Discussion

on

Neutrino Mass Models

December 17, 2018

Neutrino Workshop, IIT Bombay

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  - Quark-Lepton unification?
  - Other UV dynamics: Flux compactification, Clockwork,… ?

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  - UV dynamics?
- UV origin of radiative neutrino mass schemes?

BACKUP SLIDES

Unbroken Residual Symmetries

Discrete Flavour Group G_f

G_f invariant $\mathcal{L}$

$$\mathcal{L} \ni Y_{ij}\bar{L}_L^i E_R^j H + \frac{Y'_{ij}}{\Lambda} L_L^i H L_L^j H + \text{ h.c.}$$

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$$G_f$$

$$G_l = Z_m$$

$$T_l \in G_l, \quad T_l^m = \mathbf{1}, \; m \geq 3$$

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$$|c_0| = |V_l^\dagger c_\nu|$$

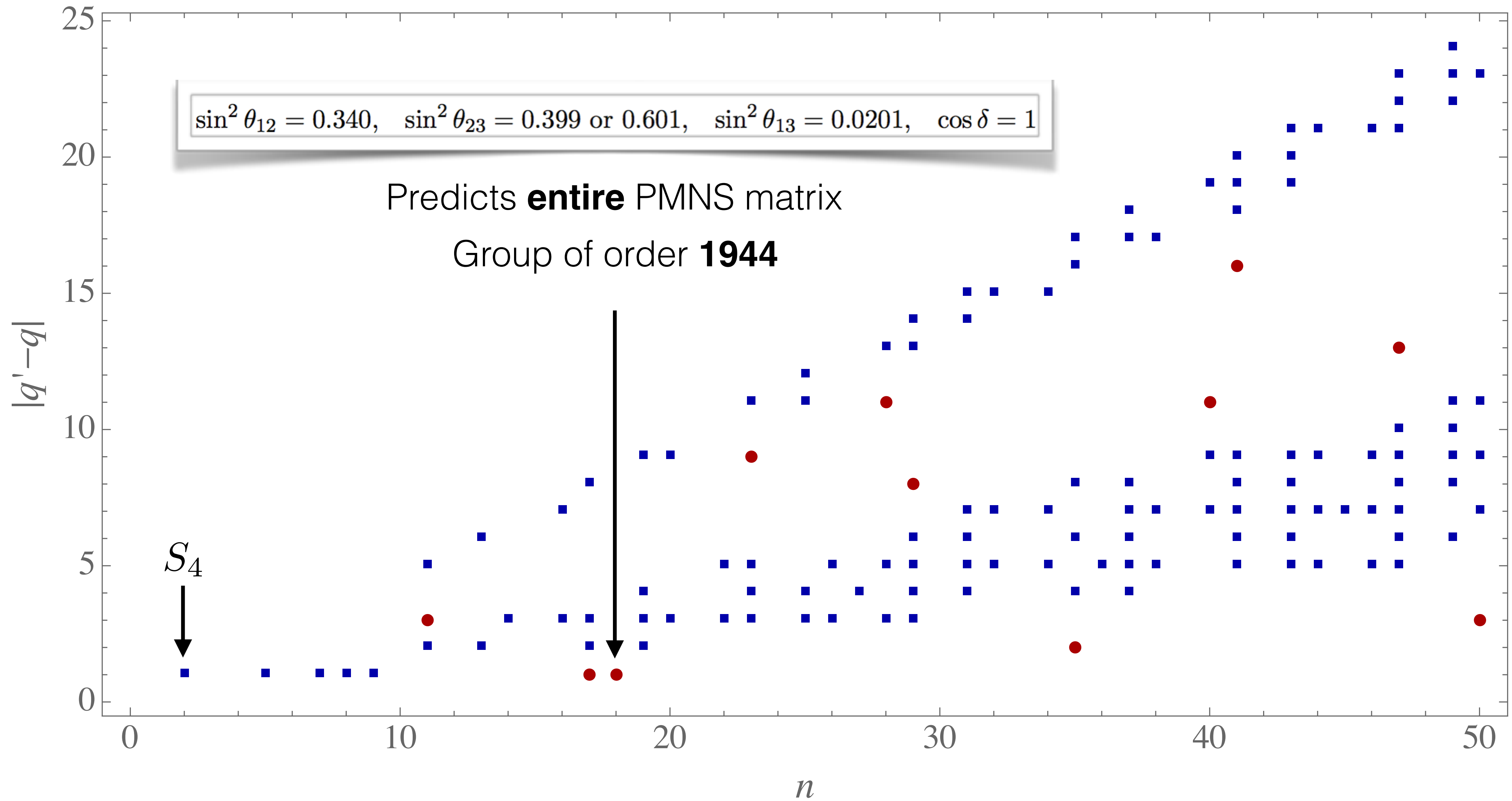
fixed column of PMNS matrix

Unbroken Residual Symmetries

Predictions for $G_f = \Delta(6n^2)$

$$|c_0|^2 = \frac{1}{6}\left(|1-\epsilon^{q'-q}|^2,\ |1-\omega\epsilon^{q'-q}|^2,\ |1-\omega^2\epsilon^{q'-q}|^2\right)$$

$$\epsilon = e^{2\pi i/n};\ q, q' = 0, 1, 2, ..., n-1.$$



can predict viable first or third column of PMNS matrix

Unbroken Residual Symmetries

$$|U| = \begin{pmatrix} 0.800 \to 0.844 & 0.515 \to 0.581 & 0.139 \to 0.155 \\ 0.229 \to 0.516 & 0.438 \to 0.699 & 0.614 \to 0.790 \\ 0.249 \to 0.528 & 0.462 \to 0.715 & 0.595 \to 0.776 \end{pmatrix}$$

Explicit model?

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Smallest Group: $\Delta(6 \times 2^2) = S_4$ $\quad \Delta(3 \times 2^2) = A_4$ $\quad \Delta(6 \times 11^2)$

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For entire PMNS matrix

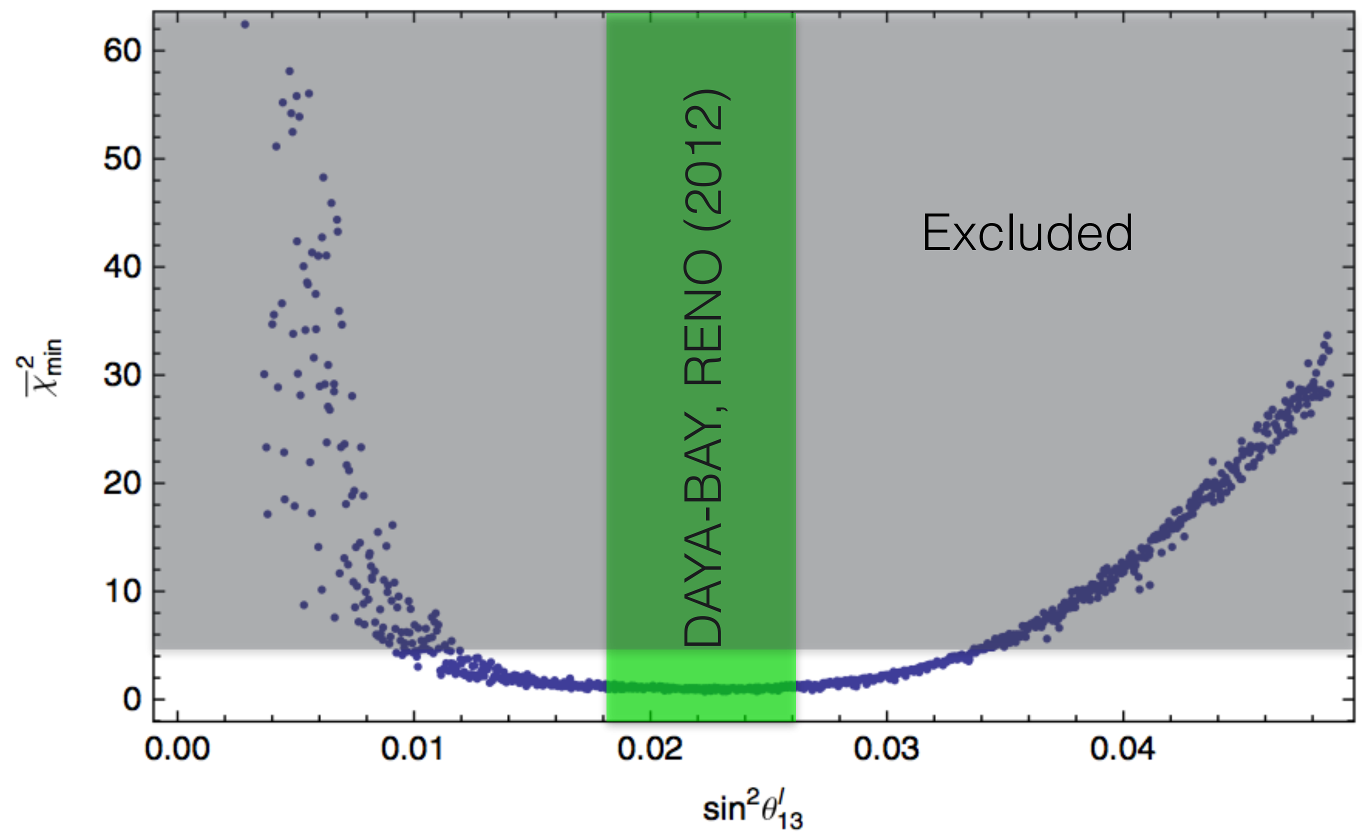
Smallest Group: $\Delta(6 \times 18^2)$

Explicit model?

An example: minimal SO(10) model

$$-\mathcal{L}_Y = 16_i \left(Y_{10}^{ij}\, 10_H + Y_{126}^{ij}\, \overline{126}_H \right) 16_j$$

$$Y_d^{ij} = c_1 Y_{10}^{ij} + c_2 Y_{126}^{ij}$$
$$Y_e^{ij} = c_1 Y_{10}^{ij} - 3c_2 Y_{126}^{ij}$$
$$Y_u^{ij} = d_1 Y_{10}^{ij} + d_2 Y_{126}^{ij}$$
$$Y_\nu^{ij} = d_1 Y_{10}^{ij} - 3d_2 Y_{126}^{ij}$$
$$M_{\nu^c}^{ij} = v_{B-L} Y_{126}^{ij}$$



Quark Lepton Unification

Effective 4D theory

$$\mathcal{Y}_u = F_{10} Y_u F_{10} \ , \qquad \mathcal{Y}_d = F_{10} Y_d F_{\bar{5}} \ , \qquad \mathcal{Y}_e = F_{\bar{5}} Y_e F_{10} \qquad m_\nu \propto F_{\bar{5}} \ Y_\nu Y_R^{-1} Y_\nu^T \ F_{\bar{5}}$$

$$Y_{u,d,l,\nu,R} = \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}$$

$$F_{10} = \begin{pmatrix} \lambda^a & 0 & 0 \\ 0 & \lambda^b & 0 \\ 0 & 0 & \lambda^c \end{pmatrix} \qquad F_{\bar{5}} = \begin{pmatrix} \lambda^x & 0 & 0 \\ 0 & \lambda^y & 0 \\ 0 & 0 & \lambda^z \end{pmatrix}$$

Quark Lepton Unification

Observable	Normal ordering Fitted value	Normal ordering Pull	Inverted ordering Fitted value	Inverted ordering Pull
y_t	0.51	0	0.54	1.00
y_b	0.37	0	0.37	0
y_τ	0.51	0	0.47	-1.00
m_u/m_c	0.0027	0	0.0031	0.67
m_d/m_s	0.051	0	0.045	-0.86
m_e/m_μ	0.0048	0	0.0048	0
m_c/m_t	0.0023	0	0.0023	0
m_s/m_b	0.016	0	0.015	-0.50
m_μ/m_τ	0.050	0	0.049	-0.50
$\lvert V_{us}\rvert$	0.227	0	0.227	0
$\lvert V_{cb}\rvert$	0.037	0	0.038	1.00
$\lvert V_{ub}\rvert$	0.0033	0	0.0030	-0.50
J_{CP}	0.000023	0	0.000021	-0.51
Δ_S/Δ_A	0.0309	0	0.0320	0.73
$\sin^2\theta_{12}$	0.308	0	0.309	0.06
$\sin^2\theta_{23}$	0.425	0	0.435	-0.07
$\sin^2\theta_{13}$	0.0234	0	0.0237	-0.10
$\chi^2_{\min}$		≈ 0		≈ 5.75
	Predicted value		**Predicted value**	
$m_{\nu_{\text{lightest}}}$ [meV]	0.08		2.15	
$\lvert m_{\beta\beta}\rvert$ [meV]	1.63		30.4	
$\sin\delta^l_{CP}$	0.265		0.510	
M_{N_1} [GeV]	3.85×10^6		1.13×10^4	
M_{N_2} [GeV]	9.31×10^7		3.06×10^6	
M_{N_3} [GeV]	2.19×10^{14}		2.02×10^{13}	
v_R [GeV]	0.05×10^{16}		0.18×10^{16}	

Fitted Profiles

Normal Ordering

$$F_{10}=\lambda^{0.3}\begin{pmatrix}\lambda^{3.7}&0&0\\0&\lambda^{2.4}&0\\0&0&1\end{pmatrix}$$

$$F_{\bar{5}}=\lambda^{0.3}\begin{pmatrix}\lambda^{1.5}&0&0\\0&\lambda^{0.9}&0\\0&0&1\end{pmatrix}$$

$$F_{1}=\lambda^{0.4}\begin{pmatrix}\lambda^{6.2}&0&0\\0&\lambda^{4.8}&0\\0&0&1\end{pmatrix}$$

Inverted Ordering

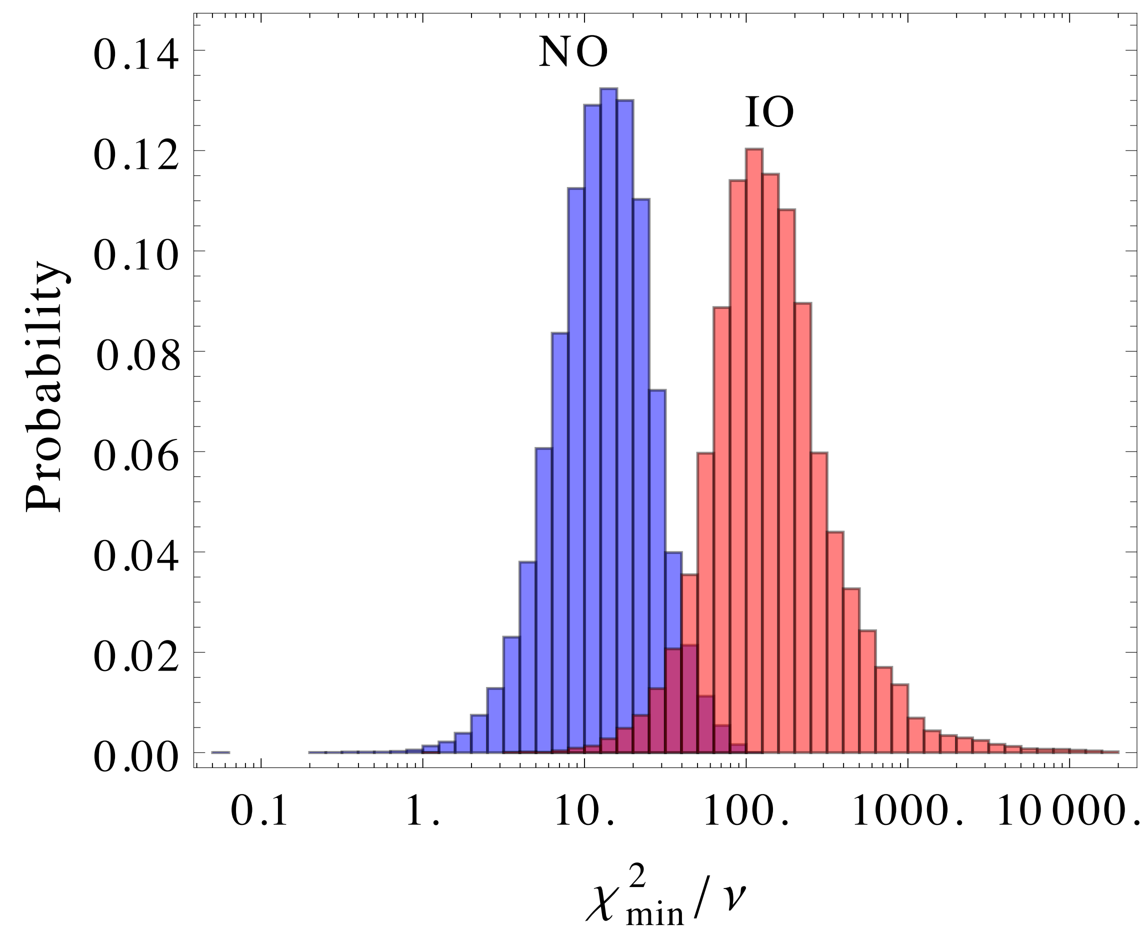
$$F_{10}=\lambda^{0.7}\begin{pmatrix}\lambda^{3.9}&0&0\\0&\lambda^{2.2}&0\\0&0&1\end{pmatrix}$$

$$F_{\bar{5}}=\lambda^{0.4}\begin{pmatrix}\lambda^{0.8}&0&0\\0&\lambda^{0.4}&0\\0&0&1\end{pmatrix}$$

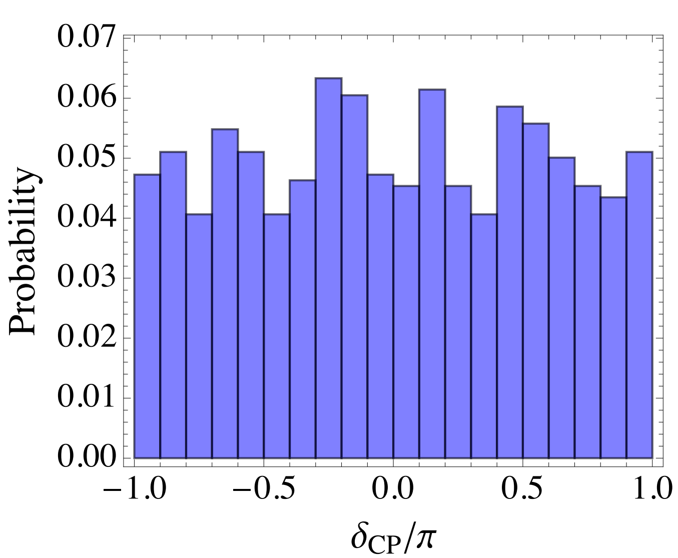
$$F_{1}=\lambda^{1.5}\begin{pmatrix}\lambda^{7.4}&0&0\\0&\lambda^{5.5}&0\\0&0&1\end{pmatrix}$$

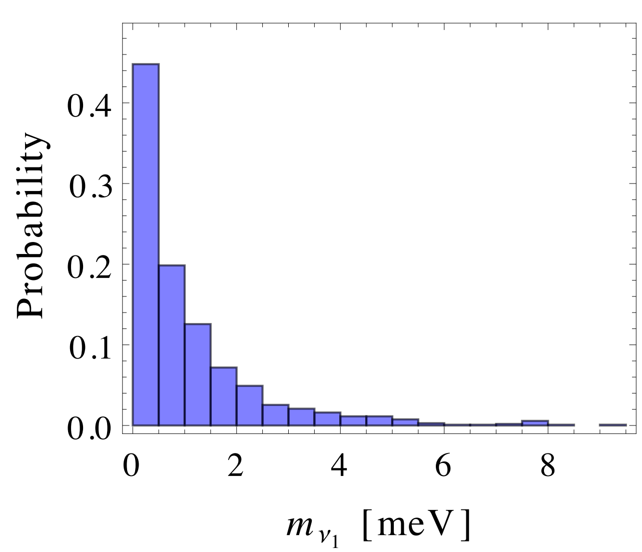
Quark Lepton Unification

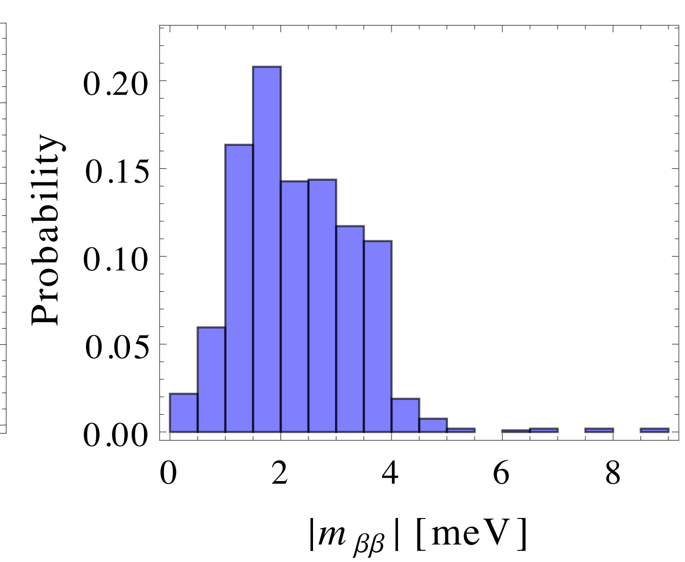
Random $\mathcal{O}(1)$ Yukawas

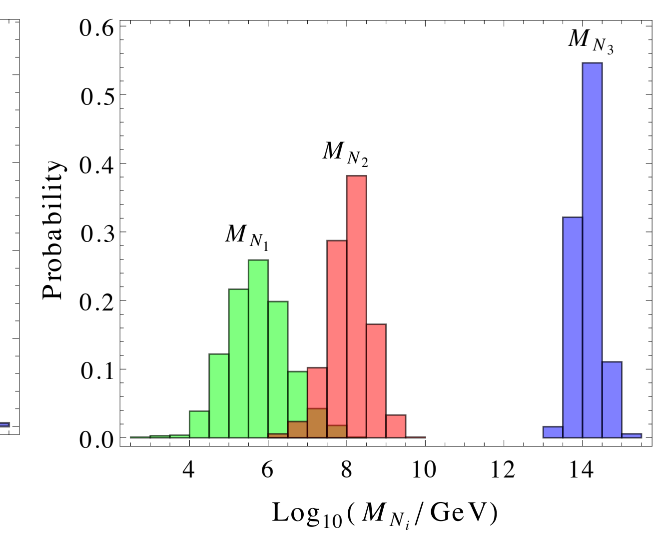


$$F_{10} \simeq \lambda^{0.4}\, \texttt{diag.}(\lambda^{4.1}, \lambda^{2.2}, 1)$$
$$F_{\bar{5}} \simeq \lambda^{0.3}\, \texttt{diag.}(\lambda^{1.5}, \lambda^{0.7}, 1)$$
$$F_{1} \simeq \lambda^{0.6}\, \texttt{diag.}(\lambda^{6.8}, \lambda^{4.9}, 1)$$









Abelian magnetic flux

- An additional U(1) gauge symmetry and nonzero constant flux

$$F_{56} = \partial_5 A_6 - \partial_6 A_5 \equiv f$$

$$A_5(y) = -f y_2$$

$$A_5(t_m y) = A_5(y) - \partial_5 \Lambda(y)$$

$$\Lambda(t_m y) - \Lambda(y) = \int_y^{y+L} \partial_5 \Lambda(y) = \int_0^L f L dy_1 \equiv \frac{2\pi N}{gq}$$

$$\frac{qg}{2\pi} L^2 f = N \, , \quad N \in \mathbb{Z}$$

Flux is quantised!

- Bulk 16-plet with charge q and flux f give rise to N massless modes

Flavour from Flux Compactifiation

The massless modes

$$\psi \to (q_i, u_i^c, d_i^c, l_i, e_i^c, n_i^c) \quad + \quad \text{parity even fields}$$

$$\psi = \sum_{i=1}^{N} \left[q_i \psi_{-+}^{(i)} + l_i \psi_{--}^{(i)} + (d_i^c + n_i^c)\, \psi_{+-}^{(i)} \right] + \sum_{\alpha=1}^{N+1} (u_\alpha^c + e_\alpha^c)\, \psi_{++}^{(\alpha)}$$

$$\psi^{(j)}_{\eta_{\rm PS}\eta_{\rm GG}}(y) = \mathcal{N}'\, e^{-2\pi N y_2^2} \sum_{n\in\mathbb{Z}} e^{-2\pi N\left(n-\frac{j}{2N}\right)^2 - i\pi\left(n-\frac{j}{2N}\right)(ik_{\rm PS}-k_{\rm GG})}$$
$$\times \cos\left[2\pi\left(-2nN + j + \frac{k_{\rm PS}}{2}\right)(y_1 + iy_2)\right] \,,$$
$$\eta_{PS} = e^{i\pi k_{\rm PS}}\,, \quad \eta_{\rm GG} = e^{i\pi k_{GG}}\,, \quad k_{\rm PS}, k_{\rm GG} = 0, 1\,.$$

Quark-Lepton unification is preserved!

Yukawa sector

$$
\begin{aligned}
W_{\mathrm{Y}} = \delta_{\mathrm{I}} \Bigg[& \left(\frac{1}{2} y^{\mathrm{I}}_{ua}\, \psi\psi + y^{\mathrm{I}}_{ub}\, \psi\chi + \frac{1}{2} y^{\mathrm{I}}_{uc}\, \chi\chi\right) H_1 \\
& + \left(\frac{1}{2} y^{\mathrm{I}}_{da}\, \psi\psi + y^{\mathrm{I}}_{db}\, \psi\chi + \frac{1}{2} y^{\mathrm{I}}_{dc}\, \chi\chi\right) H_2 \\
& \Bigg] \\
& + \delta_{\mathrm{PS}} \\
& + \delta_{\mathrm{GG}} \left(\frac{1}{2} y^{\mathrm{GG}}_{ua} 10_{\psi} 10_{\psi} H_5 + y^{\mathrm{GG}}_{db} 10_{\psi} 5^*_{\chi} H_{5^*} \right) \\
& + \delta_{\mathrm{fl}} \left(y^{\mathrm{fl}}_{ea} \tilde{5}^*_{\psi} \tilde{1}_{\psi} H_{\tilde{5}} + y^{\mathrm{fl}}_{ub} \tilde{5}^*_{\psi} \tilde{10}_{\chi} H_{\tilde{5}^*} + \frac{1}{2} y^{\mathrm{fl}}_{dc} \tilde{10}_{\chi} \tilde{10}_{\chi} H_{\tilde{5}} \right) ,
\end{aligned}
$$

Yukawa sector

$$W_{\mathrm{Y}} = \delta_{\mathrm{I}} \left[\left(\frac{1}{2} y^{\mathrm{I}}_{ua} \psi\psi + y^{\mathrm{I}}_{ub} \psi\chi + \frac{1}{2} y^{\mathrm{I}}_{uc} \chi\chi \right) H_1 + \left(\frac{1}{2} y^{\mathrm{I}}_{da} \psi\psi + y^{\mathrm{I}}_{db} \psi\chi + \frac{1}{2} y^{\mathrm{I}}_{dc} \chi\chi \right) H_2 \right]$$

$$+ \delta_{\mathrm{PS}}$$

$$+ \delta_{\mathrm{GG}} \left(\frac{1}{2} y^{\mathrm{GG}}_{ua} 10_\psi 10_\psi H_5 + y^{\mathrm{GG}}_{db} 10_\psi 5^*_\chi H_{5^*} \right)$$

$$+ \delta_{\mathrm{fl}} \left(y^{\mathrm{fl}}_{ea} \tilde{5}^*_\psi \tilde{1}_\psi H_{\tilde{5}} + y^{\mathrm{fl}}_{ub} \tilde{5}^*_\psi \tilde{10}_\chi H_{\tilde{5}^*} + \frac{1}{2} y^{\mathrm{fl}}_{dc} \tilde{10}_\chi \tilde{10}_\chi H_{\tilde{5}} \right),$$

11 complex parameters,
Consistent with
charged fermion spectrum

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& + \left(\frac{1}{2} y^{\mathrm{I}}_{da}\, \psi\psi + y^{\mathrm{I}}_{db}\, \psi\chi + \frac{1}{2} y^{\mathrm{I}}_{dc}\, \chi\chi \right) H_2 \\
& + \left(\frac{1}{2} y^{\mathrm{I}}_{na}\, \psi\psi + y^{\mathrm{I}}_{nb}\, \psi\chi + \frac{1}{2} y^{\mathrm{I}}_{nc}\, \chi\chi \right) \Psi^c \Psi^c \Bigg] \\
+ \delta_{\mathrm{PS}} & \left(\frac{1}{2} y^{\mathrm{PS}}_{na}\, 4^*_\psi 4^*_\psi + y^{\mathrm{PS}}_{nb}\, 4^*_\psi 4^*_\chi + \frac{1}{2} y^{\mathrm{PS}}_{nc}\, 4^*_\chi 4^*_\chi \right) F F \\
+ \delta_{\mathrm{GG}} & \left(\frac{1}{2} y^{\mathrm{GG}}_{ua} 10_\psi 10_\psi H_5 + y^{\mathrm{GG}}_{db} 10_\psi 5^*_\chi H_{5^*} + y^{\mathrm{GG}}_{\nu c} 5^*_\chi 1_\chi H_5 + \frac{1}{2} y^{\mathrm{GG}}_{nc} 1_\chi 1_\chi N N \right) \\
+ \delta_{\mathrm{fl}} & \left(y^{\mathrm{fl}}_{ea} \tilde{5}^*_\psi \tilde{1}_\psi H_{\tilde{5}} + y^{\mathrm{fl}}_{ub} \tilde{5}^*_\psi \tilde{10}_\chi H_{\tilde{5}^*} + \frac{1}{2} y^{\mathrm{fl}}_{dc} \tilde{10}_\chi \tilde{10}_\chi H_{\tilde{5}} + \frac{1}{2} y^{\mathrm{fl}}_{nc} \tilde{10}_\chi \tilde{10}_\chi \tilde{T}^* \tilde{T}^* \right) ,
\end{aligned}
$$

11 complex parameters,
Consistent with
charged fermion spectrum

Yukawa sector

$$
\begin{aligned}
W_{\rm Y} = \delta_{\rm I}\, \Bigg[& \left(\frac{1}{2} y^{\rm I}_{ua}\, \psi\psi + y^{\rm I}_{ub}\, \psi\chi + \frac{1}{2} y^{\rm I}_{uc}\, \chi\chi\right) H_1 \\
& + \left(\frac{1}{2} y^{\rm I}_{da}\, \psi\psi + y^{\rm I}_{db}\, \psi\chi + \frac{1}{2} y^{\rm I}_{dc}\, \chi\chi\right) H_2 \\
& + \left(\frac{1}{2} y^{\rm I}_{na}\, \psi\psi + y^{\rm I}_{nb}\, \psi\chi + \frac{1}{2} y^{\rm I}_{nc}\, \chi\chi\right) \Psi^c \Psi^c \Bigg] \\
& + \delta_{\rm PS} \left(\frac{1}{2} y^{\rm PS}_{na}\, 4^*_\psi 4^*_\psi \,+\, y^{\rm PS}_{nb}\, 4^*_\psi 4^*_\chi \,+\, \frac{1}{2} y^{\rm PS}_{nc}\, 4^*_\chi 4^*_\chi\right) FF \\
& + \delta_{\rm GG} \left(\frac{1}{2} y^{\rm GG}_{ua} 10_\psi 10_\psi H_5 \,+ y^{\rm GG}_{db} 10_\psi 5^*_\chi H_{5^*} \,+ y^{\rm GG}_{\nu c} 5^*_\chi 1_\chi H_5 \,+ \frac{1}{2} y^{\rm GG}_{nc} 1_\chi 1_\chi N N\right) \\
& + \delta_{\rm fl} \left(y^{\rm fl}_{ea} \tilde{5}^*_\psi \tilde{1}_\psi H_{\tilde{5}} \,+ y^{\rm fl}_{ub} \tilde{5}^*_\psi \widetilde{10}_\chi H_{\tilde{5}^*} \,+ \frac{1}{2} y^{\rm fl}_{dc} \widetilde{10}_\chi \widetilde{10}_\chi H_{\tilde{5}} \,+ \frac{1}{2} y^{\rm fl}_{nc} \widetilde{10}_\chi \widetilde{10}_\chi \tilde{T}^* \tilde{T}^*\right) \ ,
\end{aligned}
$$

Yukawa sector

$$
\begin{aligned}
W_{\mathrm{Y}} = \delta_{\mathrm{I}} \Bigg[& \left(\frac{1}{2} y^{\mathrm{I}}_{ua}\, \psi\psi + y^{\mathrm{I}}_{ub}\, \psi\chi + \frac{1}{2} y^{\mathrm{I}}_{uc}\, \chi\chi \right) H_1 \\
& + \left(\frac{1}{2} y^{\mathrm{I}}_{da}\, \psi\psi + y^{\mathrm{I}}_{db}\, \psi\chi + \frac{1}{2} y^{\mathrm{I}}_{dc}\, \chi\chi \right) H_2 \\
& + \left(\frac{1}{2} y^{\mathrm{I}}_{na}\, \psi\psi + y^{\mathrm{I}}_{nb}\, \psi\chi + \frac{1}{2} y^{\mathrm{I}}_{nc}\, \chi\chi \right) \Psi^c \Psi^c \Bigg] \\
+ \delta_{\mathrm{PS}} & \left(\frac{1}{2} y^{\mathrm{PS}}_{na}\, 4^*_\psi 4^*_\psi \; + \; y^{\mathrm{PS}}_{nb}\, 4^*_\psi 4^*_\chi \; + \; \frac{1}{2} y^{\mathrm{PS}}_{nc}\, 4^*_\chi 4^*_\chi \right) \; FF \\
+ \delta_{\mathrm{GG}} & \left(\frac{1}{2} y^{\mathrm{GG}}_{ua} 10_\psi 10_\psi H_5 \; + y^{\mathrm{GG}}_{db} 10_\psi 5^*_\chi H_{5^*} \; + y^{\mathrm{GG}}_{\nu c} 5^*_\chi 1_\chi H_5 \; + \frac{1}{2} y^{\mathrm{GG}}_{nc} 1_\chi 1_\chi N N \right) \\
+ \delta_{\mathrm{fl}} & \left(y^{\mathrm{fl}}_{ea} \tilde{5}^*_\psi \tilde{1}_\psi H_{\tilde{5}} \; + y^{\mathrm{fl}}_{ub} \tilde{5}^*_\psi \tilde{10}_\chi H_{\tilde{5}^*} \; + \frac{1}{2} y^{\mathrm{fl}}_{dc} \tilde{10}_\chi \tilde{10}_\chi H_{\tilde{5}} \; + \frac{1}{2} y^{\mathrm{fl}}_{nc} \tilde{10}_\chi \tilde{10}_\chi \tilde{T}^* \tilde{T}^* \right) \; ,
\end{aligned}
$$

$$
W_{\mathrm{mix}} = \sum_{p=\mathrm{I,PS,GG,fl}} \delta_p \left(\mu^p_a \psi^c \psi + \mu^p_b \psi^c \chi + \mu^p_c \chi^c \chi + \mu^p_d \chi^c \psi \right)
$$

Fermion mass spectrum

Observables	O^{th}	O^{exp}	Deviations (in %)
m_u [GeV]	0.00048	0.00048	0
m_c [GeV]	0.23	0.23	0
m_t [GeV]	74.1	74.1	0
m_d [GeV]	0.00096	0.00113	-15
m_s [GeV]	0.018	0.021	-18
m_b [GeV]	1.16	1.16	0
m_e [GeV]	0.00051	0.00044	16
m_μ [GeV]	0.094	0.093	1
m_τ [GeV]	1.61	1.61	0
m^2_{sol} [eV^2]	0.000075	0.000075	0
m^2_{atm} [eV^2]	0.0025	0.0025	0
V_{us}	0.23	0.23	0
V_{cb}	0.041	0.041	0
V_{ub}	0.0035	0.0035	0
$\sin^2\theta_{12}$	0.31	0.31	0
$\sin^2\theta_{23}$	0.44	0.44	0
$\sin^2\theta_{13}$	0.022	0.022	0
J^Q_{CP}	0.000030	0.000030	0
δ_{MNS} [^{o}]	279	261	7
η_B	6.1×10^{-10}	6.1×10^{-10}	0

Predictions			
α_{21} [^{o}]	129	M_{N_1} [GeV]	1.3×10^{12}
α_{31} [^{o}]	353	M_{N_2} [GeV]	2.0×10^{14}
m_{ν_1} [eV]	0.0017	M_{N_3} [GeV]	3.5×10^{14}
$m_{\beta\beta}$ [eV]	0.0026	M_{N_4} [GeV]	3.7×10^{14}
m_β [eV]	0.0089	M_{N_5} [GeV]	4.6×10^{14}

Higgs Naturalness

$$|\delta\mu^2| \approx \frac{1}{4\pi^2}\sum_{i,\alpha}|(Y_D)_{i\alpha}|^2 M_{N_\alpha}^2$$

$$|(Y_D)_{i\alpha}| M_{N_\alpha} \leq \mathcal{O}(\mathrm{TeV})$$

$$\frac{M_N^3 m_\nu}{4\pi^2 \langle\phi\rangle^2} \lesssim (\mathrm{TeV})^2 \quad \Rightarrow \quad M_N \lesssim 2.9\times 10^7 \times \left(\frac{\sqrt{m_{\mathrm{atm}}^2}}{m_\nu}\right)^{1/3} \ \mathrm{GeV}$$

$$Y_D = \begin{pmatrix} y_1 & \pm i y_1 & 0 \\ y_2 & \pm i y_2 & 0 \\ y_3 & \pm i y_3 & 0 \end{pmatrix} \quad \text{and} \quad M_N = \mathrm{Diag.}(M, M, M_3)$$