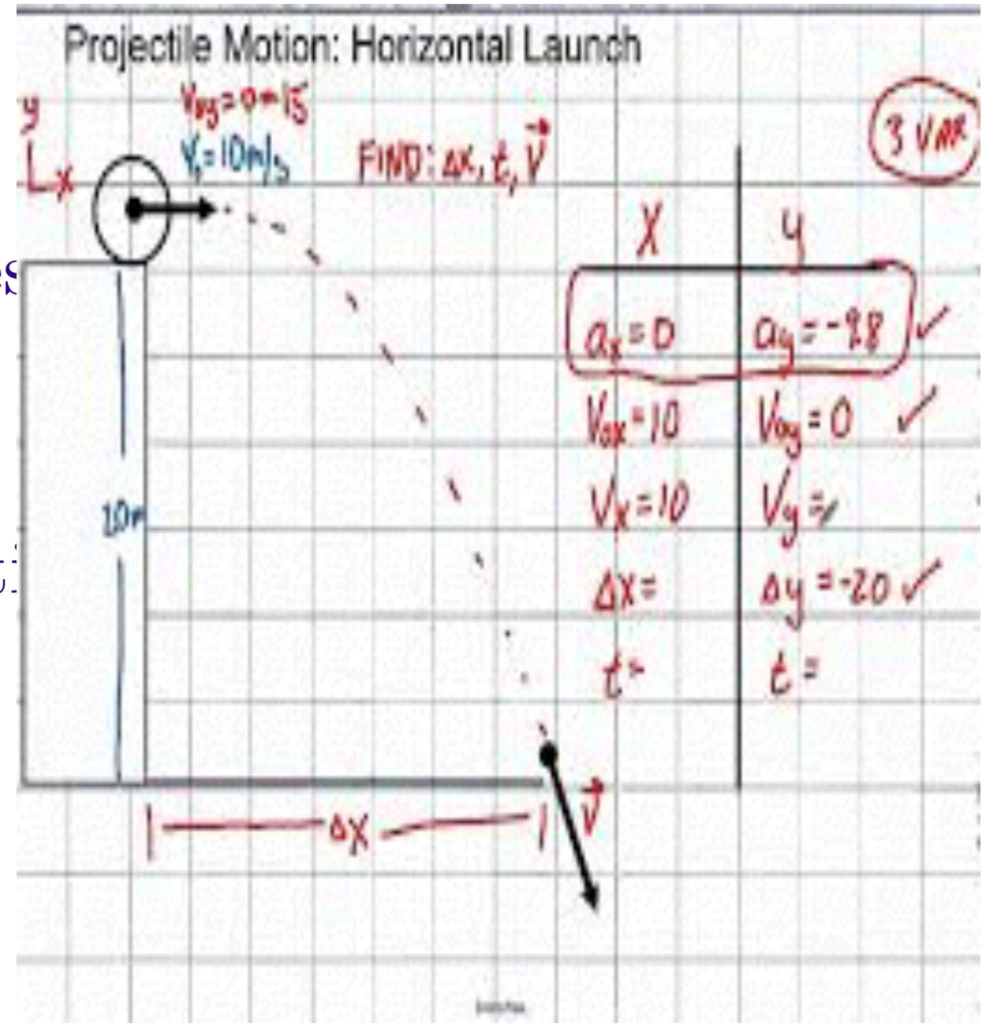
- Large background

- Large number of $\gamma, l^{\pm}, Z, W^{\pm}$
- Jets
- Large number of $t\bar{t}, b\bar{b}$

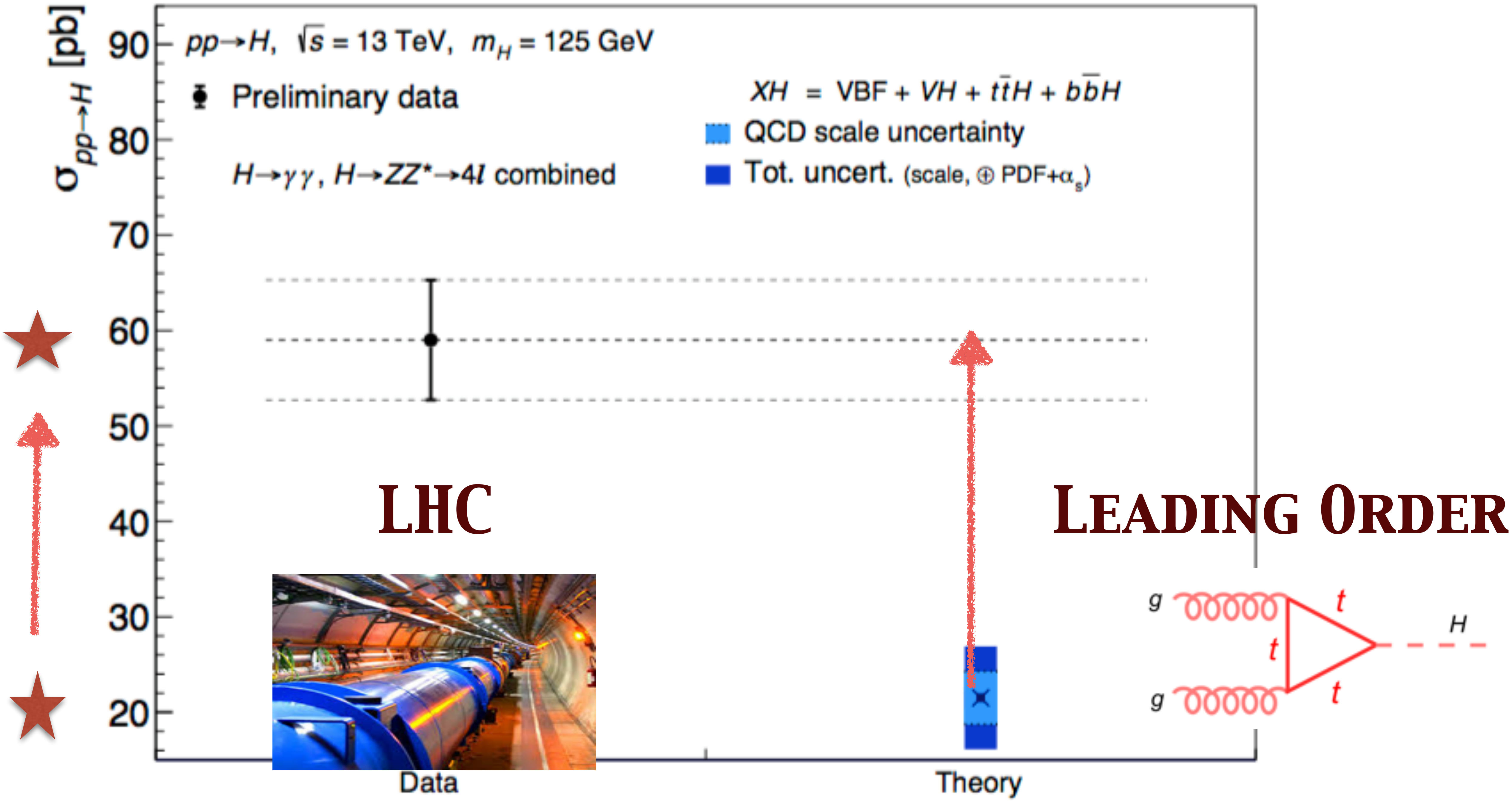


- Issues to be tackled

- Kinematics
- Normalisation
- Renormalisation and Factorisation Scales
- Parton distribution functions
- Phase space boundary effects, resummati

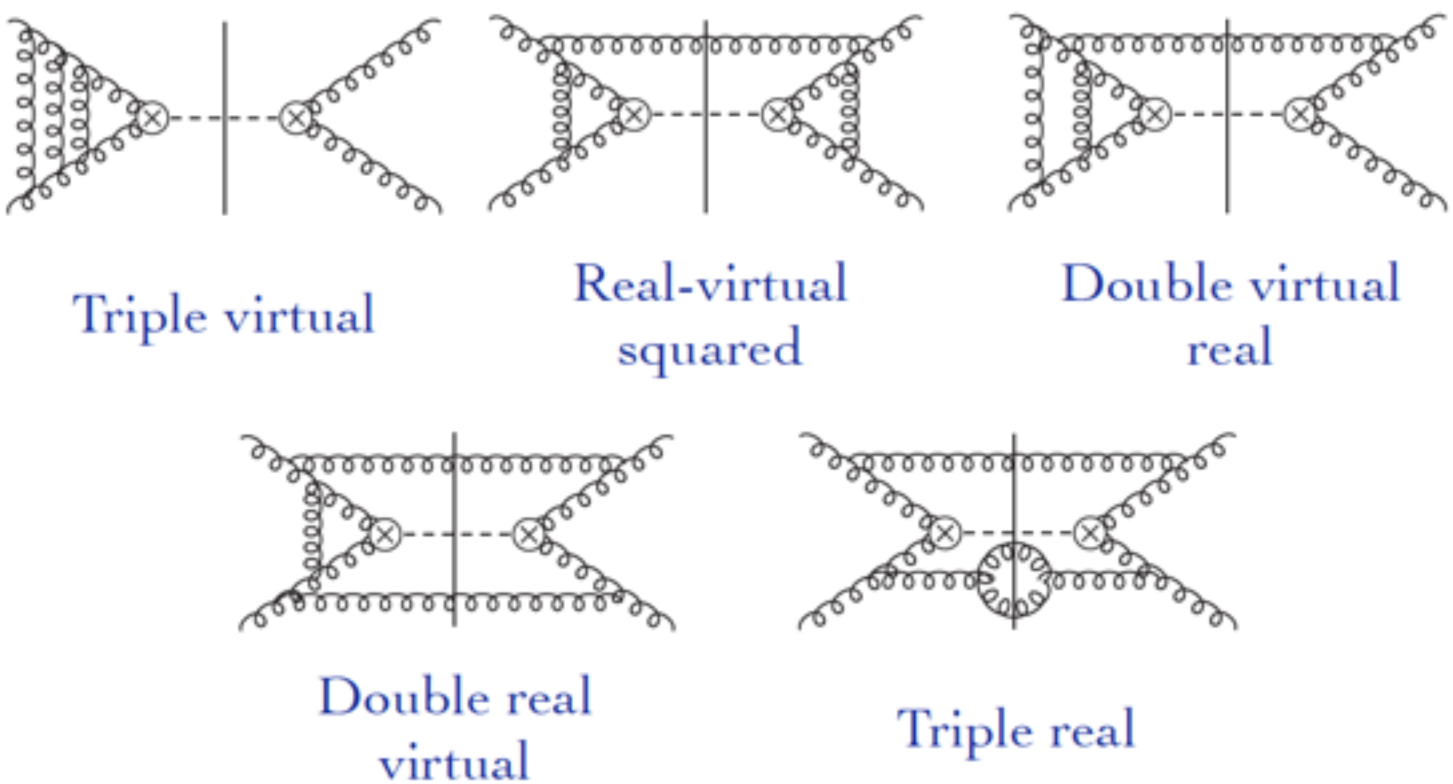


Leading order is often Crude



Higgs production to N^3LO in QCD at the LHC

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger



100 000 diagrams	Integrals	Master Integrals
NNLO	50 000	27
N3LO	517 531 178	1028

Methods

↓

Integration By Parts

$$\int \frac{d^d k_1}{(2\pi)^d} \cdots \int \frac{d^d k_3}{(2\pi)^d} \frac{\partial}{\partial k_i} \cdot \left(v_j \frac{1}{\prod_l D_l^{n_l}} \right) = 0$$

Lorentz Invariance

$$p_i^\mu p_j^\nu \left(\sum_k p_{k[\nu} \frac{\partial}{\partial p_k^{\mu]}} \right) J(\vec{n}) = 0.$$

↓

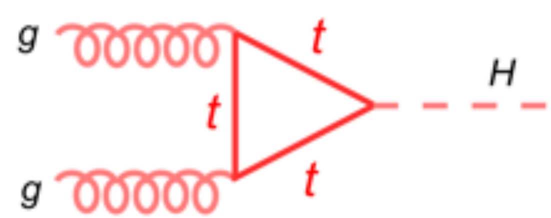
Master Integrals

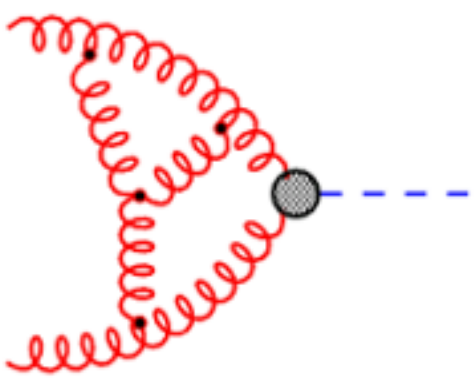
True Result for Higgs

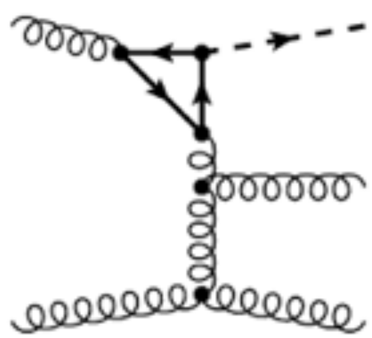
Anastasiou,Melnikov,Harlander,Kilgore,VR, van Neerven, Smith, Anastasiou, Mistelberger, Dulat et al]

$$\hat{\sigma}(z) = \hat{\sigma}^{LO}(z) + \alpha_S \hat{\sigma}^{NLO}(z) + \alpha_S^2 \hat{\sigma}^{NNLO}(z) + \alpha_S^3 \hat{\sigma}^{N3LO}(z) + \mathcal{O}(\alpha_S^4)$$

✓ ✓ ✓ ✓

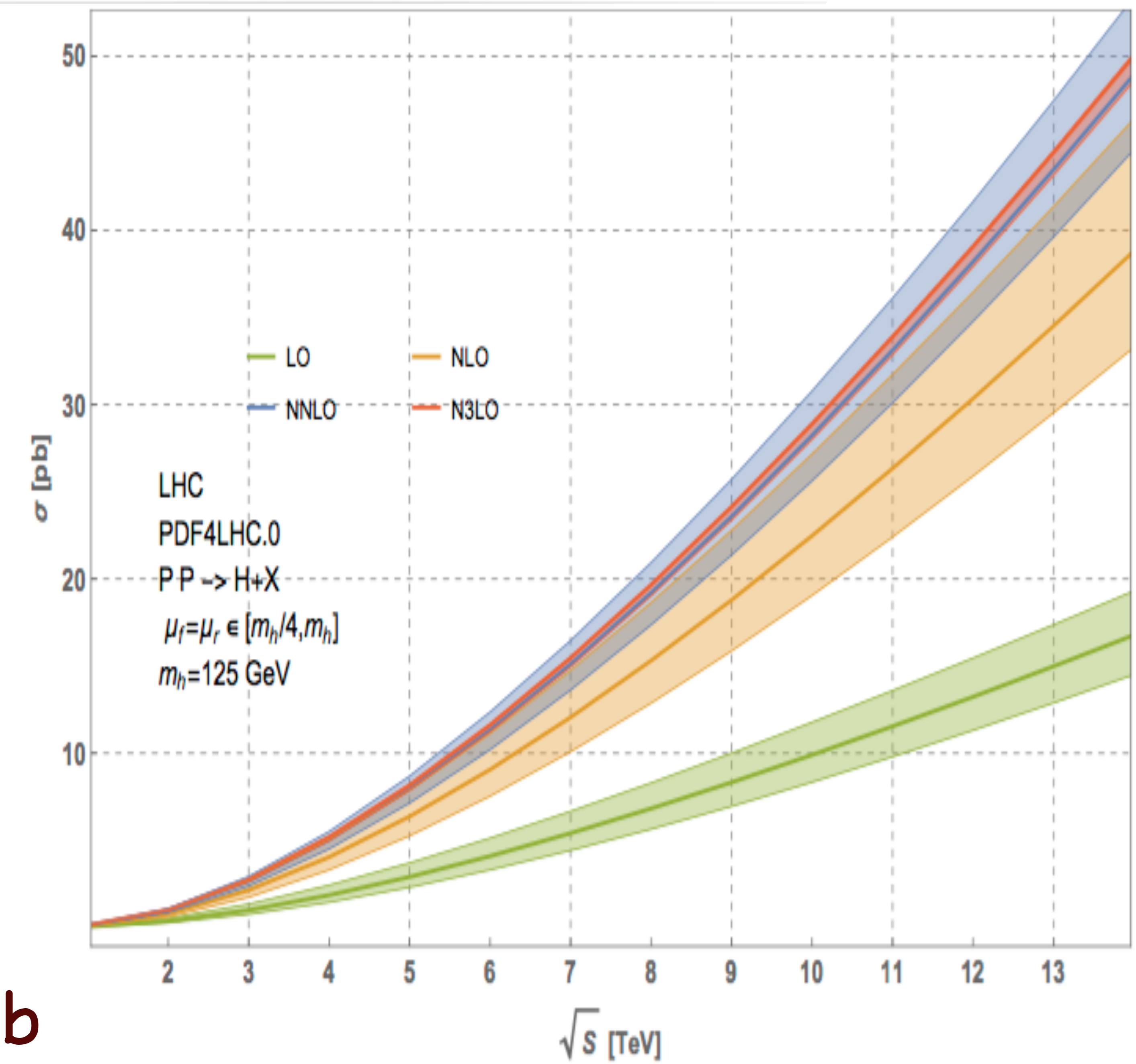






LO	$15.05 \pm 14.8\%$
NLO	$38.2 \pm 16.6\%$
NNLO	$45.1 \pm 8.8\%$
N3LO	$45.2 \pm 1.9\%$

pb



Form Factor - Building blocks

Form Factor : On-shell matrix elements of composite operators

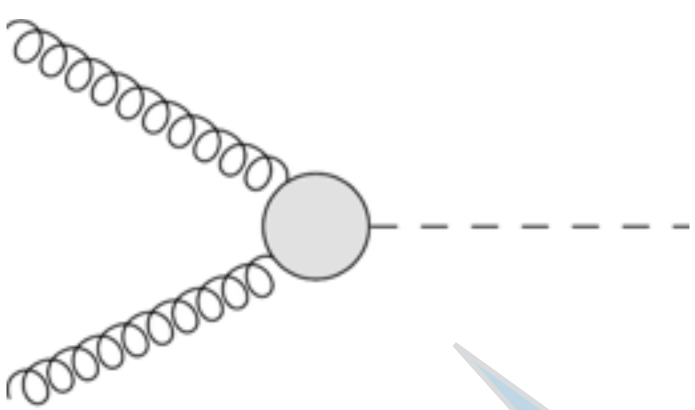
$$< p'|\mathcal{O}|p >$$

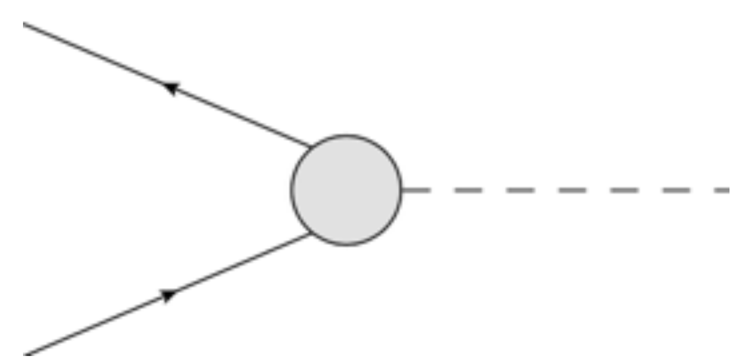
Gauge boson form factor

$$< g(p')|G^a_{\mu\nu}G^{\mu\nu a}|g(p) >$$

Fermion form factor

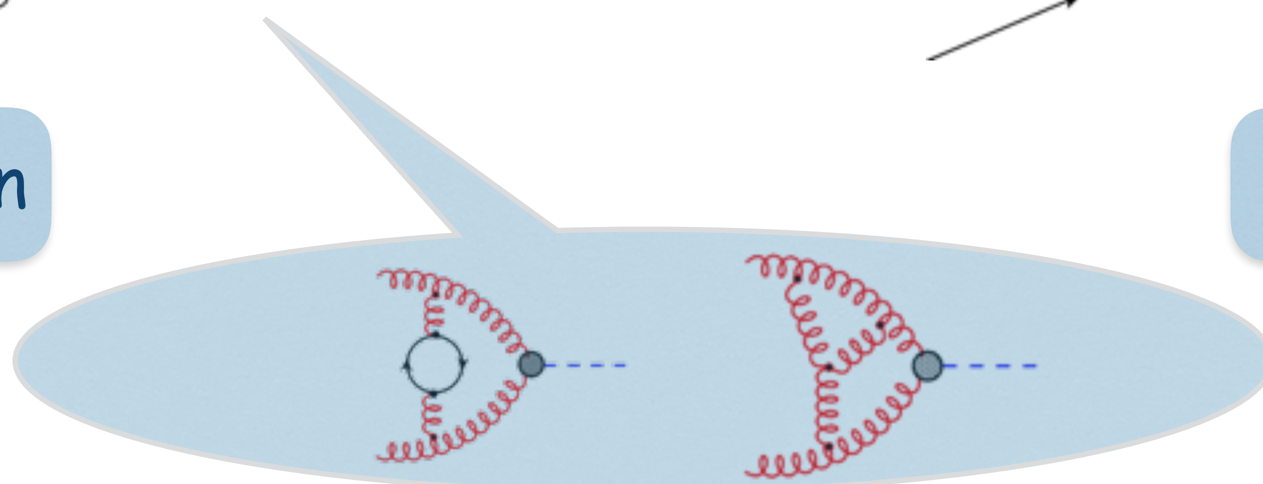
$$< e(p')|\overline{\psi}\gamma_\mu\psi|e(p) >$$





Higgs production

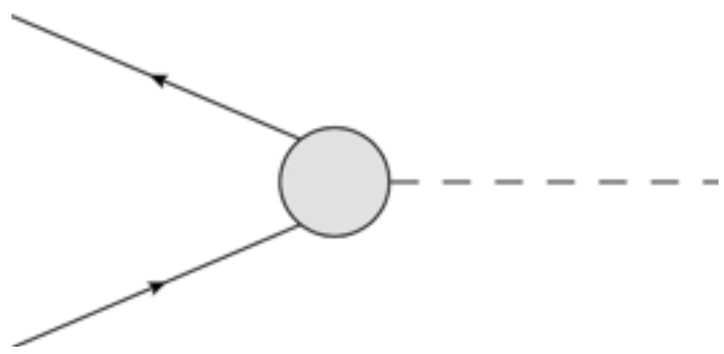
Drell-Yan



Sudakov Form Factor

[Sudakov, Sen, Sterman, Collins, Magnea]

Large $q^2 = (p+p')^2$ behaviour of form factors



Sudakov: $< e(p')|\overline{\psi}\gamma_\mu\psi|e(p) >$

$$\exp\left(-\frac{g^2}{8\pi^2}\ln^2(\frac{q^2}{m^2})\right)$$

Sen: QCD Leading and subleading logs exponentiate:

$$\left(\frac{g_s^2}{8\pi^2}\right)^n \ln^\nu\left(\frac{q^2}{m^2}\right),$$

$< q(p')|\overline{\psi}\gamma_\mu\psi|q(p) >$

Infrared divergences

[Bloch, Nordsieck, Kinoshita,: Lee, Nauenberg]

Quantum Field Theories with massless particles encounter two kinds of divergences:

Soft :

On-shell amplitudes in gauge theories contain Soft divergences due to massless gauge bosons.

Collinear :

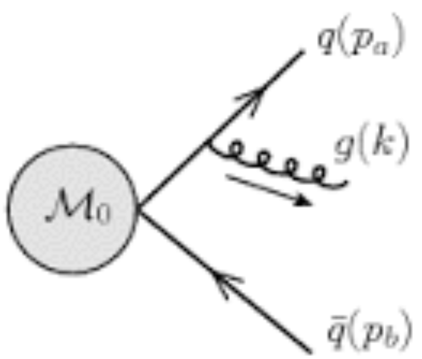
If the matter fields in the theory are light (mass of the particles are negligible compared to hard scale of the process), there will be mass singularities, called Collinear divergences

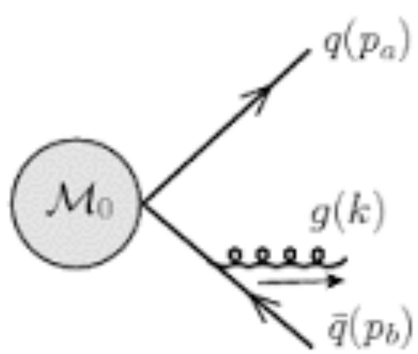
Infrared divergences

[Bloch, Nordsieck, Kinoshita,: Lee, Nauenberg]

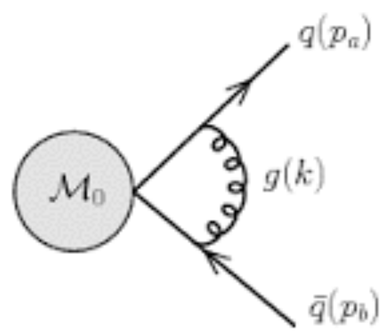
In the Limit $k \to p \quad (p_a \text{ or } \mathrm{p_b})$ $\quad m_a, m_b << Q$

Real emission





Virtual



$$\frac{1}{(p+k)^2} = \frac{1}{2p^0 k^0 \left(1 - \cos\theta\right)}$$

$k^0 \to 0$	Soft divergence
$\cos\theta \to 0$	Collinear divergence

Infrared divergences

[S Weinberg]

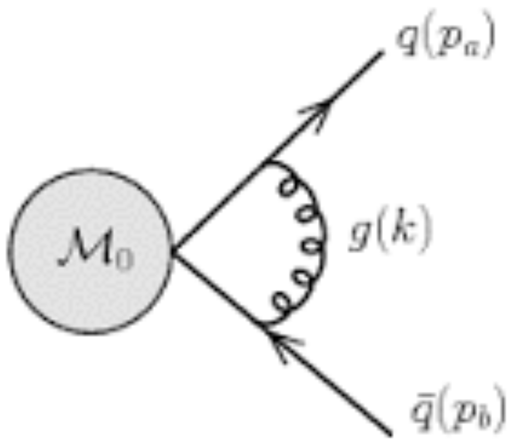
"In [Yang-Mills theory] a soft photon (**gluon**) *emitted from an external line can itself emit a pair of soft charged massless particles, which themselves emit soft photons* (**gluons**), *and so on, building up a cascade of soft massless particles each of which contributes an infra-red divergence. The elimination of such complicated interlocking infra-red divergences would certainly be a Herculean task, and might not even be possible. "*

S. Weinberg, Phys. Rev. 140B (1965)

Sudakov Form Factor

One loop on-shell form factor

$$(p-k)^2 = 0,$$
$$p_a^2 = p_b^2 = m^2 << q^2$$



Soft

$k \to 0$

Collinear

$p_a || k \ or \ p_b || k$

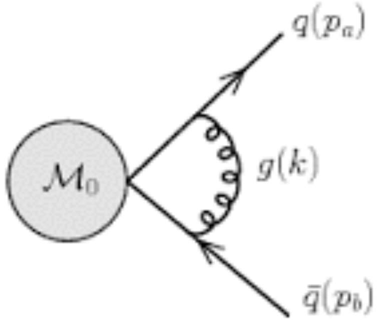
$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2((p_a+k)^2 - m^2)((p_b-k)^2 - m^2)} \to \infty$$

Ill defined

Virtual effect

One loop on-shell form factor

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2((p_a+k)^2-m^2)((p_b-k)^2-m^2)} \to \infty$$



$$p_a^2 = p_b^2 = m^2 << q^2$$

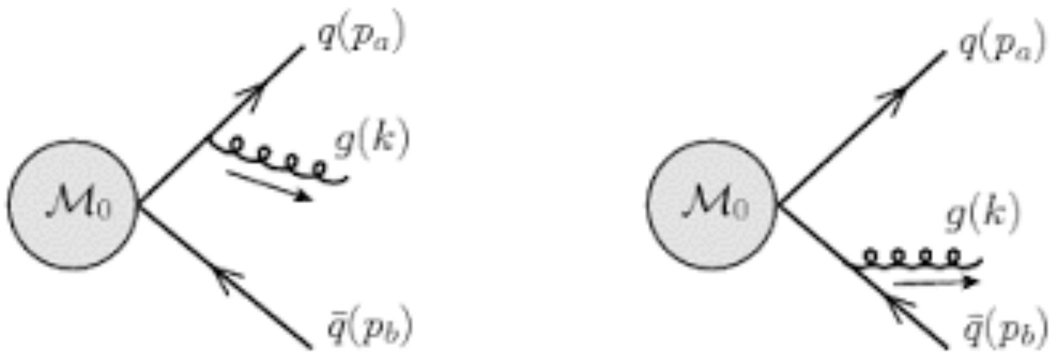
Summing to all orders in g^2

$$1 - g^2\left(\infty\right) + \frac{1}{2!}g^4\left(\infty\right) - \frac{1}{3!}g^6\left(\infty\right) + \cdots = exp(-g^2\infty)$$

Probability to happen this is ZERO

Real emission

Real photon emission:



$$\int \frac{d^4k}{(2\pi)^4} \frac{\delta^+(k^2)}{((p_a+k)^2-m^2)((p_b-k)^2-m^2)} \to \infty$$

Summing multiple emissions

$$p_a^2 = p_b^2 = m^2 << q^2$$

$$1 + g^2\infty + g^4\infty + \cdots$$

Probability grows uncontrollably

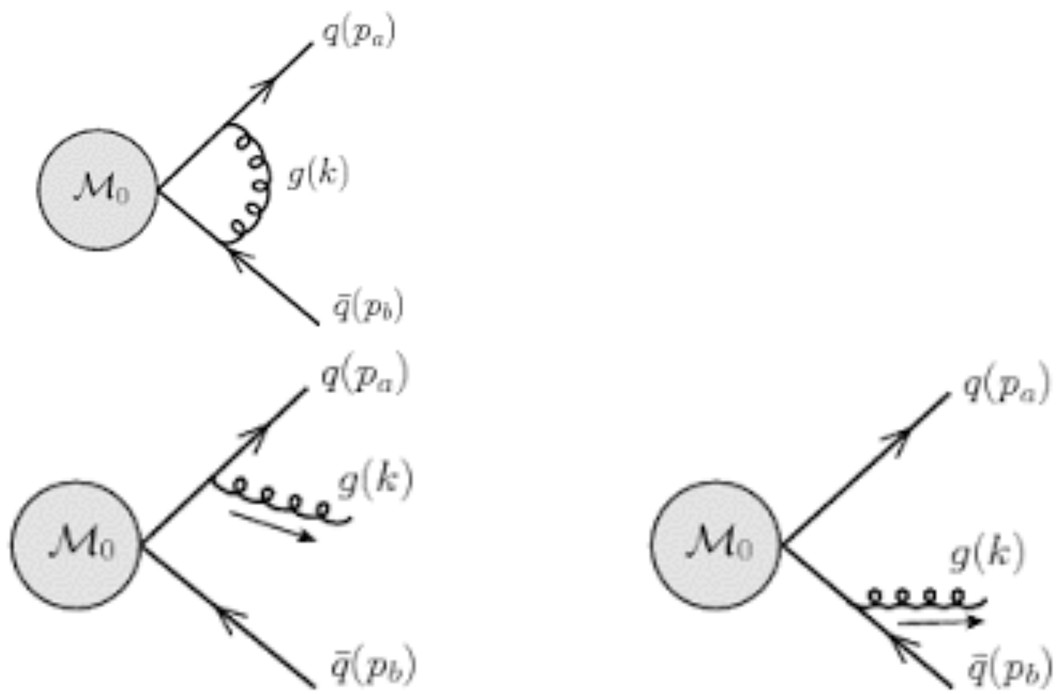
"Weinberg Fear"

Indistinguishable states

[Bloch, Nordsieck, Kinoshita,: Lee, Nauenberg]

If the detector is not sensitive to photons below certain energy Es (soft ones)

Below this energy the Detector can not distinguish these two processes when the gluons are soft/collinear



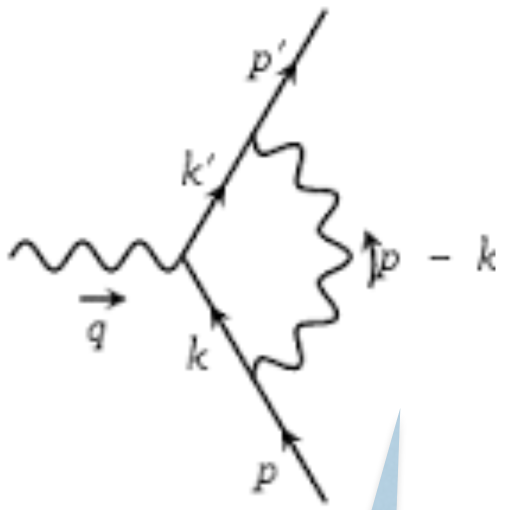
Indistinguishable when soft or collinear

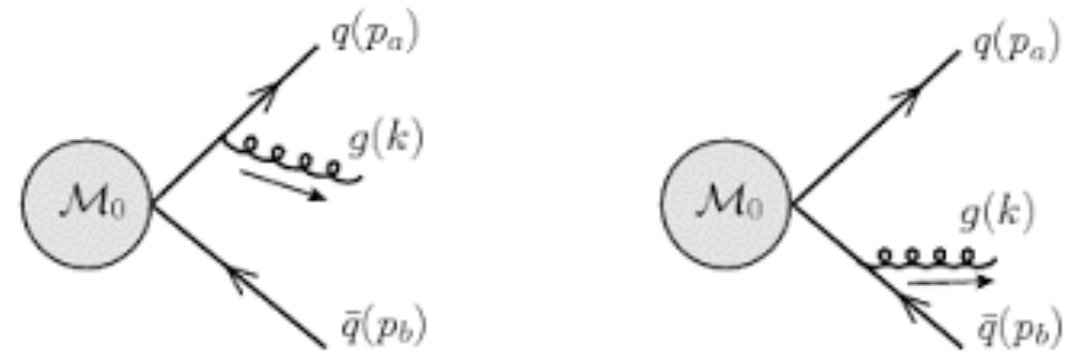
Sum their contributions and it is finite but dependent on Es !

IR contribution

[Bloch, Nordsieck,Yannie, Suura]

If the detector is not sensitive to photons below certain energy Es (soft ones)





$$|\mathcal{B}|^2 \exp\left\{-\frac{\alpha}{\pi}\mathcal{K}\ln\left(\frac{E}{\lambda}\right)\right\}$$

$$\exp\left\{+\frac{\alpha}{\pi}\mathcal{K}\ln\left(\frac{\Delta E}{\lambda}\right)\right\}$$

$$\exp\left\{-\frac{\alpha}{\pi}\mathcal{K}\ln\left(\frac{E}{\lambda}\right)\right\}\exp\left\{+\frac{\alpha}{\pi}\mathcal{K}\ln\left(\frac{\Delta E}{\lambda}\right)\right\} \longrightarrow \left(\frac{\Delta E}{E}\right)^{\alpha\mathcal{K}/\pi}$$

Probability with no energy loss is Zero

Infrared Safety

[Bloch, Nordsieck, Kinoshita,: Lee, Nauenberg]

Physical processes that happen at Long distances are responsible for these divergences.

Measurable quantities are not sensitive to soft and Collinear divergences

REASON

Long distance physics is associated to configurations that are experimentally indistinguishable

Infrared Safety

[Bloch, Nordsieck, Kinoshita,: Lee, Nauenberg]

Bloch and Nordsieck Theorem

Soft Singularities cancel between real and virtual processes when one adds up all states which are indistinguishable by virtue of the energy resolution of the apparatus.

$$\exp\left\{-\frac{\alpha}{\pi}\mathcal{K}\ln\left(\frac{E}{\lambda}\right)\right\}\exp\left\{+\frac{\alpha}{\pi}\mathcal{K}\ln\left(\frac{\Delta E}{\lambda}\right)\right\} \longrightarrow \left(\frac{\Delta E}{E}\right)^{\alpha\mathcal{K}/\pi}$$

Kinoshita, Lee and Nauenberg Theorem

Both soft and collinear singularities cancel when the summation is carried out among all the mass degenerate states.

Infrared Safety

[Kulish, Fadeev]

Alternate formalism in QED was proposed by Kulish and Fadeev:

Evolution operator can be factorised into Asymptotic and Regular ones

Fock states are dressed with soft photons giving Coherent states.

S-matrix elements between these Coherent states give IR finite results.

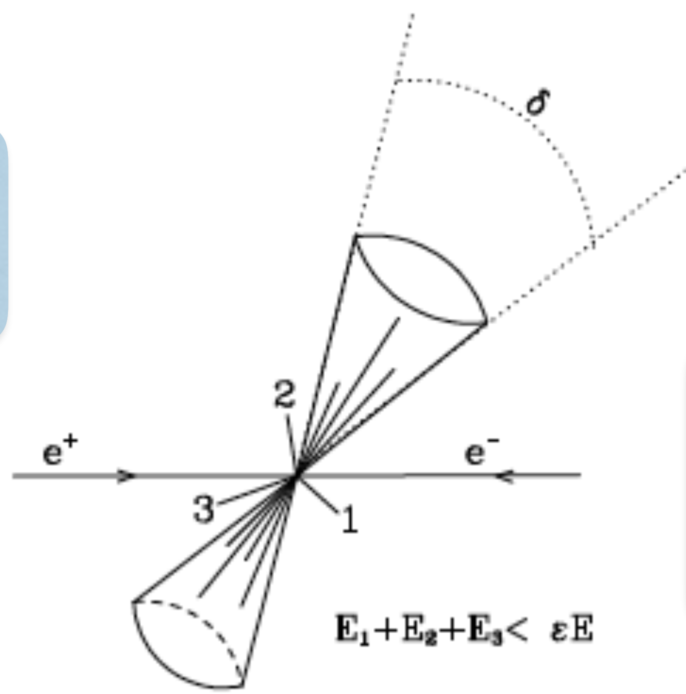
Sterman-Weinberg Jet in QCD

[Sterman, Weinberg]

Any event in electron-positron collision containing

Two cones of opening angle δ that contain all the energy of the event, excluding atmost ϵ fraction of the total.

Infra-red Safe

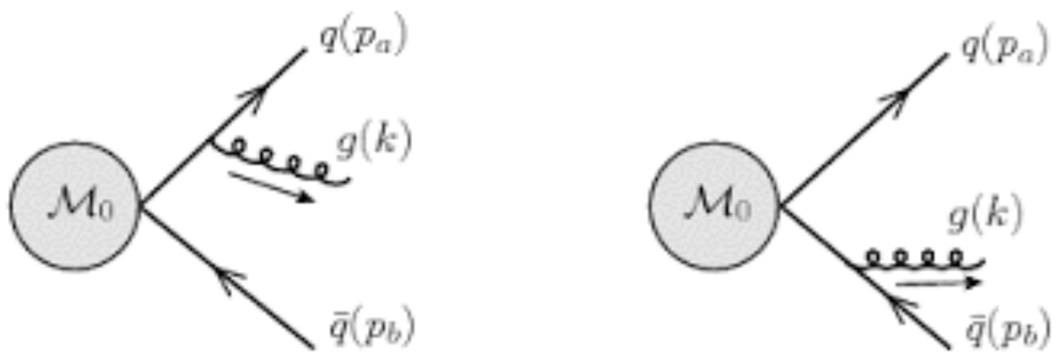


$$= \sigma_0 \left(1 - \frac{4\alpha_s C_F}{2\pi} \log \epsilon \log \delta^2 \right)$$

IR factorisation

[Yennie, Frautschi, Suura, Weinberg]

Eikonal approximation:



$$g_s T^a \frac{p_\mu}{p \cdot k + i\epsilon} \mathcal{M}_0^{\mu a}$$

Universal current

Born amplitude

Infrared divergences FACTORISE

Sudakov Equation (K+G Eqn.)

$$\mathcal{F}^{\lambda}_{\beta} =< \beta|\mathcal{O}^{\lambda}|\beta >$$

[Sen, Sterman,Magnea]

$$d = 4 + \varepsilon$$

$$Q^2 \frac{d}{dQ^2} \ln \mathcal{F}^{\lambda}_{\beta}(\hat{a}_s, Q^2, \mu^2, \epsilon) = \frac{1}{2}\left[K^{\lambda}_{\beta}(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon) + G^{\lambda}_{\beta}(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon)\right]$$

poles

No poles

RG invariance

$$\mu_R^2 \frac{d}{d\mu_R^2} K^{\lambda}_{\beta}(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon) = -\mu_R^2 \frac{d}{d\mu_R^2} G^{\lambda}_{\beta}(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon) = -A^{\lambda}_{\beta}(a_s(\mu_R^2))$$

Cusp (soft) Anomalous dim.

Casimir Duality

$$A_q = \frac{C_F}{C_A} A_g$$

Upto 3 loops

Single Pole mystery

[Ravindran, Smith, van Neerven; Moch et. al.]

UV Anomalous dim.

$$C^{\lambda}_{\beta,i} = \sum_j s_j C^{\lambda}_{\beta,j}, j < i$$

$$G^{\lambda}_{\beta,i}(\epsilon) = 2\left(B^{\lambda}_{\beta,i} - \gamma^{\lambda}_{\beta,i}\right) + f^{\lambda}_{\beta,i} + C^{\lambda}_{\beta,i} + \sum_{k=1}^{\infty} \epsilon^k g^{\lambda,k}_{\beta,i}$$

Collinear Anomalous dim.

Soft Anomalous dim.

Casimir Duality

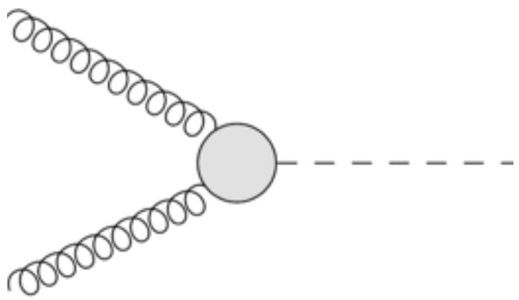
$$f_q = \frac{C_F}{C_A} f_g$$

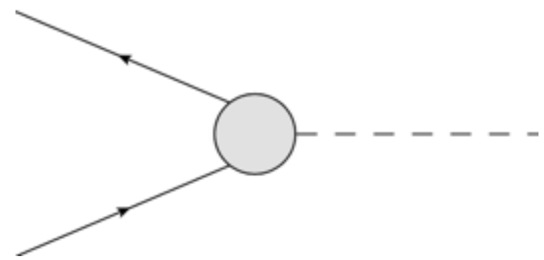
Upto 3 loops

Sudakov Form Factor

[Sudakov, Sen, Sterman, Collins, Magnea]

Large $q^2 = (p + p')^2$ behaviour of form factors





$$< e(p')|\overline{\psi}\gamma_\mu\psi|e(p) >$$

Sudakov:

$$\exp\left(-\frac{g^2}{8\pi^2}\ln^2(\frac{q^2}{m^2})\right)$$

Sen: QCD Leading and subleading logs exponentiate:

$$\left(\frac{g_s^2}{8\pi^2}\right)^n \ln^\nu\left(\frac{q^2}{m^2}\right),$$

$2\nu \leq n$

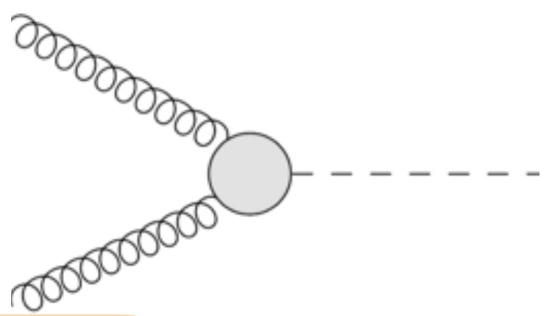
Coefficients are Universal

Form Factor

[Moch, Vogt, Vermaseren,VR, Smith, v Neerven]

Gluon form factor

$$< g(p')|G^a_{\mu\nu}G^{\mu\nu a}|g(p) >$$



At 3-loops

$$\gamma_q, \qquad \gamma_g$$

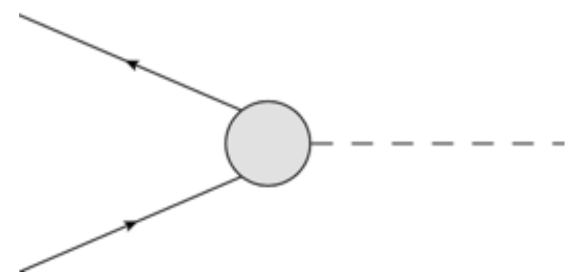
$$A_q = \frac{C_F}{C_A} A_g$$

$$f_q = \frac{C_F}{C_A} f_g$$

$$B_q, \qquad B_g$$

Quark form factor

$$< q(p')|\overline{\psi}\gamma_\mu\psi|q(p) >$$



Anomalous dimension

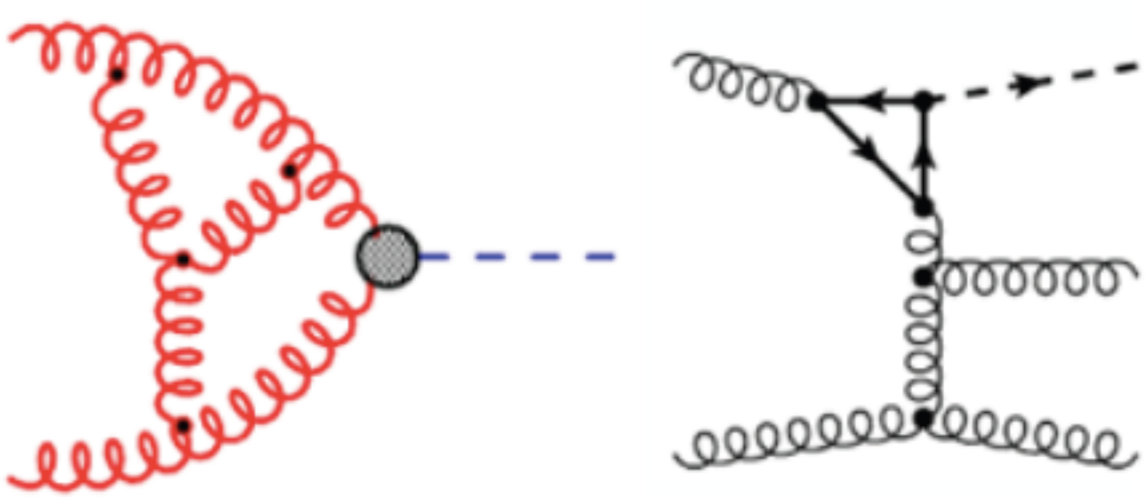
UV

Cusp

Soft

Collinear

Multi-loops and Multi-legs

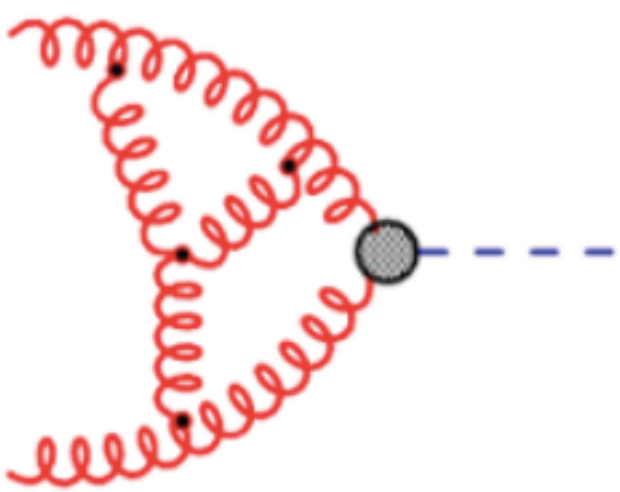


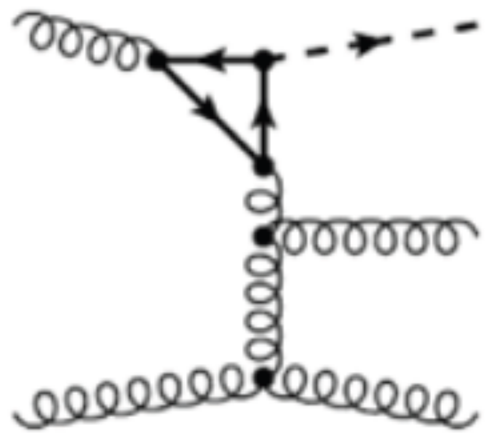
Catani's proposal

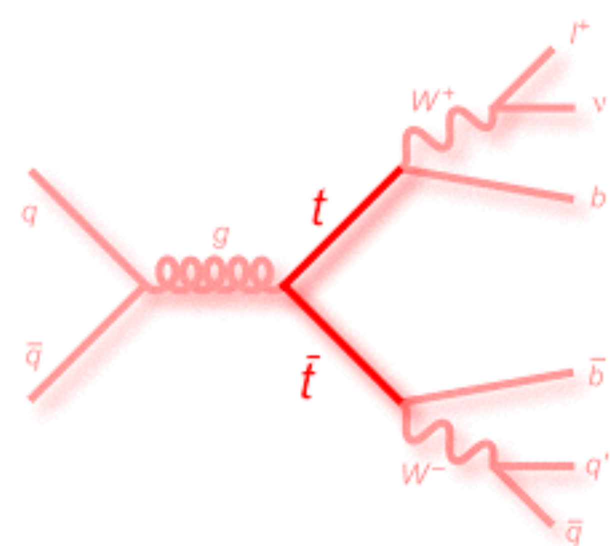
[Yennie, Frautschi, Subram; Weinberg]

UV Renormalised on-shell QCD amplitudes

$$|\mathcal{M}_n(\epsilon, \{p\})\rangle$$







Universal Infrared Structure

Catani's proposal

[Catani]

Upto Two loop !

$$\left[1-\frac{\alpha_s}{2\pi}\,\boldsymbol{I}^{(1)}(\epsilon)-\left(\frac{\alpha_s}{2\pi}\right)^2\boldsymbol{I}^{(2)}(\epsilon)\right]|\mathcal{M}_n(\epsilon,\{p\})\rangle$$

Universal IR Subtraction Operators

depend only on

Process independent

Soft and Collinear

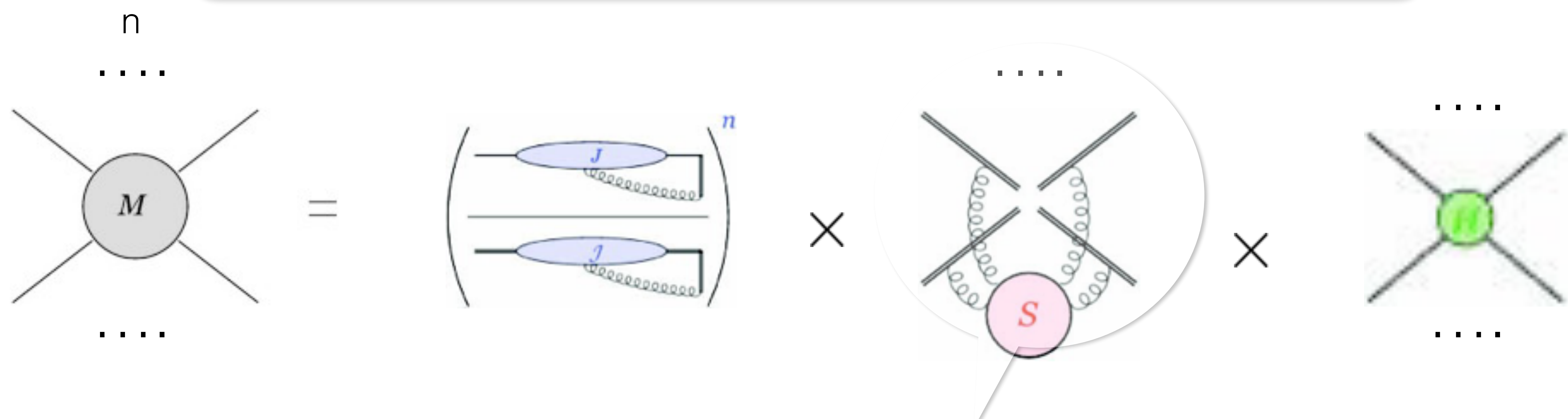
Anomalous Dimensions

Sterman's proof using factorisation

On-shell QCD amplitude in color basis:

[G. Sterman, M Tejeda-Yeomans]

$$\mathcal{M}^{[\mathrm{f}]}_{\{r_i\}}\left(\beta_j, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \sum_{L=1}^{N^{[\mathrm{f}]}} \mathcal{M}^{[\mathrm{f}]}_{L}\left(\beta_j, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) (c_L)_{\{r_i\}}$$



$$|\mathcal{M}_n(\epsilon, \{p\})\rangle = \prod_{i=1}^{n+2} J^{[i]}\left(\frac{Q'^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) S^{[\mathrm{f}]}_{LI}\left(\beta_j, \frac{Q'^2}{\mu^2}, \frac{Q'^2}{Q^2}, \alpha_s(\mu^2), \epsilon\right) H^{[\mathrm{f}]}_{I}\left(\beta_j, \frac{Q^2}{\mu^2}, \frac{Q'^2}{Q^2}, \alpha_s(\mu^2)\right)$$

Collinear Soft Hard

Three loop conjecture in QCD

[Becher, Neubert, Gardi, Magnea]

Matrix valued solution

$$\mathcal{Z}\left(\frac{p_i \cdot p_j}{\mu_f^2}, \alpha_s(\mu_f^2), \epsilon\right) = \mathcal{P}exp\left[-\int_0^{\mu_f^2} \frac{d\lambda}{\lambda} \Gamma\left(\frac{p_i \cdot p_j}{\lambda}, \alpha_s(\lambda)\right)\right]$$

Conjecture for IR anomalous dimension in QCD

$$\boldsymbol{\Gamma} = \sum_{(i,j)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2} \, \gamma_{\text{cusp}}(\alpha_s) \, \ln \frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)$$

Di-pole

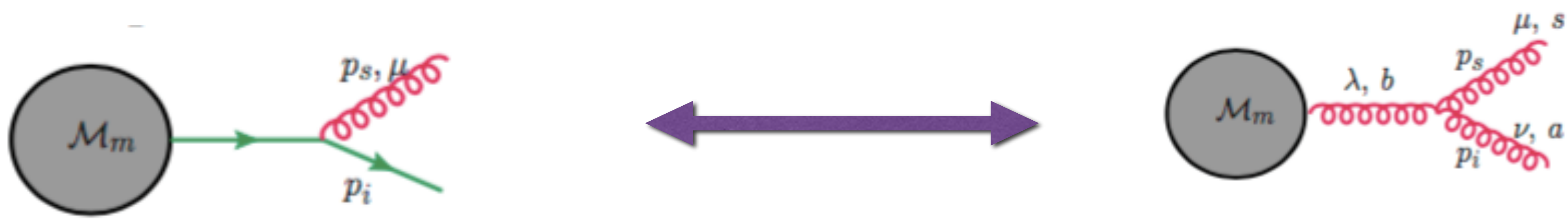
Soft

Soft +Collinear

Only Di-pole part Depends on Kinematics

Casimir Duality

Casimir Duality



Cusp Anomalous Dimension

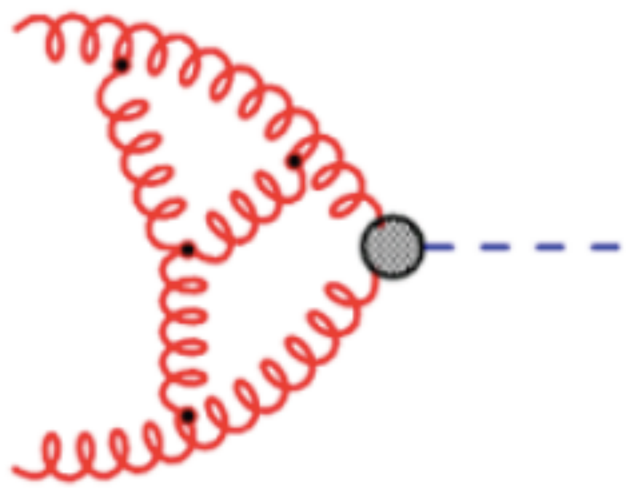
$$A_q = \frac{C_F}{C_A} A_g$$

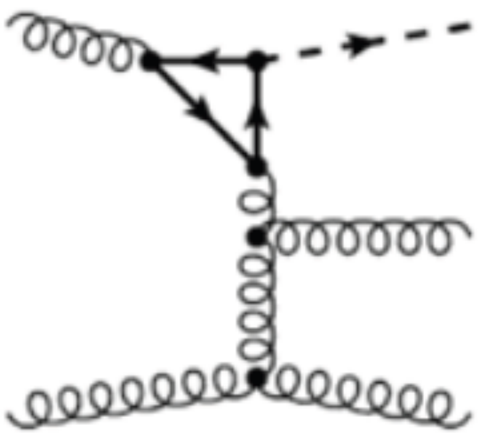
Soft Anomalous Matrix

$$\Gamma_q = \frac{C_F}{C_A} \Gamma_g$$

Upto 3- loops in QCD!

Multi-parton amplitude





Anomalous dimension

$\gamma_q, \quad \gamma_g$

$$A_q = \frac{C_F}{C_A} A_g$$

$$\Gamma_q = \frac{C_F}{C_A} \Gamma_g$$

$B_q, \quad B_g$

UV

Cusp

Soft Matrix

Collinear

Infrared to Ultraviolet

UV renormalisation of Composite operators

[Taushif,Narayan,VR]

- Even in Renormalised Quantum Field Theories Composite operators are often UV divergent:

$$\mathcal{O}(x) = \overline{\psi}(x)\psi(x),$$

$$\mathcal{O}(x) = G^a_{\mu\nu}(x)G^{a\mu\nu}(x)$$

- Multiple of fields at the same space time point gives additional short distance (UV) singularities

- Overall UV renormalisation Z is required for each composite operator

$$\mathcal{O}^R(x,\mu_R^2) = Z_{\mathcal{O}}(\alpha_s(\mu_R^2),\epsilon)\ \mathcal{O}(x)$$

UV and IR poles mix

[Taushif,Narayan,VR]

On-shell matrix elements between quark and gluon fields are relatively easy to compute, BUT

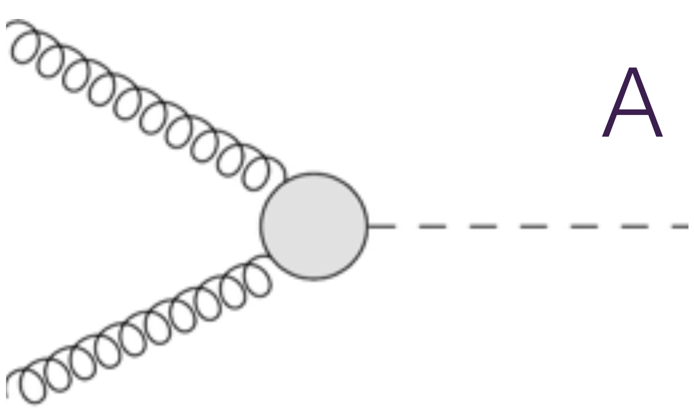
$$< a(p')|O|a(p) >, \quad a = q, \bar{q}, g$$

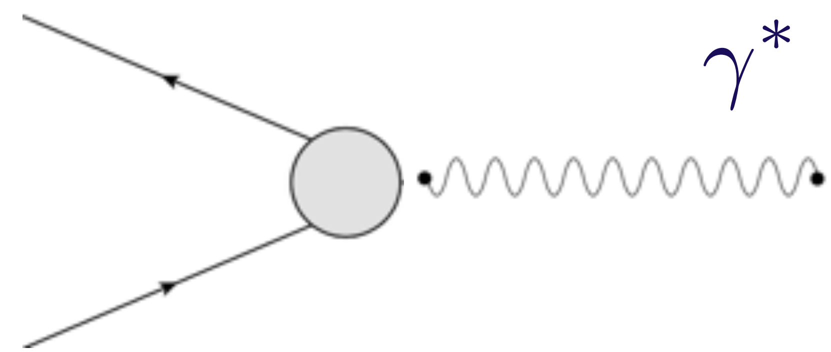
UV and IR poles mix in n-dimensions

Trick!

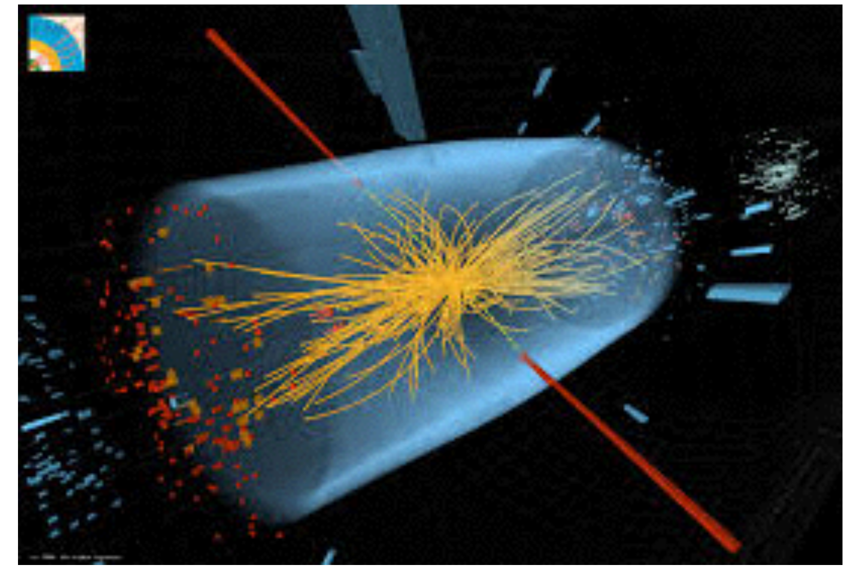
Exploit Univerality of IR poles → UV poles

Duality between Higgs and DY



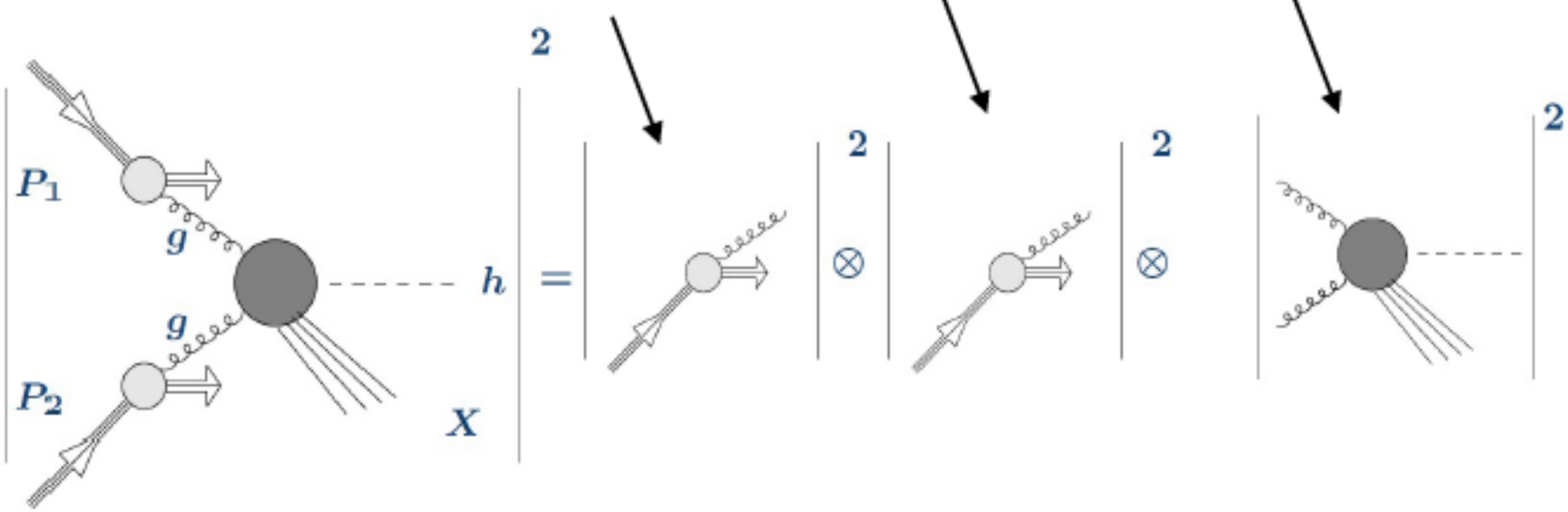


Physics at the LHC



$$P_1 + P_2 \to higgs + X$$

$$d\sigma^{P_1 P_2} = \sum \int dx_1 \int dx_2 f_{\frac{a}{P_1}}\left(x_1, \mu_F^2\right) f_{\frac{b}{P_2}}\left(x_2, \mu_F^2\right) d\hat{\sigma}^{ab}\left(x_1, x_2, \{p_i\}, \mu_F^2\right),$$



Parton Model in QCD

Inclusive cross section:

$$\sigma^A(\tau, m_A^2) = \sigma^{A,(0)}(\mu_R^2) \sum_{a,b=q,\bar{q},g} \int_\tau^1 dy\, \Phi_{ab}(y, \mu_F^2) \Delta_{ab}^A \left(\frac{\tau}{y}, m_A^2, \mu_R^2, \mu_F^2\right)$$

Partonic Flux:

$$\Phi_{ab}(y, \mu_F^2) = \int_y^1 \frac{dx}{x} f_a(x, \mu_F^2) f_b\left(\frac{y}{x}, \mu_F^2\right),$$

Partonic cross section:

$$\Delta_{ab}^A(z, q^2, \mu_R^2, \mu_F^2) = \Delta_{ab}^{A,\mathrm{SV}}(z, q^2, \mu_R^2, \mu_F^2) + \Delta_{ab}^{A,\mathrm{hard}}(z, q^2, \mu_R^2, \mu_F^2)$$

Soft + Virtual

Hard

Exponentiation

[VR]

RG invariance, K+G equation, Mass factorisation:

$$\Delta_g^{A,\mathrm{SV}}(z,q^2,\mu_R^2,\mu_F^2) = \mathcal{C}\exp\left(\Psi_g^A\left(z,q^2,\mu_R^2,\mu_F^2,\epsilon\right)\right)\Big|_{\epsilon=0}$$

α_s^3

$$\Psi_g^A\left(z,q^2,\mu_R^2,\mu_F^2,\epsilon\right) = \left(\ln\left[Z_g^A(\hat{a}_s,\mu_R^2,\mu^2,\epsilon)\right]^2 + \ln\left|\mathcal{F}_g^A(\hat{a}_s,Q^2,\mu^2,\epsilon)\right|^2\right)\delta(1-z)$$
$$+ 2\Phi_g^A(\hat{a}_s,q^2,\mu^2,z,\epsilon) - 2\mathcal{C}\ln\Gamma_{gg}(\hat{a}_s,\mu_F^2,\mu^2,z,\epsilon)\,.$$

	Divergences
• Z_g^A is operator renormalisation	UV
• $\mathcal{F}_g^A$ is the Form Factor	UV + Soft + Collinear
• Φ_g^A is the Soft distribution function	Soft
• Γ_{gg} is the Altarelli Parisi kernel	Initial state collinear

Sum Total = Finite

For Drell-Yan (DY)

[VR]

Higgs Production → Drell-Yan Production

$$\Delta_g^{A,\mathrm{SV}}(z,q^2,\mu_R^2,\mu_F^2) = \mathcal{C}\exp\left(\Psi_g^A\left(z,q^2,\mu_R^2,\mu_F^2,\epsilon\right)\right)\Big|_{\epsilon=0}$$

$$\Psi_g^A \rightarrow \Psi_q^{DY}$$

- Z_q^{DY} → Known
- $\mathcal{F}_q^{DY}$ → Known
- Γ_{qq}^{DY} → Known

α_s^3

IR Safety

$$\Phi_q^{DY^{(i)}} = \frac{C_F}{C_A}\Phi_g^{A(i)}$$

$i = 3$

Relations in $\mathcal{N} = 4$ SYM

[A.V.Kotikov,L.N.Lipatov,A.I.Onishchenko,V.N.Velizhanin,T. Gehrmann,J. Henn]

Leading Transcendentality Principle

- Set $C_A = C_F = N, T_f n_f = N/2$ for $SU(N)$
- Leading Transcendental (LT) parts of quark and gluon form factors in QCD are equal upto a factor 2^l
- LT part of quark and gluon form factors are identical to the scalar form factor in $\mathcal{N} = 4$ SYM
- LT part of pseudo scalar form factor is identical to quark and gluon form factors in QCD upto a factor 2^l also to scalar form factor in $\mathcal{N} = 4$ SYM

Conclusions

- Form Factors in Gague Theories
- Infrared Structure
  - Soft
  - Collinear
- Multi-leg, Multi-loop amplitudes
  - K+G equation
  - Catani's proposal
- Factorisation and Resummation
- Casimir Duality
- IR to UV and Drell-Yan