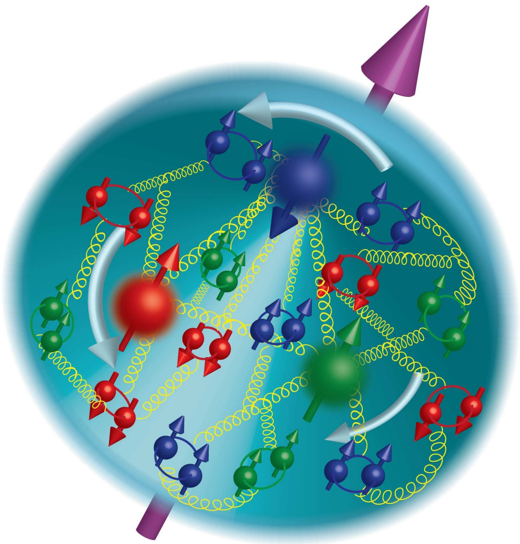

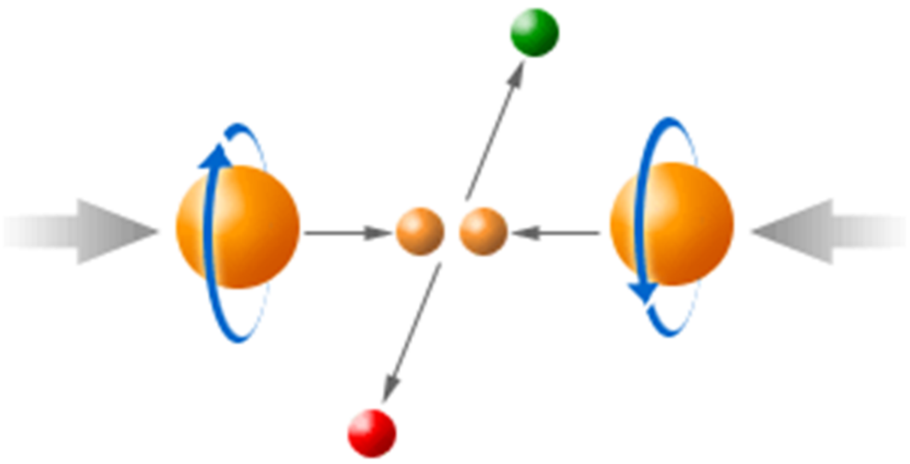

Understanding the glue that binds us all

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University of Groningen, The Netherlands

THEP group, Indian Institute of Technology Bombay, Mumbai, April 20, 2022

Overview

- Gluons - a general introduction
- Gluon PDFs - unpolarized & polarized cases
- Gluon TMDs
- QCD in high energy scattering - gauge links and loops
- Gluon GPDs
- Gluon GTMDs

Gluons - general overview

2022: 50 years of gluons

- 1972 - theoretical proposal of gluons as carriers of the strong force, birth of QCD
  Fritzsch, Gell-Mann, 1972; paper with Leutwyler, 1973

- 1979 - first experimental evidence for gluons from e^+e^- collisions at the DORIS and PETRA storage rings at DESY, Hamburg

In June 1979 the first evidence for gluons was presented at the Geneva International Conference: 3-gluon decay of the ϒ(9.46) particle (PLUTO experiment at DORIS) and 3-jet events ($q\bar{q}g$) (experiments at PETRA)

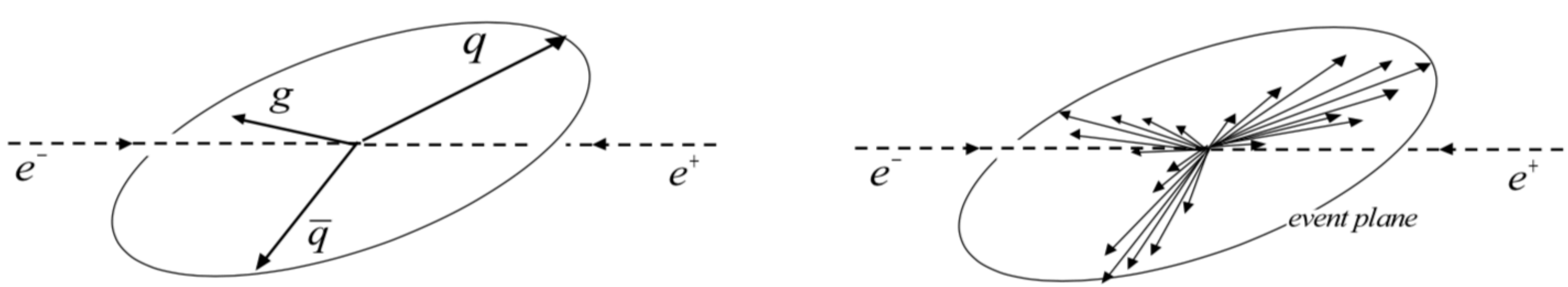
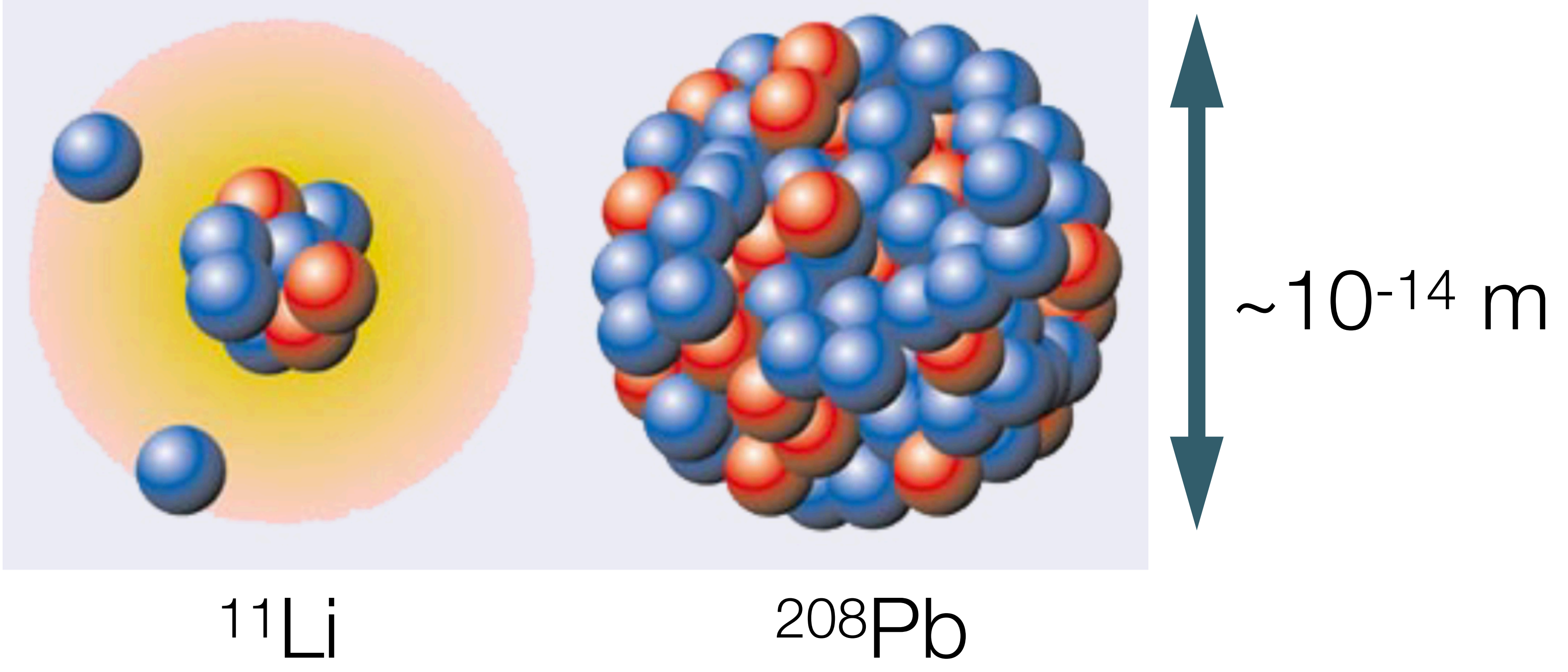


Figure from the review by P. Söding, EPJH 35 (2010) 3

The strong nuclear force

The force binding protons and neutrons into nuclei of atoms



The force is extremely short range

The gluons are the carriers of the strong force

QCD - the microscopic theory of quarks and gluons

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(i\,\not{\partial} - g\,\not{A}_a T^a - m)\psi$$

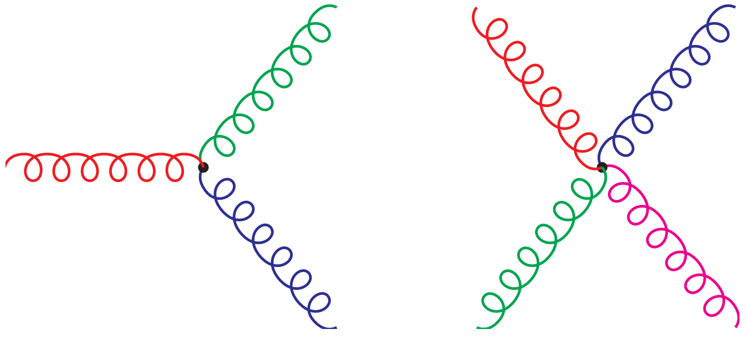
$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - g f_{abc} A_b^\mu A_c^\nu$$

a=1,...,8

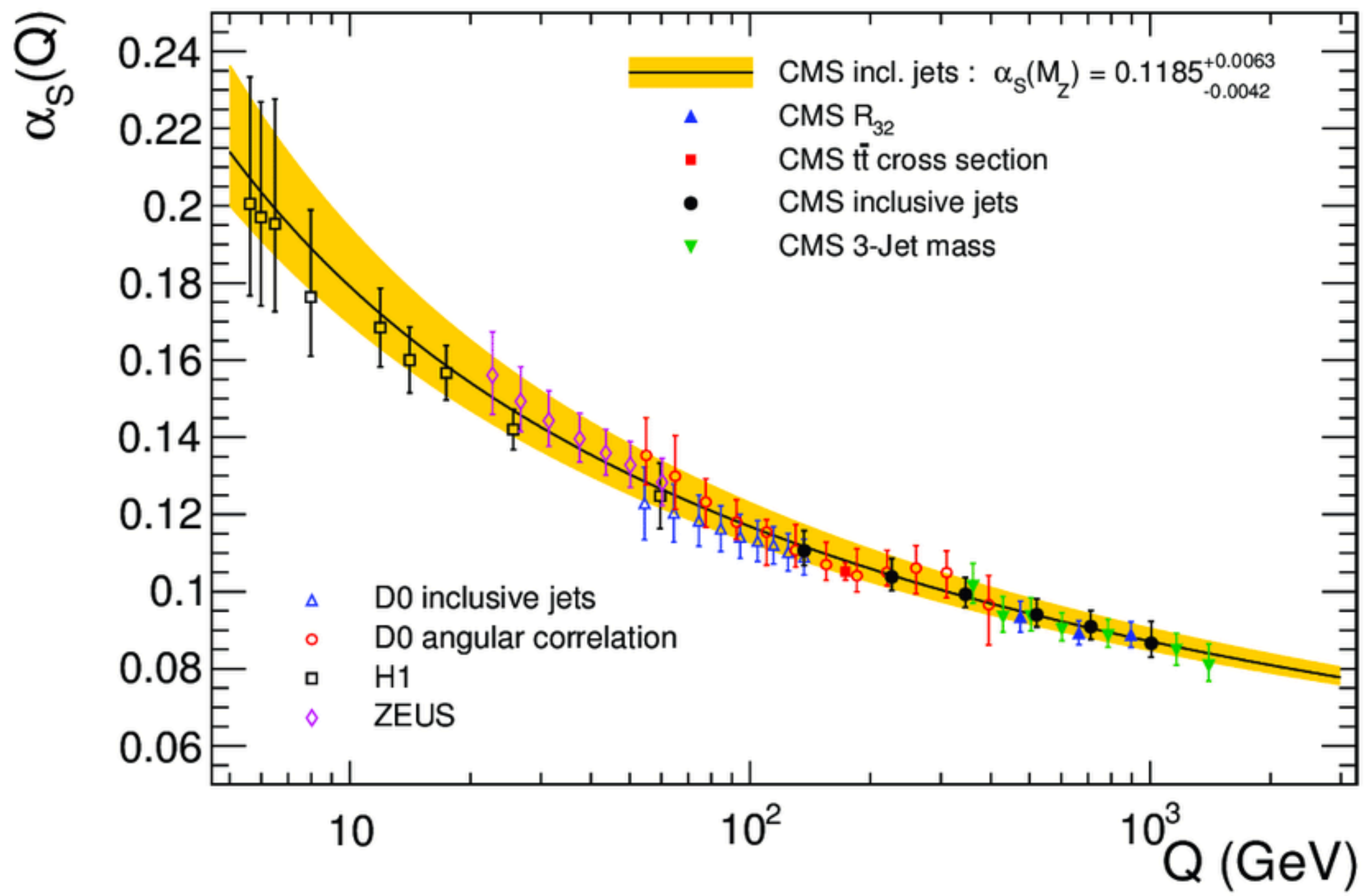
$SU(3)$ Non-Abelian gauge theory, highly nonlinear

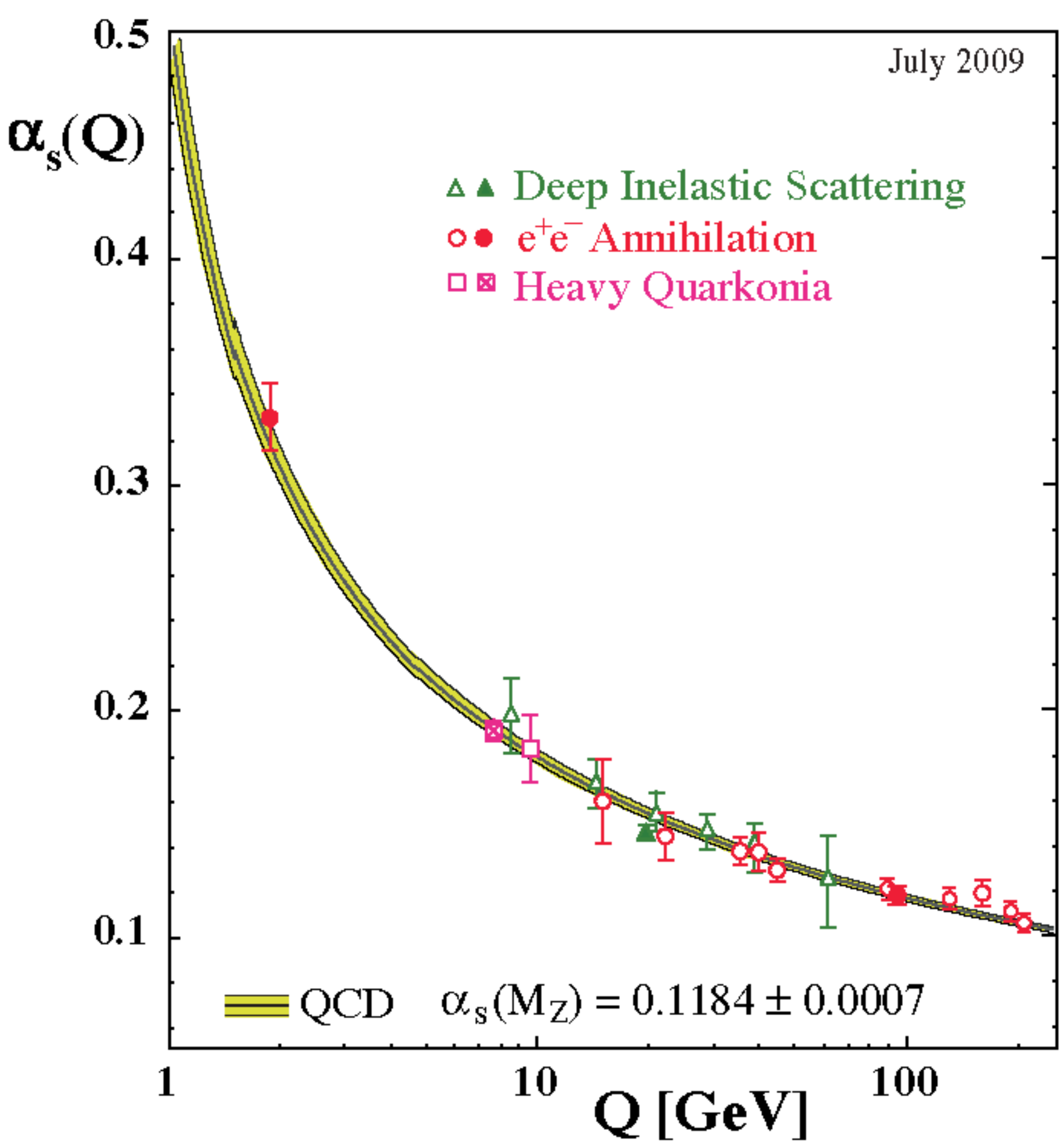
QCD has 8 “photons”, called gluons

They have color charge themselves and couple strongly to quarks and to each other



Asymptotic freedom

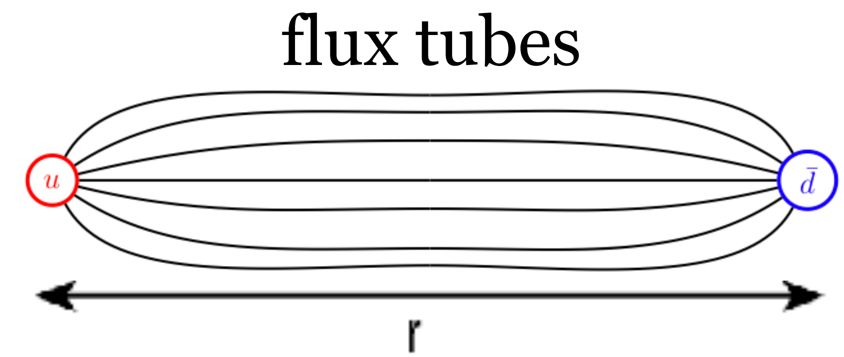




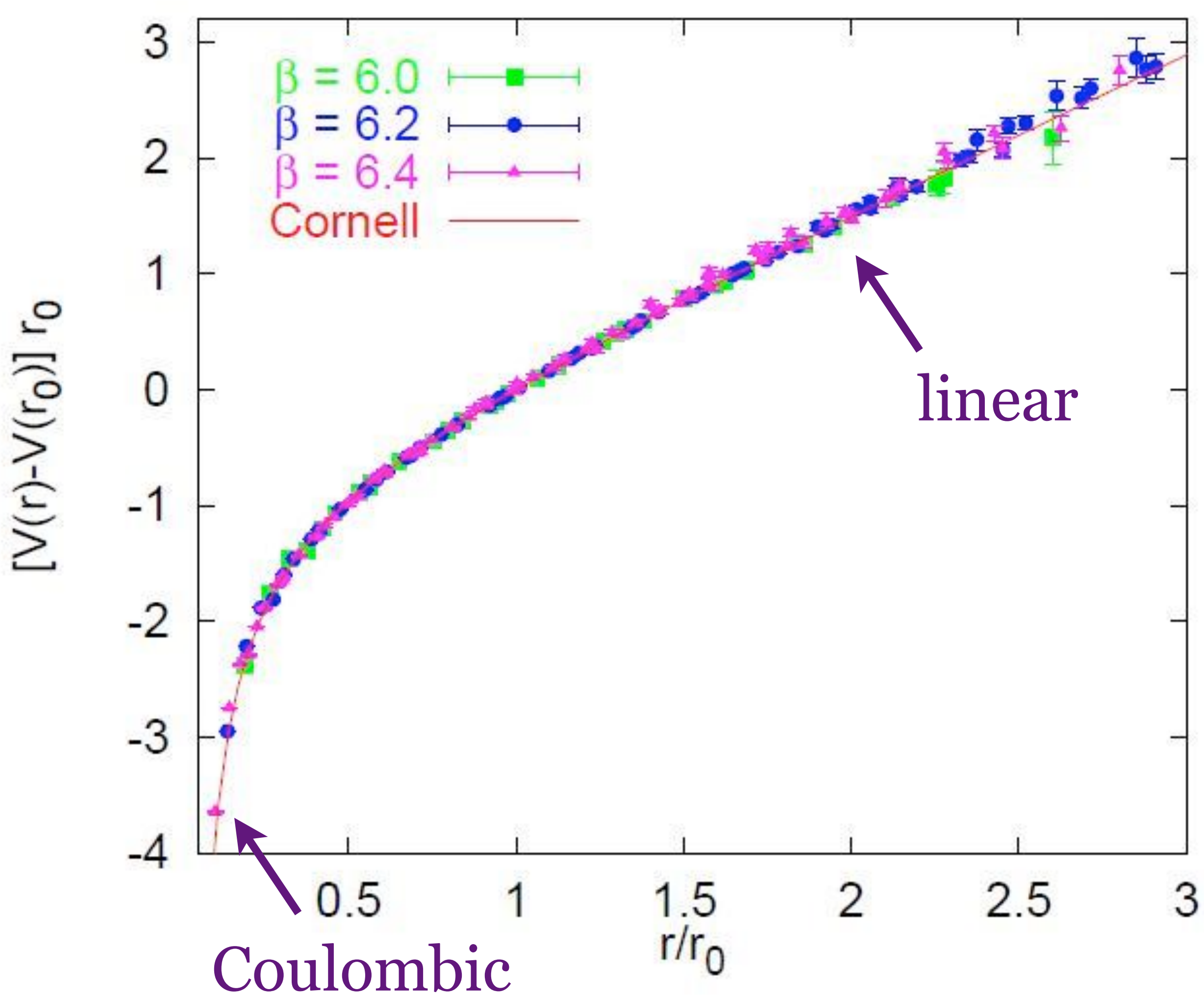
The strong force has a coupling strength that decreases with increasing energy

This allows for perturbative calculations, but when discussing hadrons the nonperturbative part *always* enters

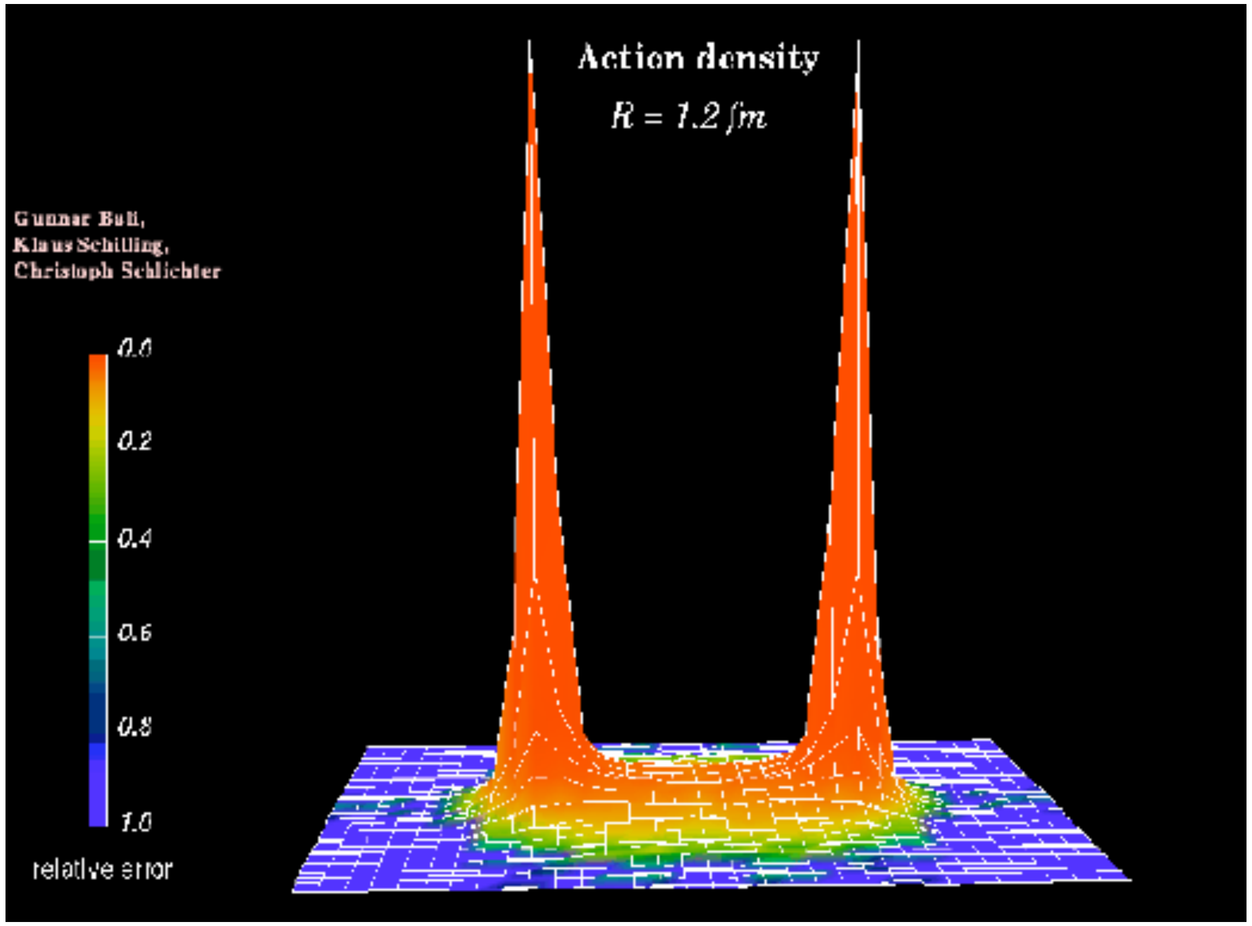
Confinement



Separating quarks costs increasingly more energy → quark confinement



From lattice QCD:



Bali, Schilling, Schlichter
Phys. Rev. D 51 (1995) 5165

Force carriers themselves subject to confinement → flux tubes

Gluon contribution to the proton mass

Lattice QCD: gluons contribute more than half the proton mass of 938 MeV

Close to 1/4 goes into confining the quarks, around 1/3 is from the energy of the gluons themselves

Yang, Liang, Bi, Chen, Draper, Keh-Fei Liu and Zhaofeng Liu, PRL 2018

Xiangdong Ji, PRL 1995

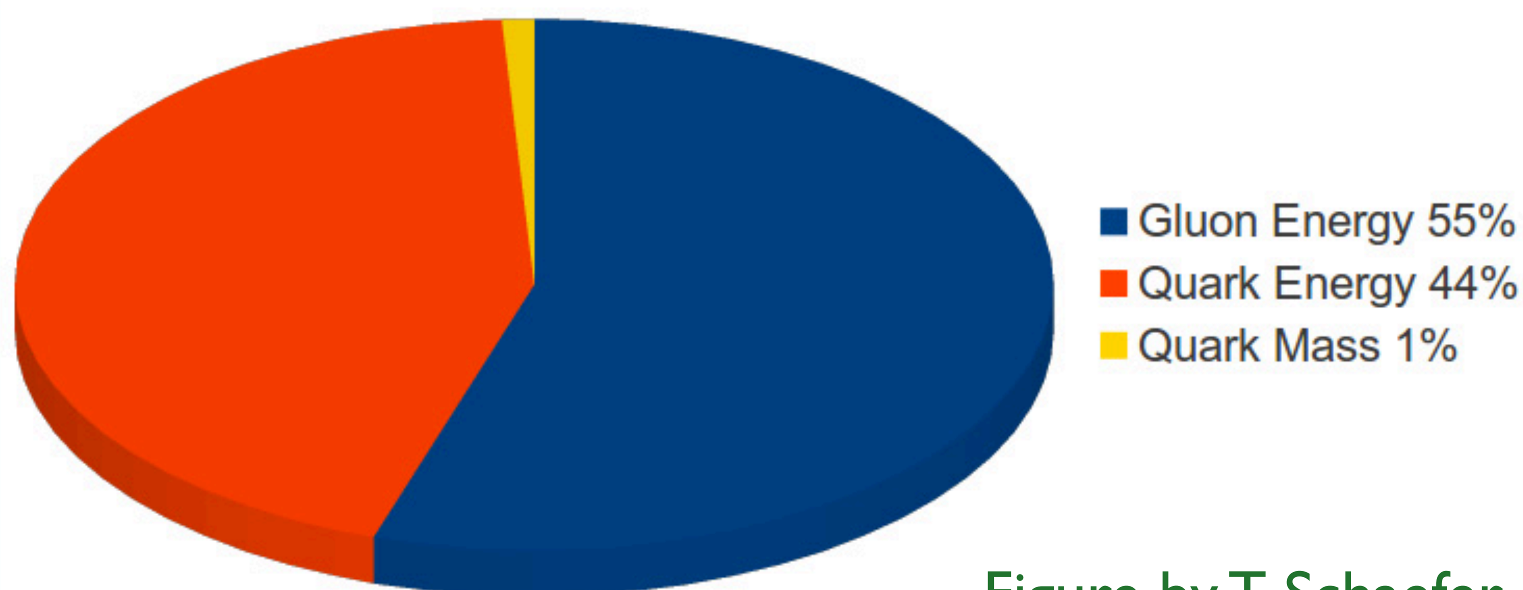


Figure by T. Schaefer

These are average numbers

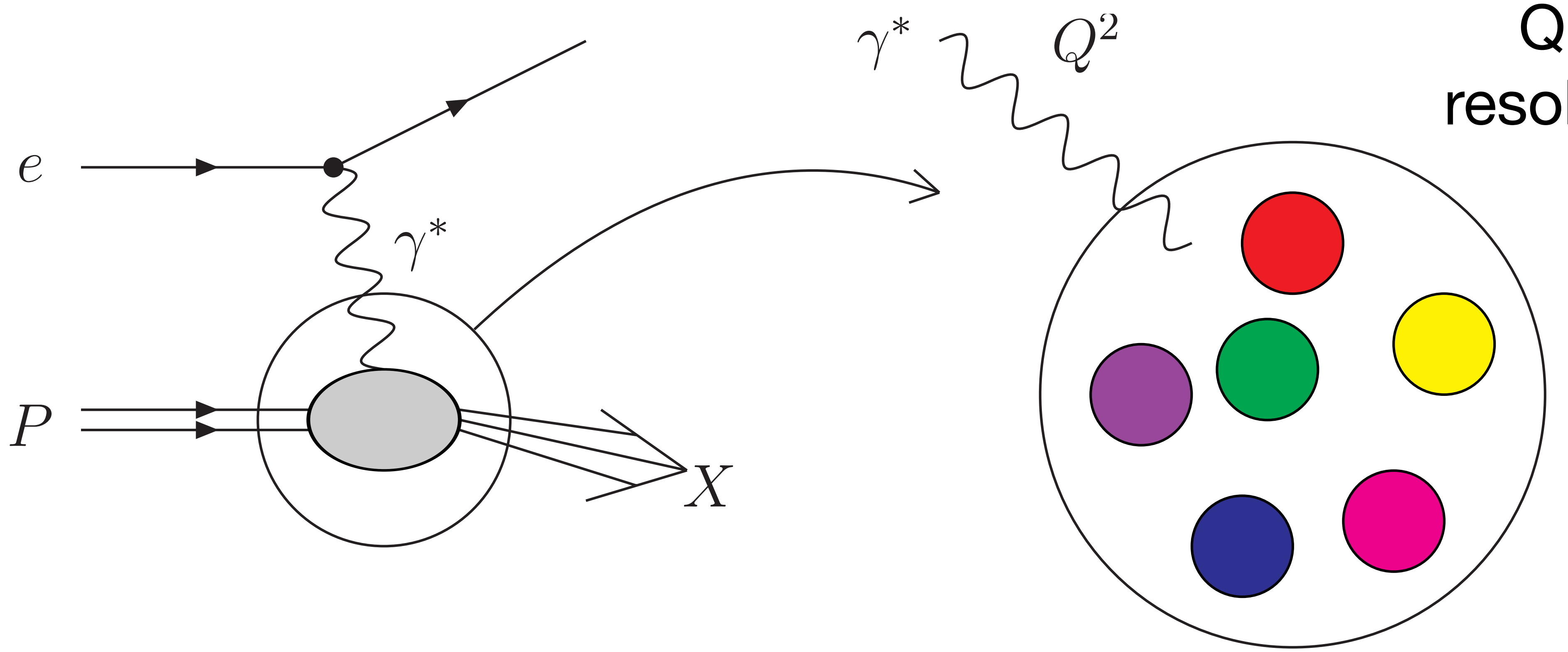
Multidimensional distributions describe in more detail how this is distributed within the proton

Probing gluons

The confinement distance is of proton size (~10^{-15} m)

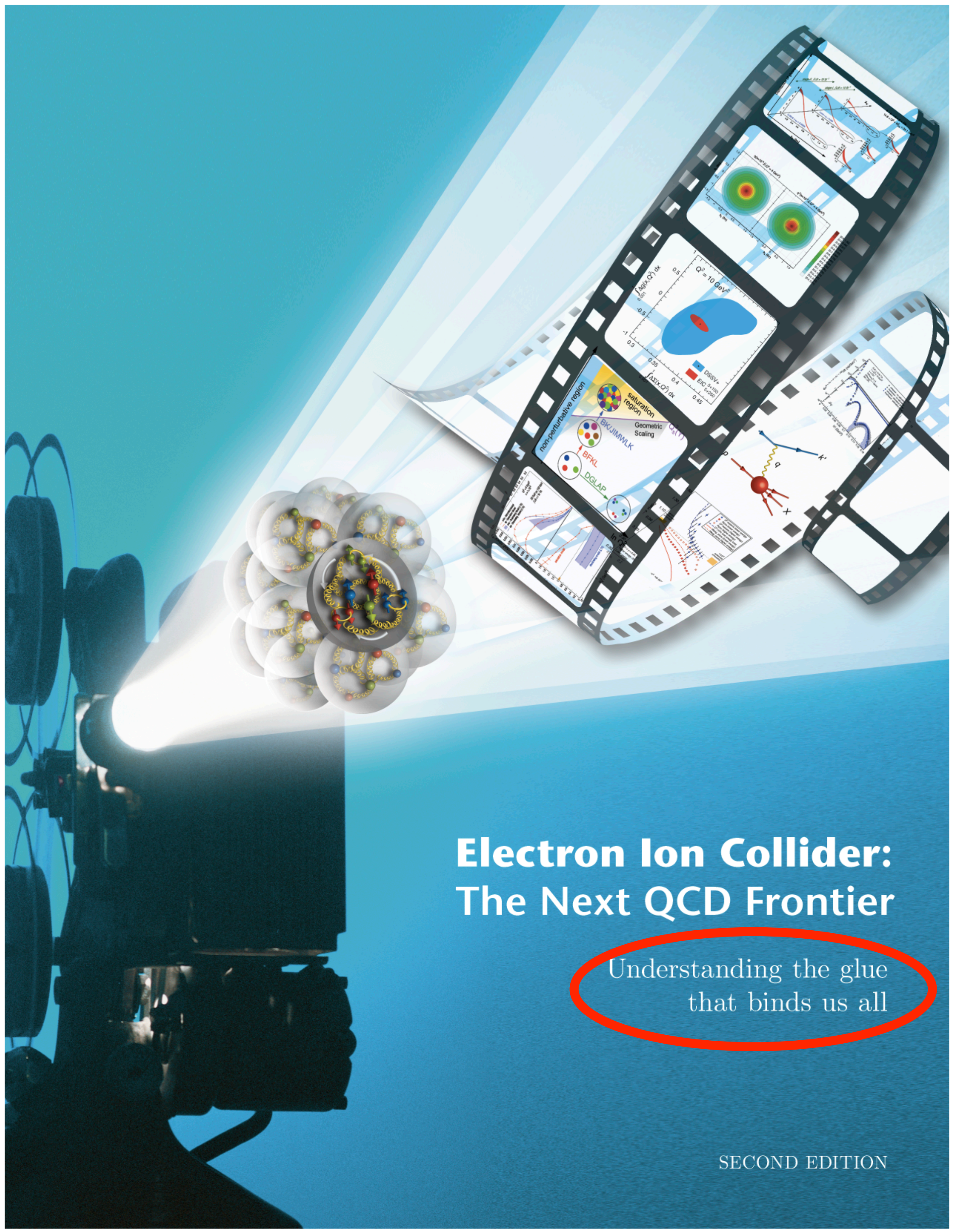
To look inside the proton requires energies larger than 200 MeV (~1 fm)

Electron-proton colliders have center of mass energies much larger than this, e.g. HERA had $\sqrt{s}$~320 GeV and EIC will have $\sqrt{s}$~20-140 GeV



Q sets the resolution scale

The future U.S.-based Electron-Ion Collider (2030+)



The Electron-Ion Collider (EIC) aims to address three key questions:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of a dense system of gluons?

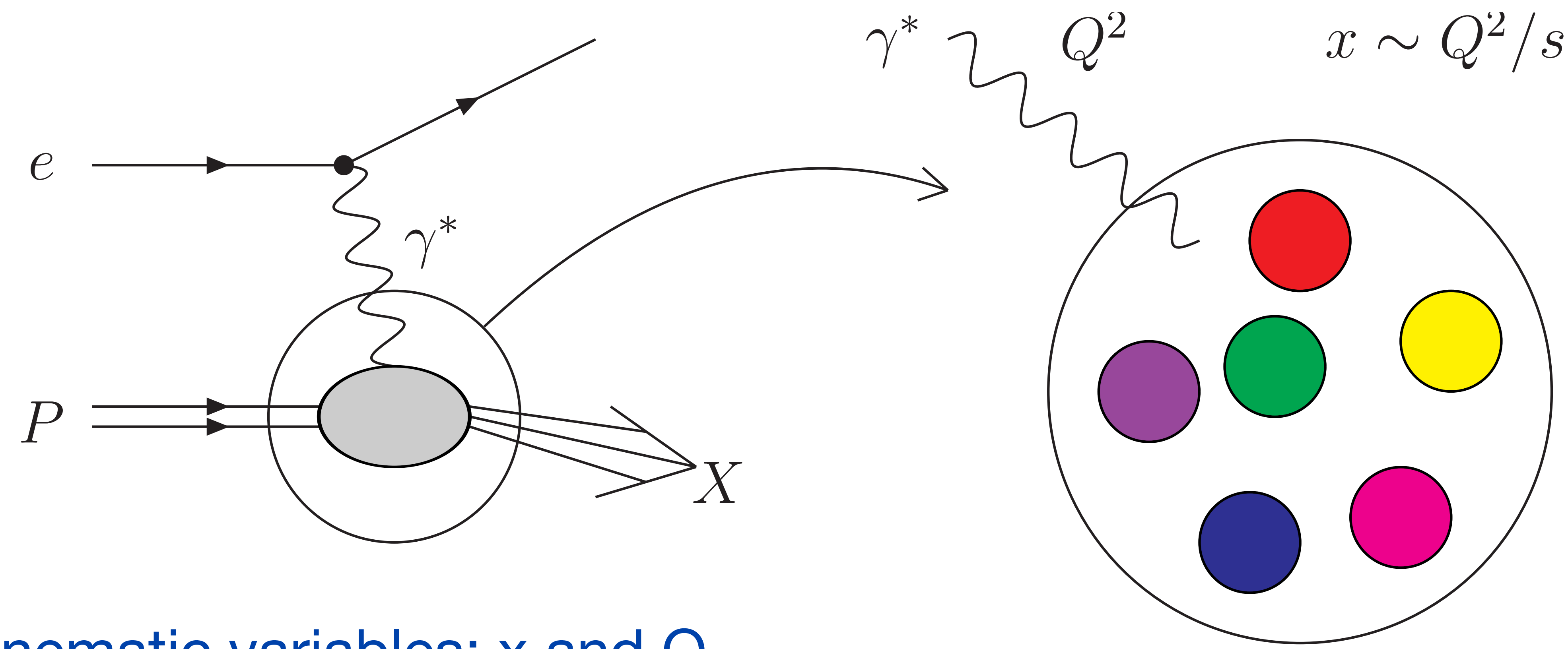
The EIC physics case is to a large extent aimed at understanding the physics of gluons

The EIC will be built at BNL, partly using the existing RHIC facility

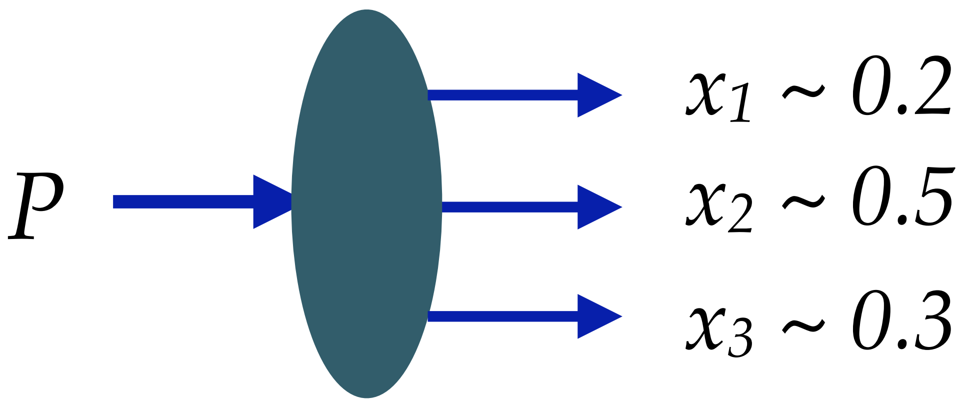
Gluon PDFs
unpolarized case

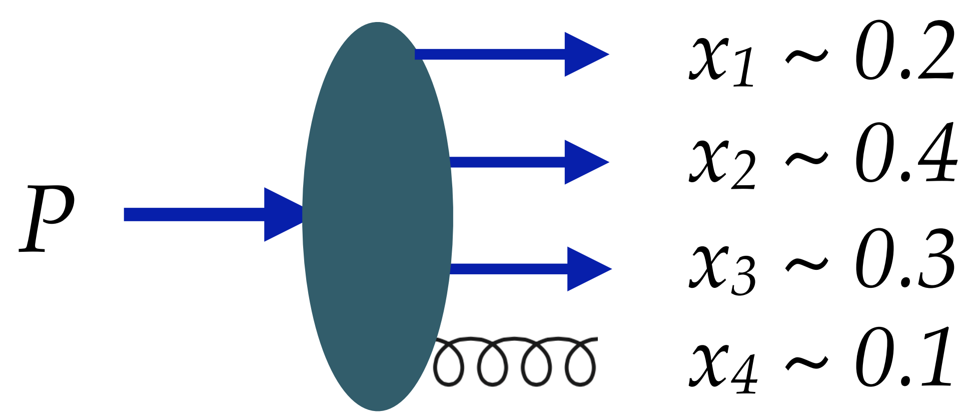
Deep inelastic scattering

Scattering off a proton at high energy = scattering off quarks and gluons



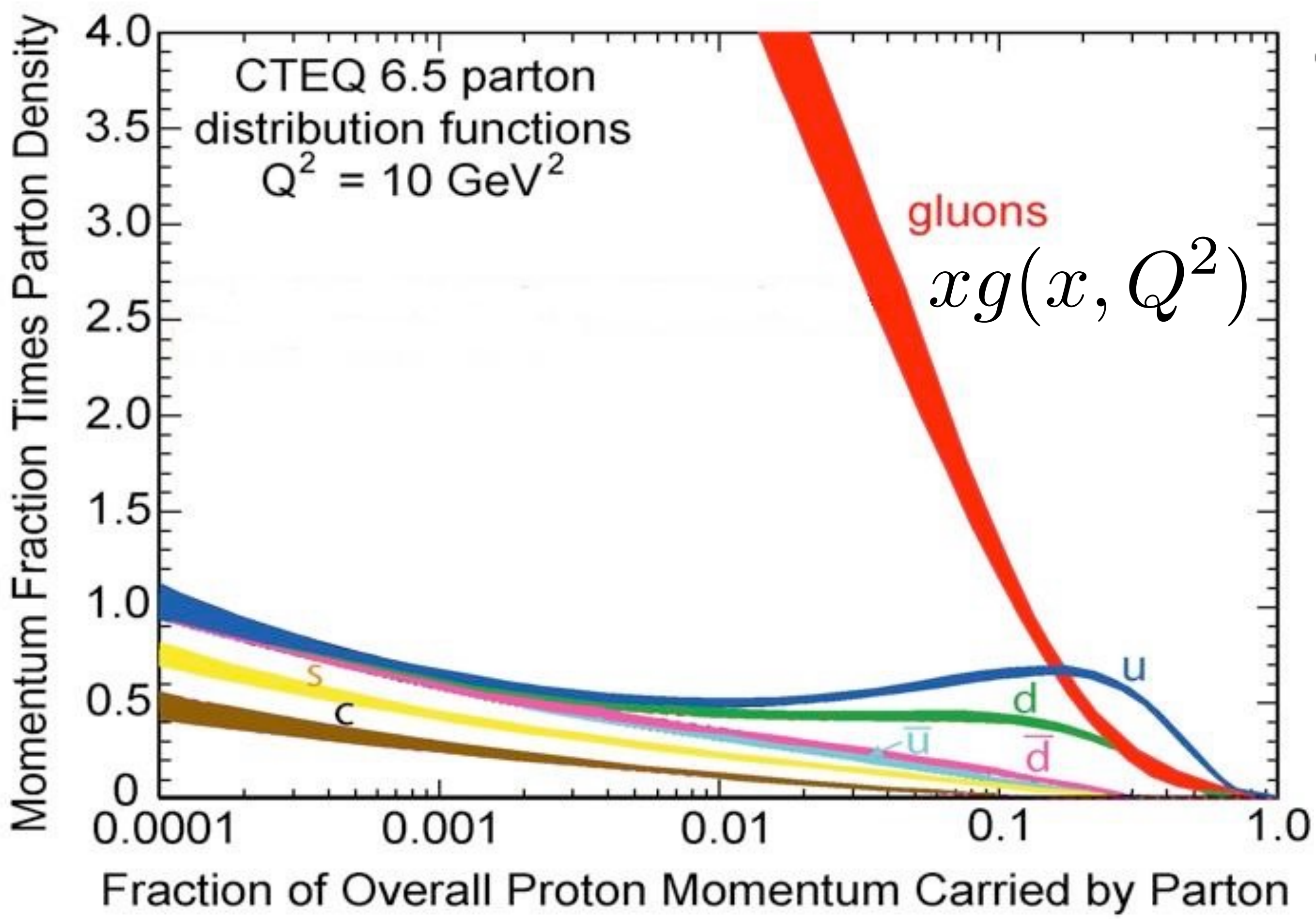
Two kinematic variables: x and Q

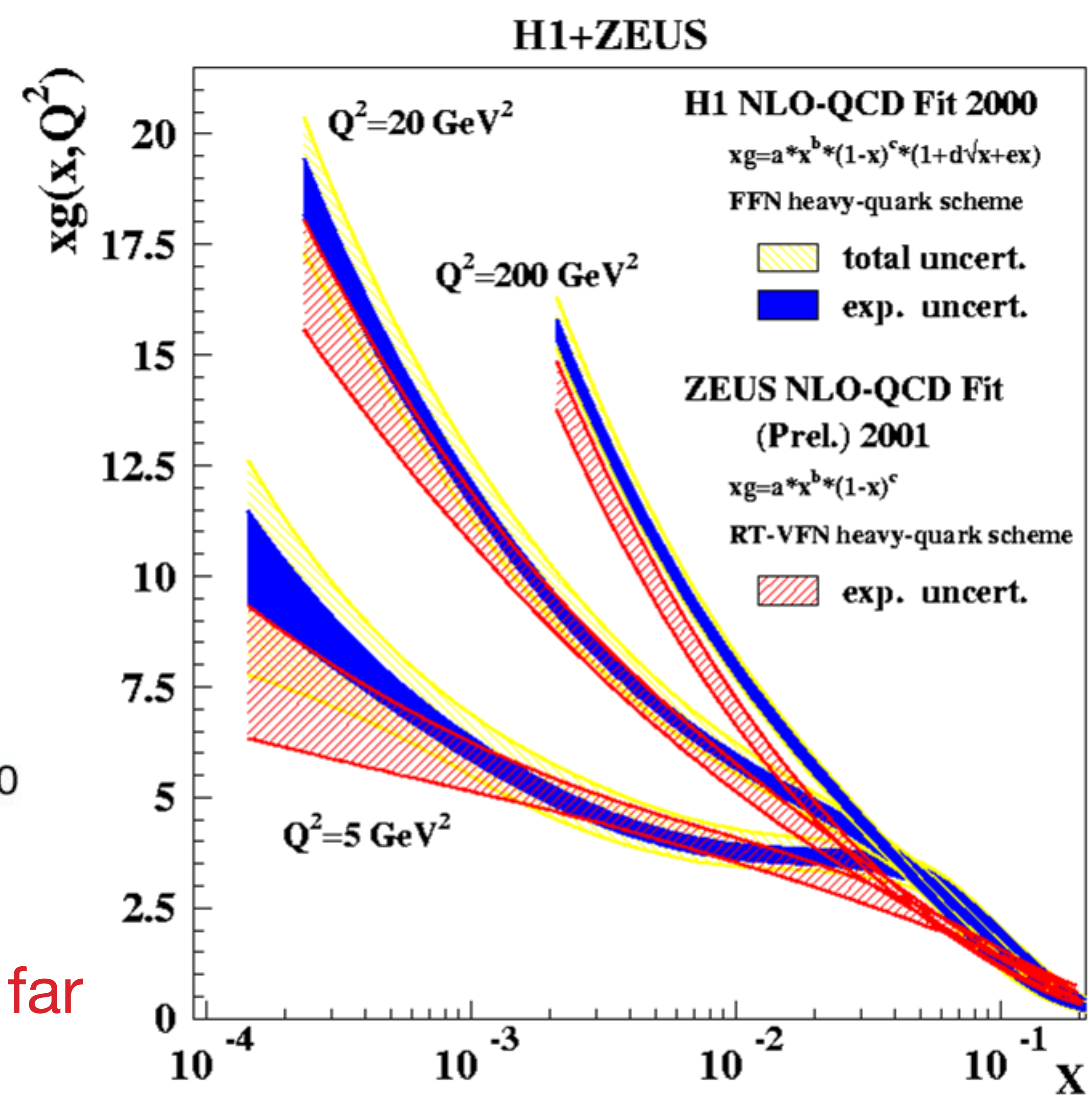




Gluon distribution

$g(x,Q^2)$ = probability of finding a gluon with momentum fraction x inside the proton at the energy scale Q





For small x values gluons dominate by far

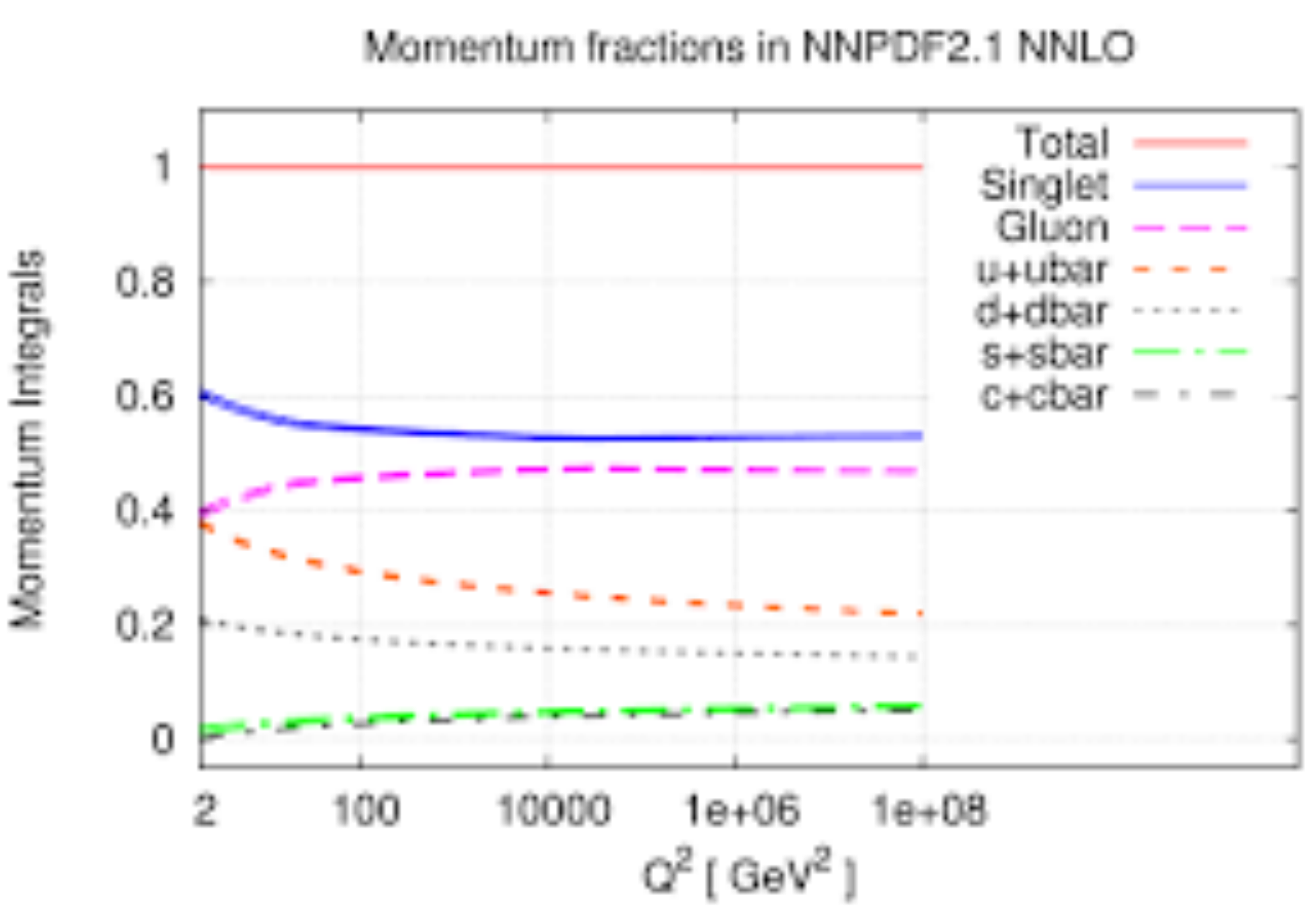
→ very high gluon density

Average momentum

Gluons carry a substantial fraction of the momentum of the proton

Asymptotically ($Q^2 \to \infty$):

$$\langle x \rangle_g \equiv \int_0^1 dx\, x\, g(x, Q^2) = \frac{16}{16 + 3N_f} - \mathcal{O}(\alpha_s(Q^2))$$



N_f = number of (active) quark flavors

Fits from experiments show that the fraction is close to 50% for perturbatively large Q values

The total number of gluons inside a proton is not bounded

NNPDF Collab arXiv:1107.2652

Evolution

The gluon distribution cannot be calculated from first principles, except integrals (Mellin moments) of it using lattice QCD

But its change with Q^2 for large Q^2 (such that $\alpha_s(Q^2) \ll 1$) can be calculated

By means of evolution equation, such as the DGLAP equation:

$$\frac{\partial g(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 dy P_{gg}\left(\frac{x}{y}\right) g(y,Q^2)$$

Gribov, Lipatov '72; Altarelli, Parisi '77; Dokshitzer '77

This describes the leading behavior in Q^2

Including higher order corrections all DIS data are well-described

Gluon distribution at small x

Fits to data show: $$xg(x,Q^2) \sim \frac{1}{x^\lambda} \qquad \text{with } \lambda \approx 0.4$$

This leads cross sections to grow too fast with s ~ Q^2/x in the limit s → ∞

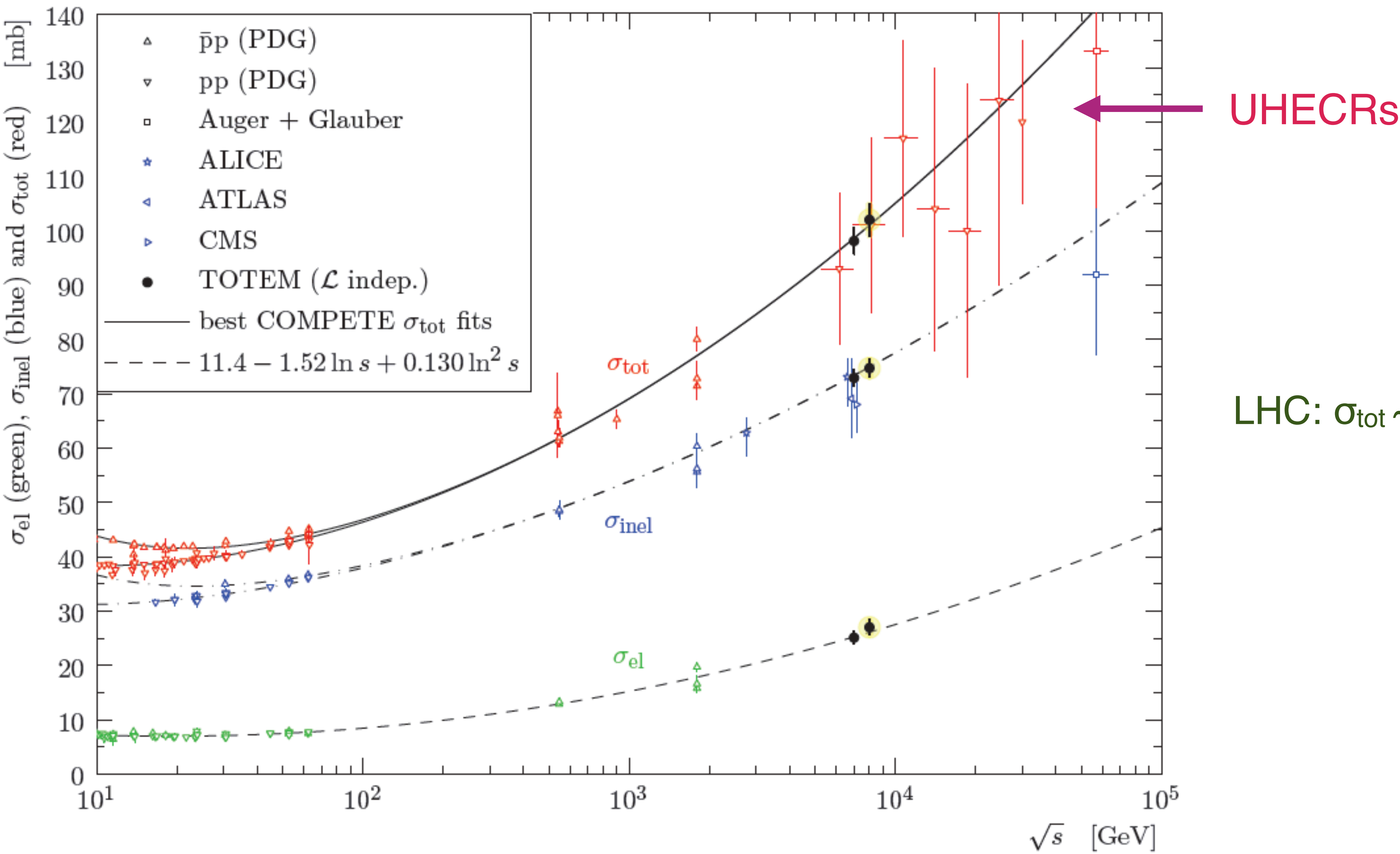
Eventually it leads to a violation of the so-called Froissart or unitarity bound:

$$\sigma_{pp,\mathrm{total}} \leq \frac{\pi}{m_\pi^2}(\ln s)^2$$

Froissart '61; Jin & Martin '64; Martin '66

Beyond this bound the probability to scatter becomes larger than 1

Total pp cross section



LHC: $\sigma_{tot} \sim 100$ mb

At LHC the Froissart bound is a few barn (1 barn=10^{-28} m^2), so no problem in practice

Gluon distribution at small x according to QCD

Fits to data show: $xg(x,Q^2) \sim \frac{1}{x^\lambda} \qquad \text{with } \lambda \approx 0.4$

Also according to linear evolution equations, like DGLAP and BFKL, $g(x,Q^2)$ will grow too fast (exponentially) toward small x

Non-linear evolution equations can moderate the growth, e.g. GLR equation:

$$\frac{\partial^2 xg(x,Q^2)}{\partial \ln 1/x \partial \ln Q^2} = \frac{\alpha_s N_c}{\pi} xg(x,Q^2) - \frac{\alpha_s^2 N_c}{R^2 Q^2}[xg(x,Q^2)]^2$$

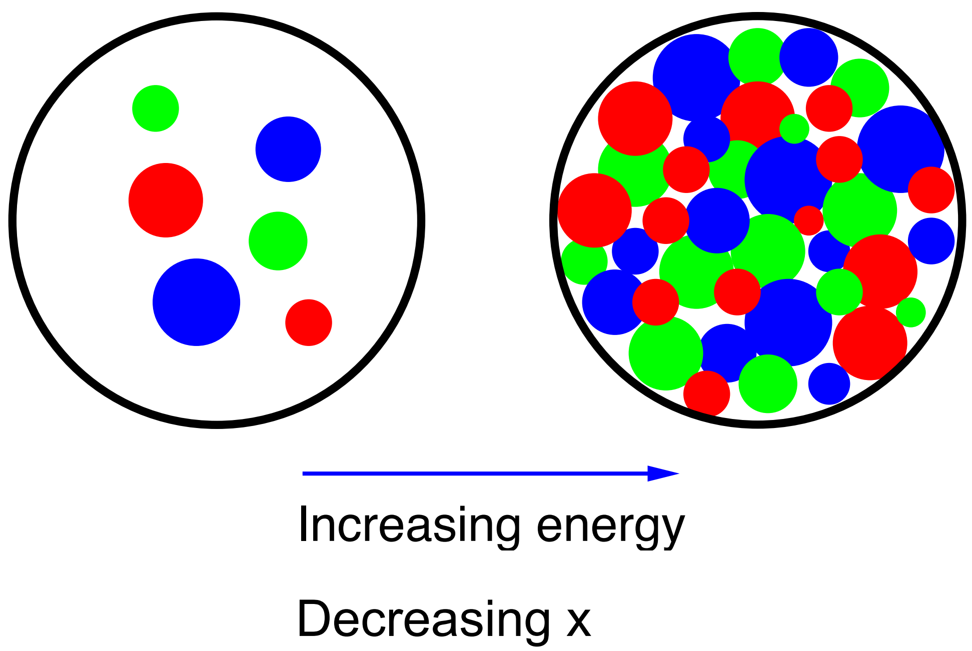
Gribov, Levin & Ryskin, 1983; Laenen, Levin, 1995

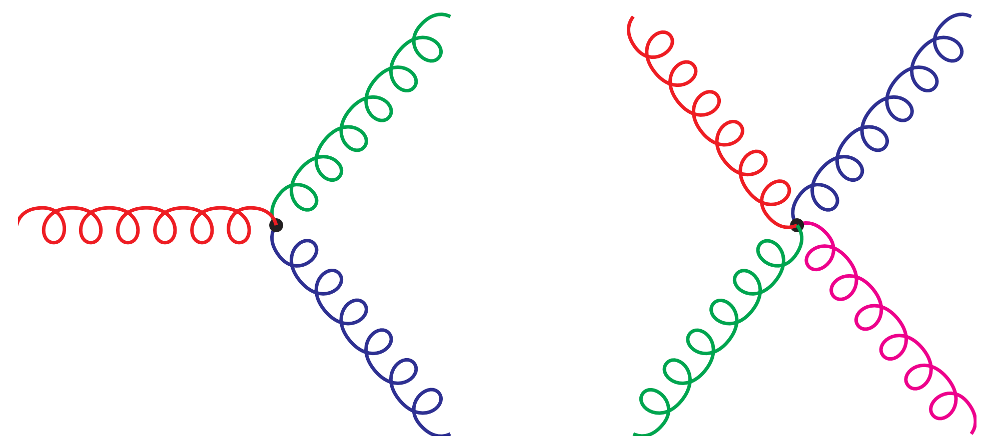
But instead of looking at the gluon density $g(x, Q^2) \propto \langle A^\dagger A \rangle$, at small x one needs to look at more general quantities since $\langle A^\dagger A \rangle^2 \neq \langle A^\dagger A^\dagger A\, A \rangle$

Generalizes to the non-linear BK equation and the JIMWLK equations

High gluon density

When x decreases, the density of gluons (n) increases



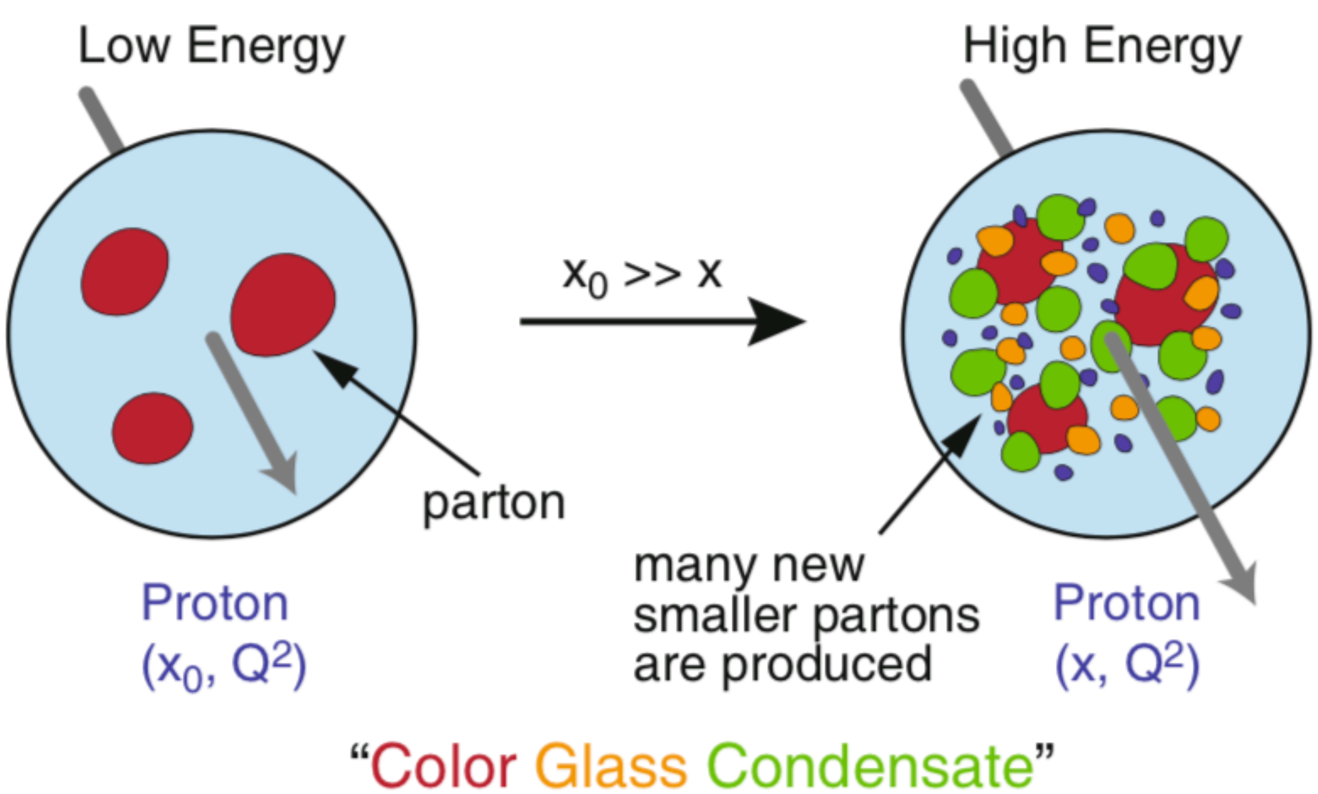


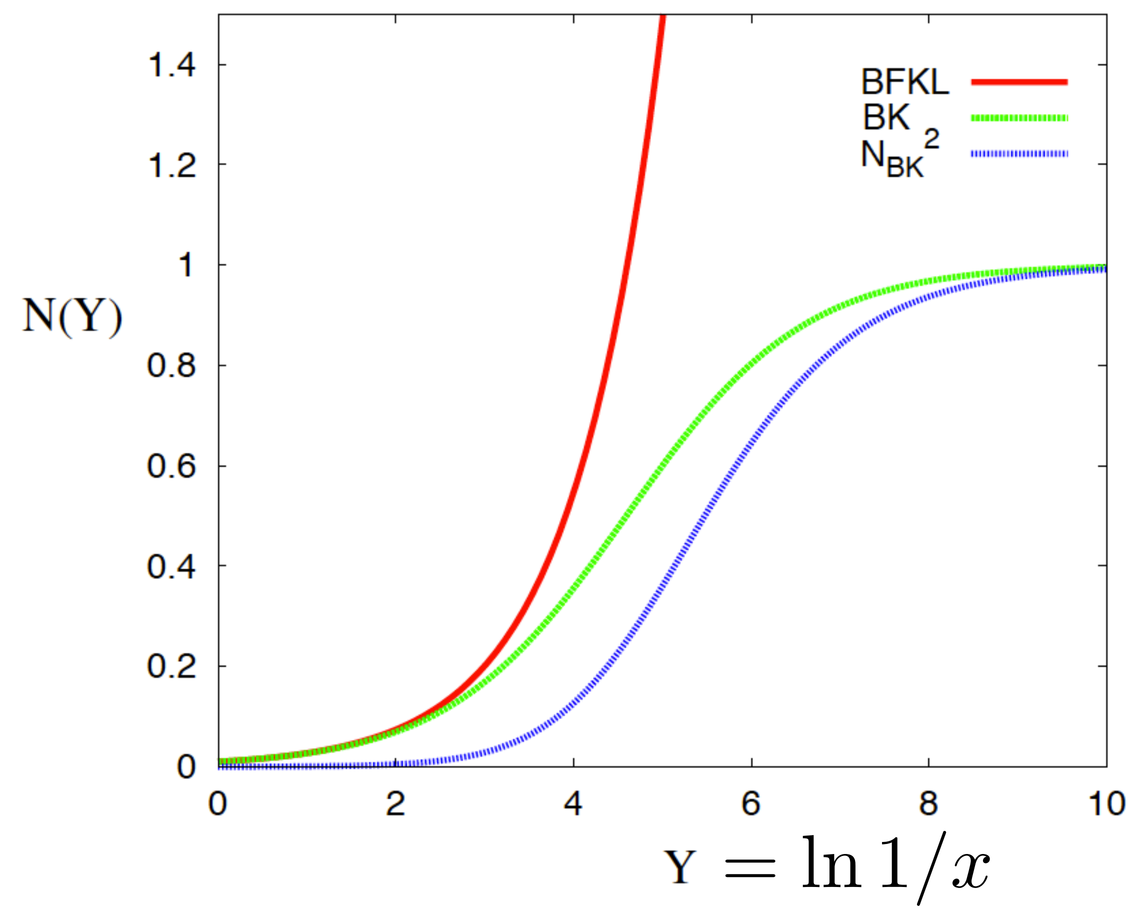
At some point n becomes so large ($n \to O(1/\alpha_s)$) that the probability for gluons to interact approaches 1 ($n \times \sigma_{gg} \to 1$) [No such effect arises for photons]

Scattering off a proton becomes scattering off multiple gluons simultaneously

It resembles more scattering off a potential

Gluon saturation





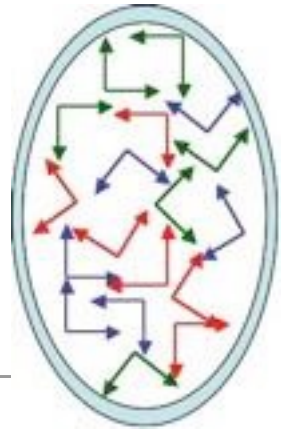
Scatter off multiple gluons simultaneously will probe their collective effect

This effect is expected to moderate the exponential growth of the gluon density

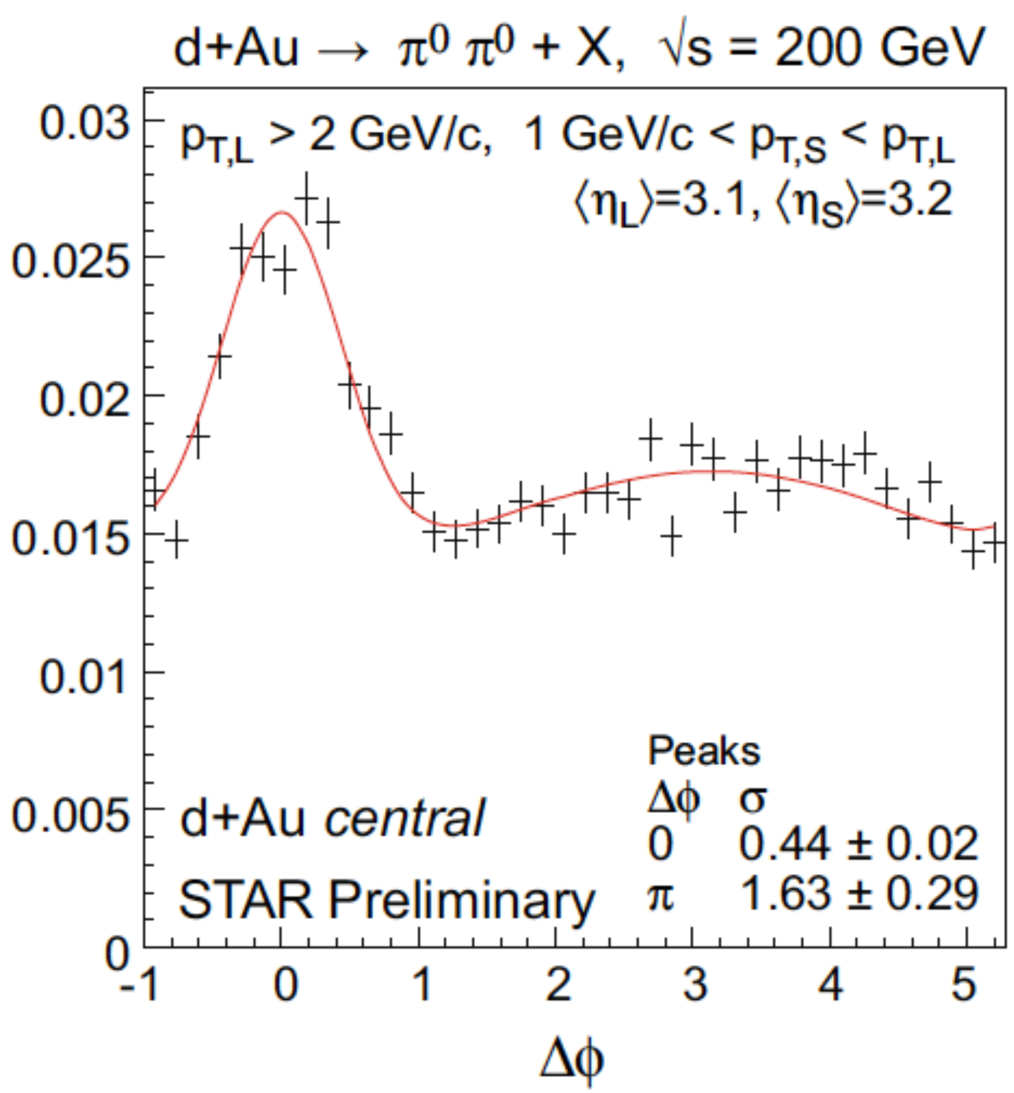
→ ultimately saturating into a state called the **Color Glass Condensate**

Never directly observed in the gluon distribution yet

CGC experimental signatures

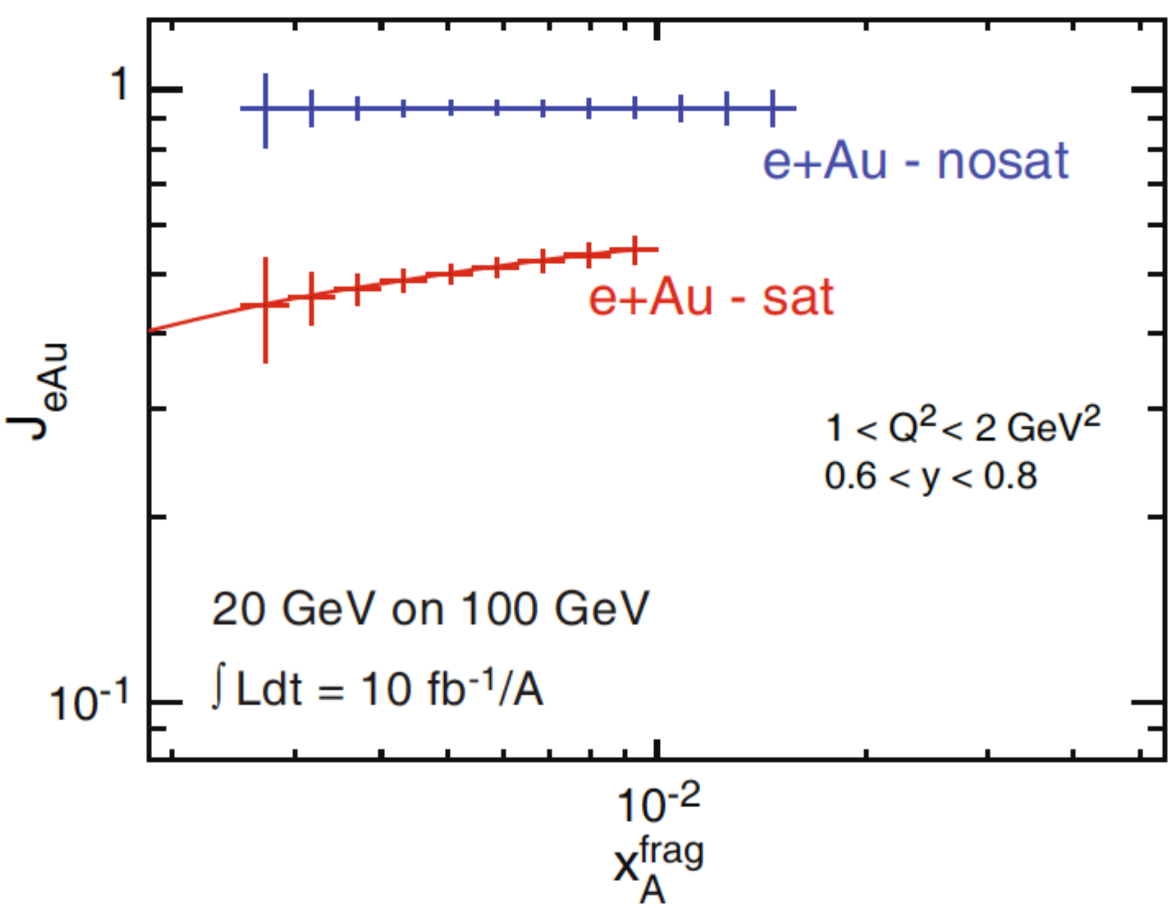


Several expected signatures of the Color Glass Condensate (CGC) have been seen in the data (HERA, RHIC and LHC), but no conclusive evidence yet



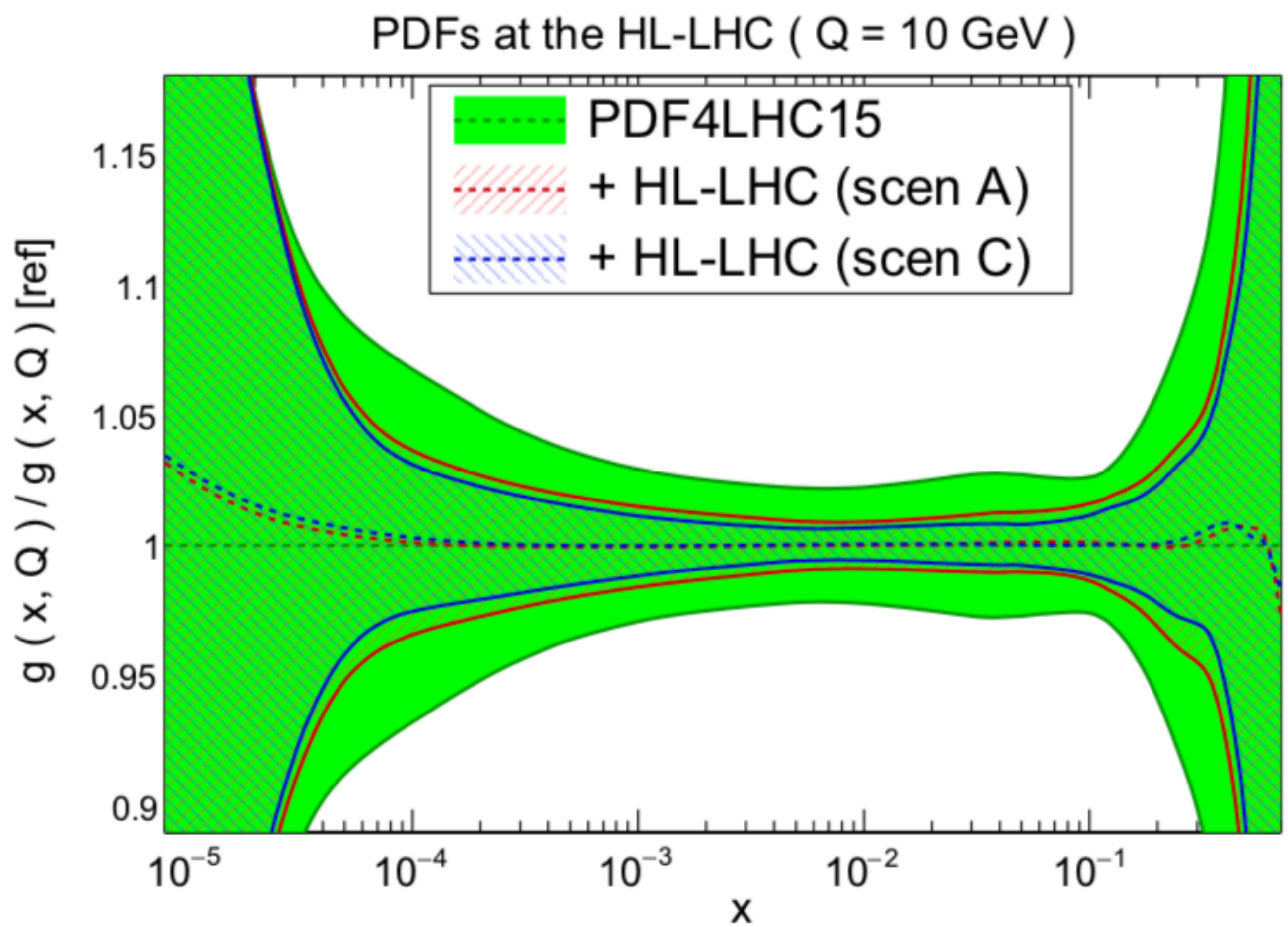
Broadening of back-to-back peak

Comparison of the relative yield of di-hadrons in eAu vs ep collisions at EIC projected with and without saturation:



Accardi *et al.*, EPJA (2016)

EIC and LHC: x range



The precision on the gluon distribution that one expects ultimately from the HL-LHC

Khalek, Bailey, Gao, Harland-Lang, Rojo, EPJC 78 (2018) 962

The EIC will span a similar range in x

The EIC's uniqueness w.r.t. HERA and LHC is in the polarization and in eA

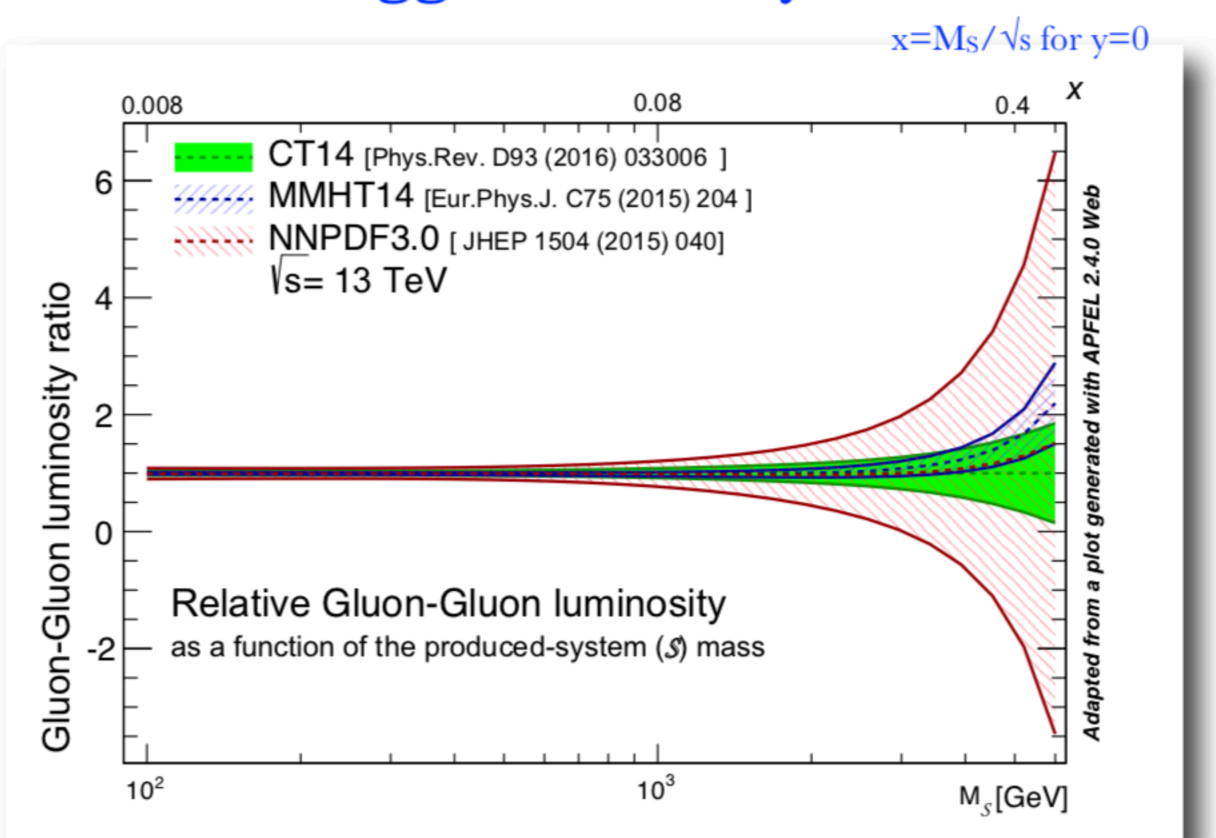
The hope is to observe the onset of gluon saturation, of nonlinear QCD effects

Gluons at large x

Nuclear dependence of gluons at large x also displays many interesting features

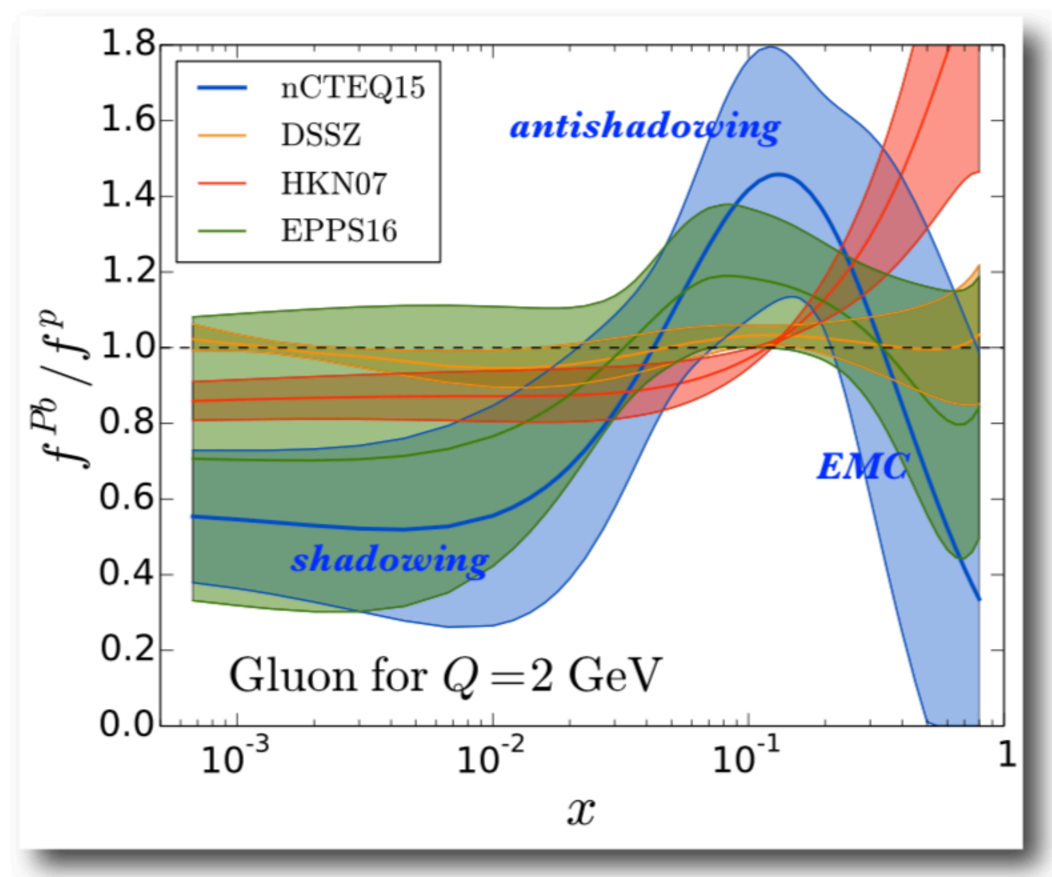
Gluons at large x matter for Beyond the Standard Model physics searches

gg luminosity



$$\frac{\partial \mathcal{L}_{ab}}{\partial \tau} = \frac{1}{s} \int_{\tau}^{1} \frac{dx}{x} f_a(x, M_S^2)\, f_b(\tau/x, M_S^2), \qquad \tau = M_S^2/s$$

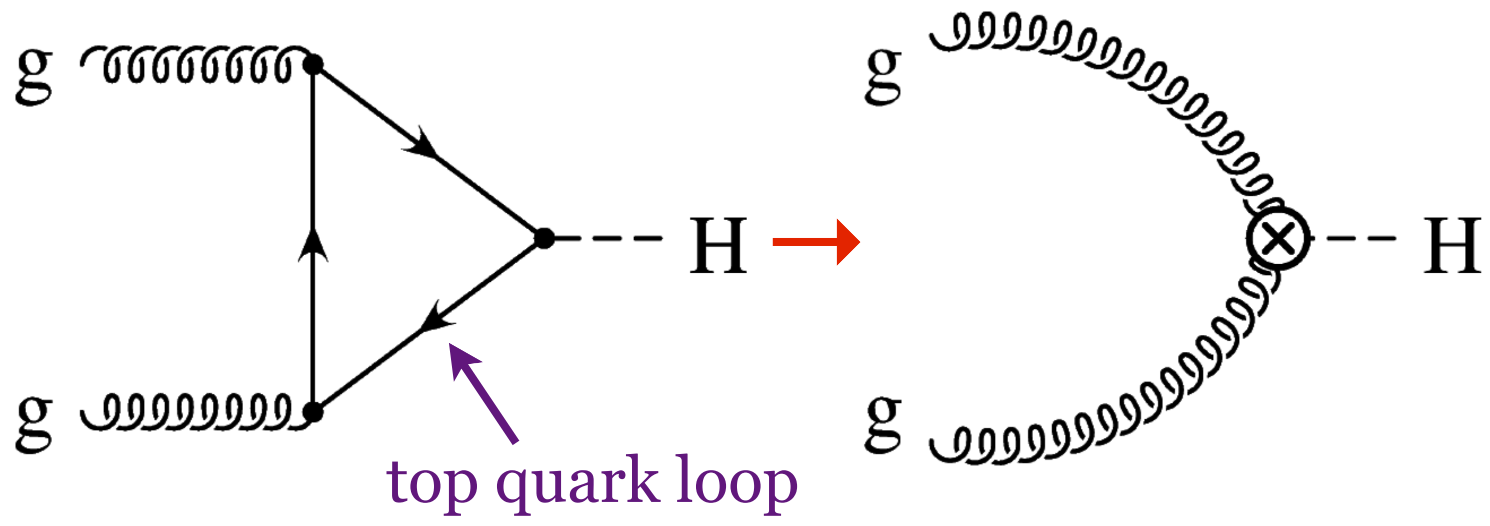
gluon nuclear PDF (Pb)



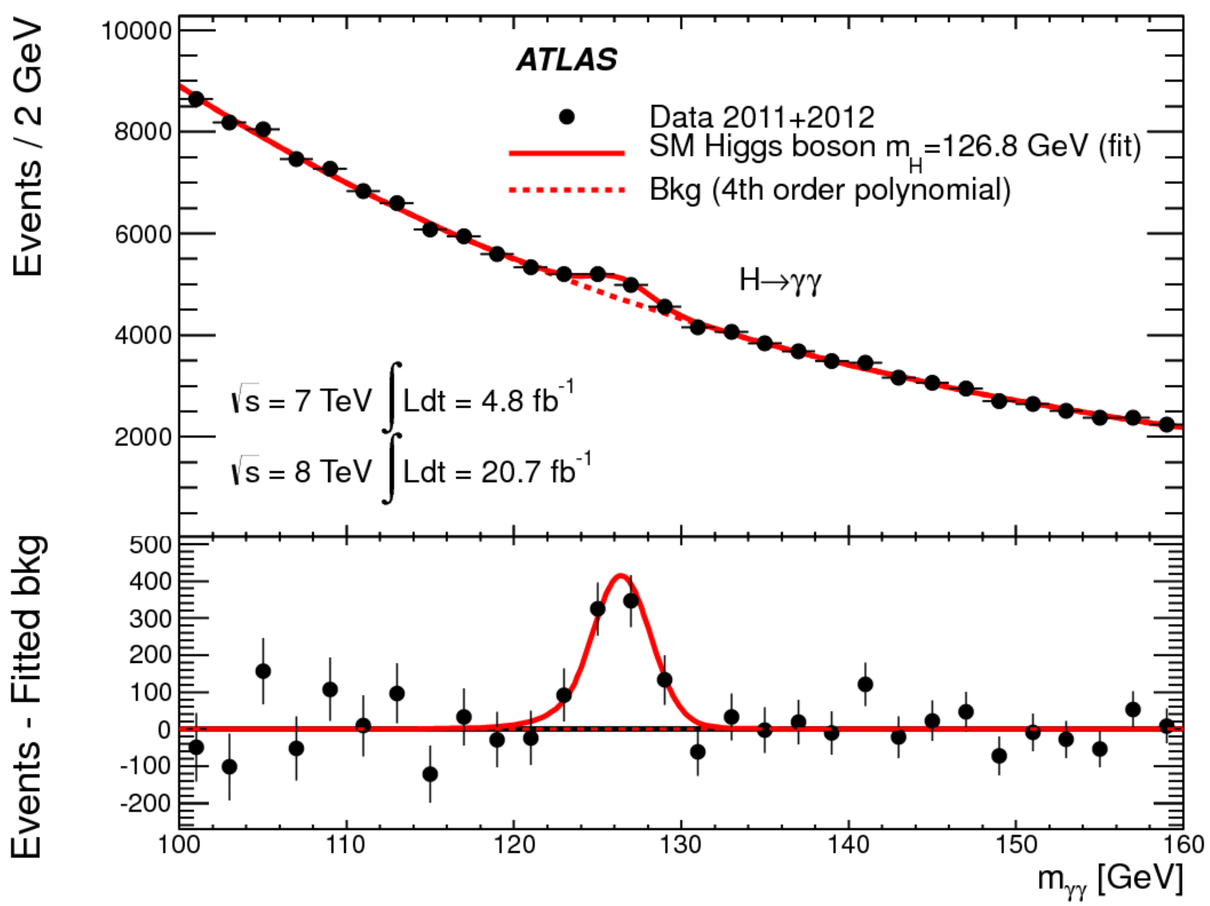
From M. Echevarria, DIS2019 & 1807.00603

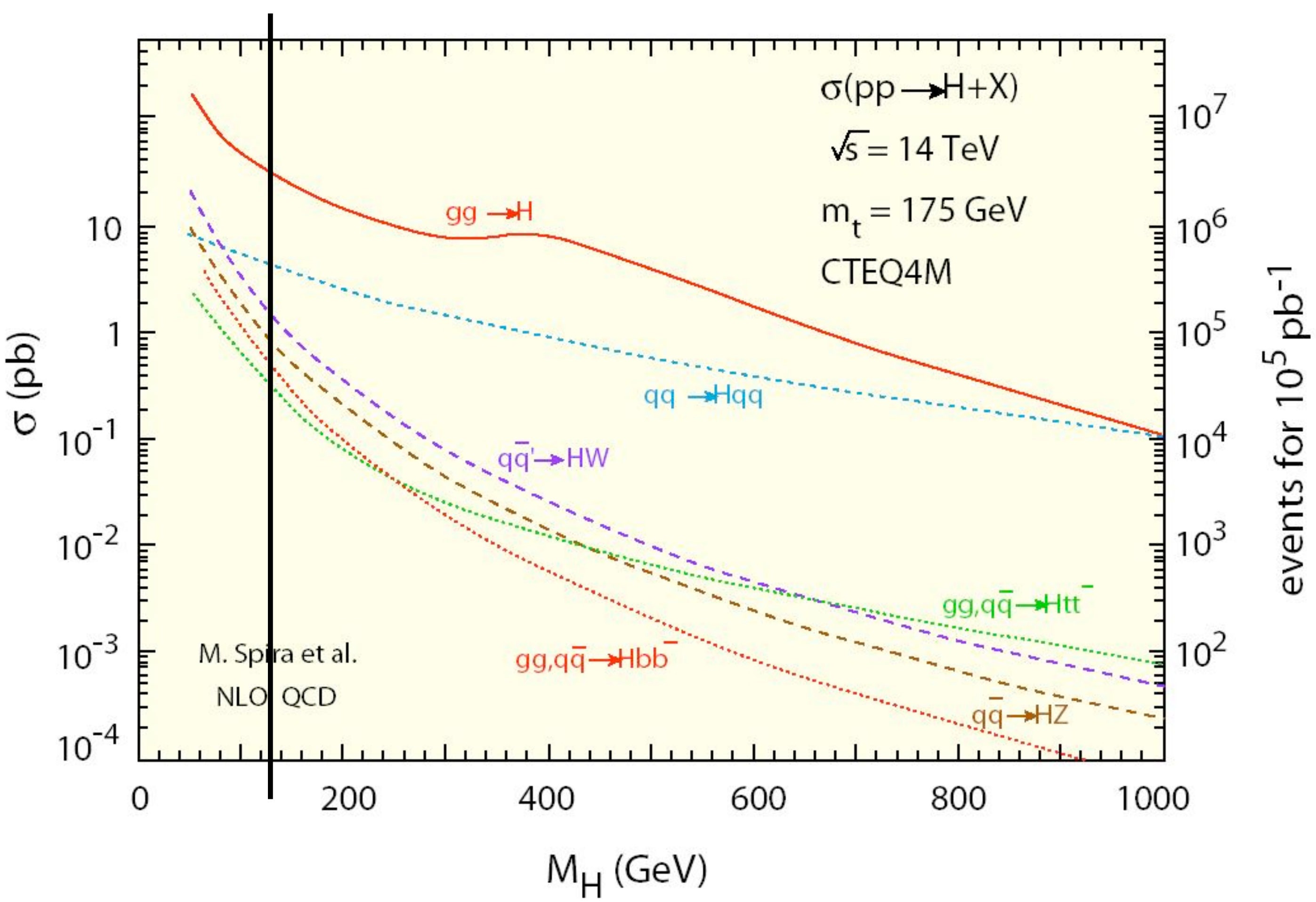
eA never studied in a collider - EIC and perhaps LHeC (no polarization)

Higgs production happens predominantly via gg → H



Gluons in Higgs production at LHC have x ~ 0.01 (in the well measured range)





Discovery of new heavy particles (bumps) does not require knowledge on gluon distributions, but to extract the properties of the new particles does

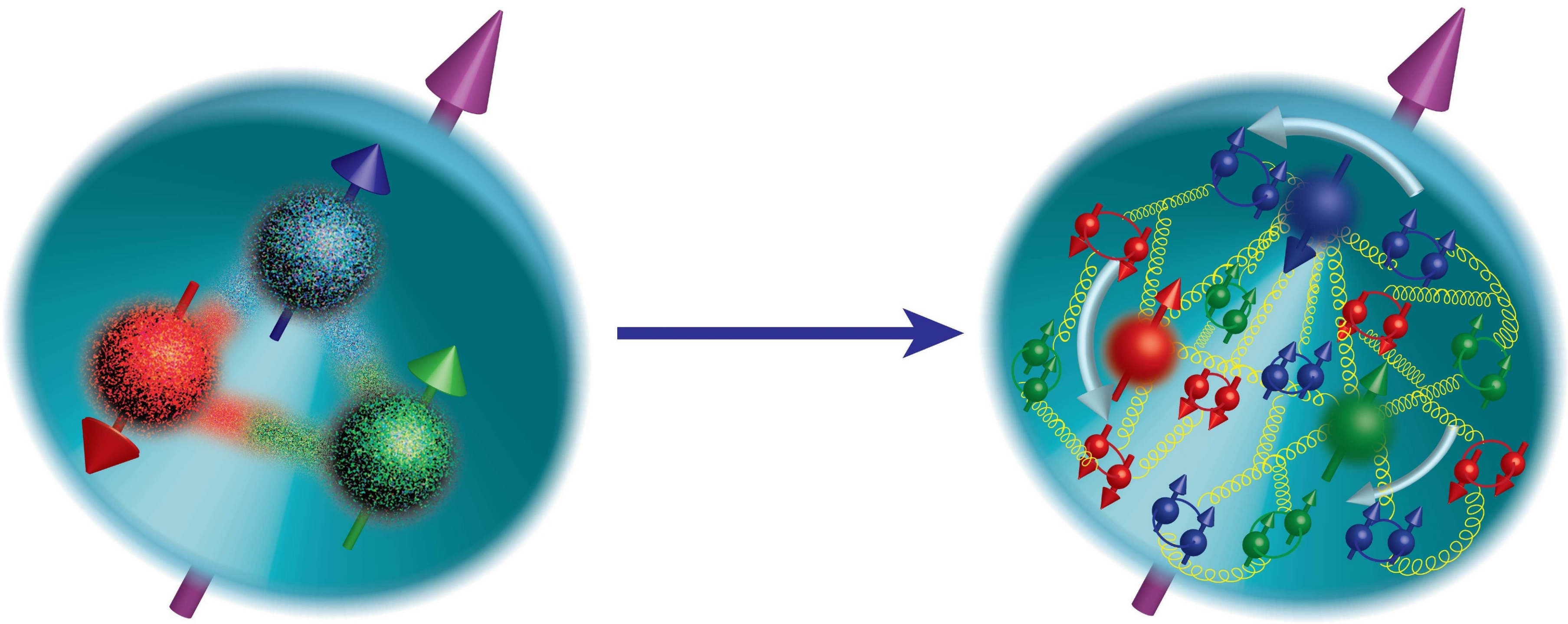
Gluon PDFs
polarized case

Proton spin decomposition

In general, one expects the following spin decomposition or “sum rule” to hold

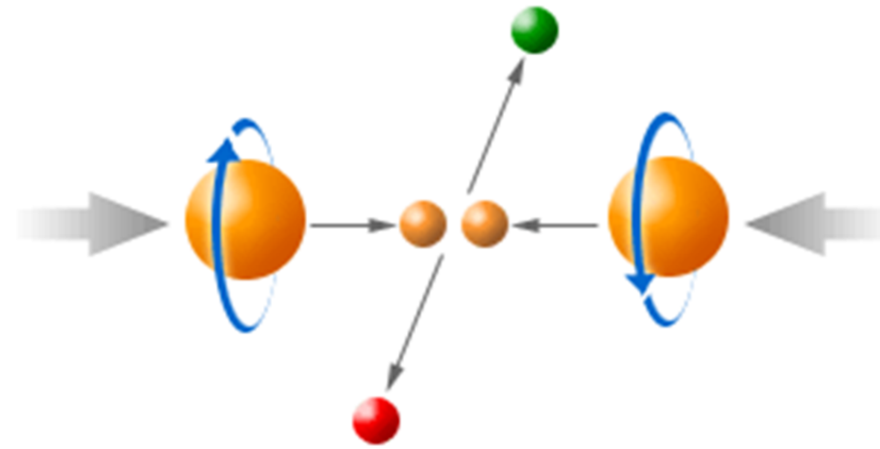
$$\text{proton spin } = \frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_z$$

Sum of the contributions to the proton spin adds up to ½

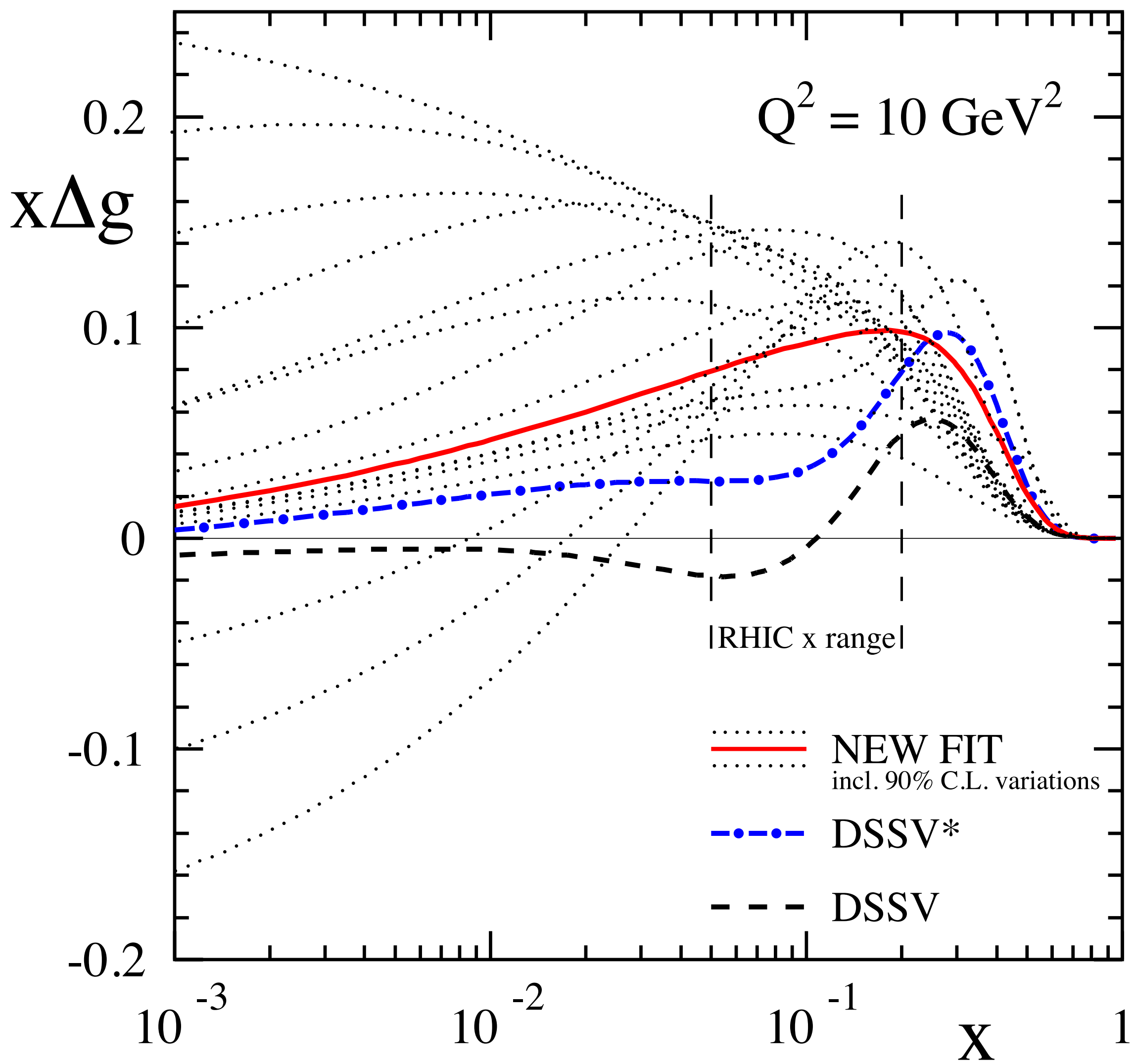


Spin puzzle - only about 1/3 of the proton spin comes from the quarks

Gluon contribution to the proton spin



RHIC - the world's only polarized proton-proton collider



At RHIC $\Delta g(x)$ is obtained from:

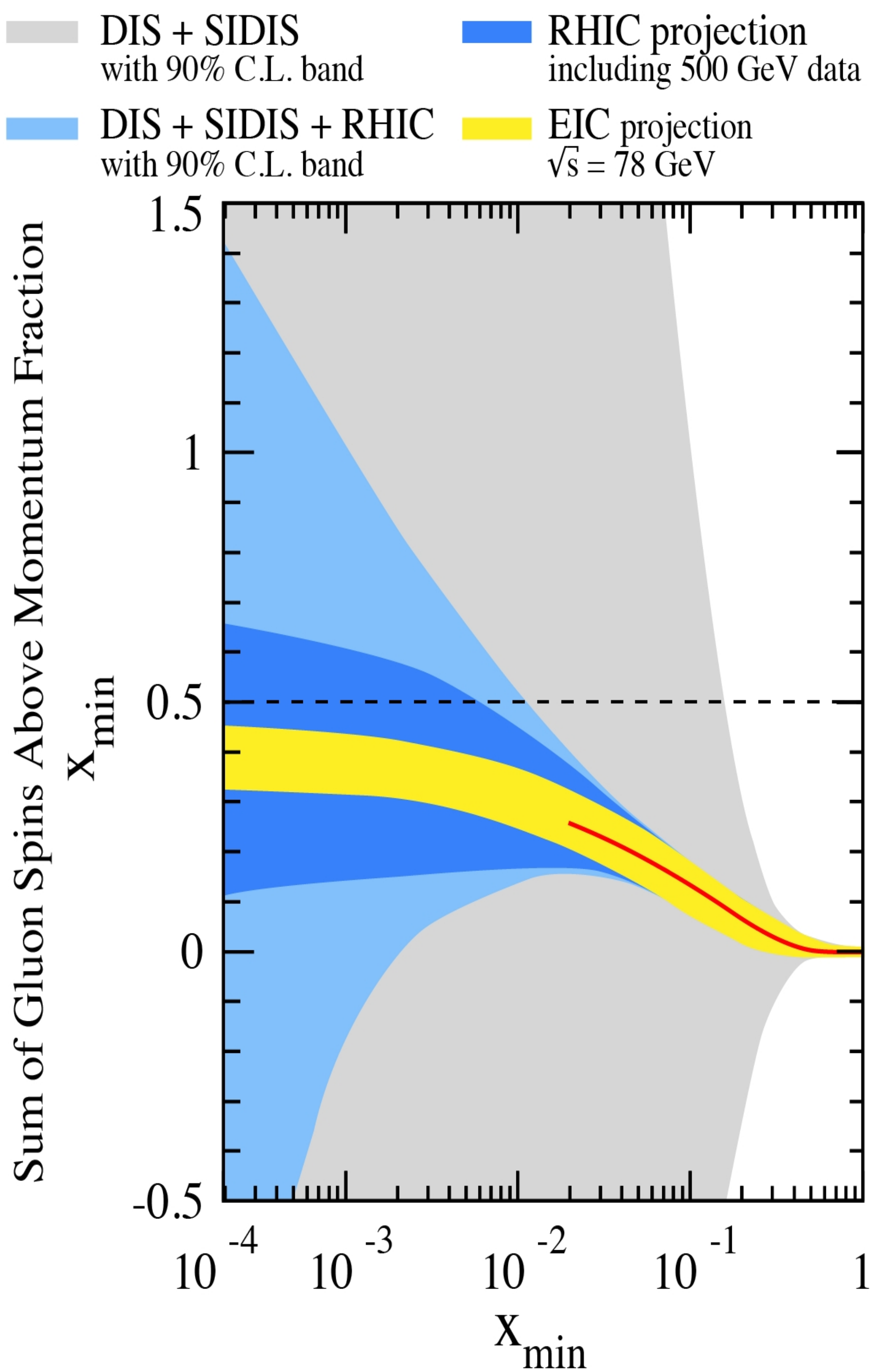
$$A_{LL} = \frac{\sigma(\vec{p}\,\vec{p} \to \text{jet } X) - \sigma(\vec{p}\,\overleftarrow{p} \to \text{jet } X)}{\sigma(\vec{p}\,\vec{p} \to \text{jet } X) + \sigma(\vec{p}\,\overleftarrow{p} \to \text{jet } X)}$$

Large uncertainties still, but ΔG is nonzero

de Florian, Sassot, Stratmann, Vogelsang, PRL 2014

$$\Delta G = \int_0^1 \Delta g(x)\, dx$$

ΔG at EIC

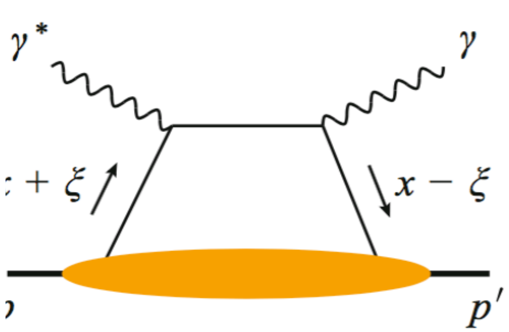


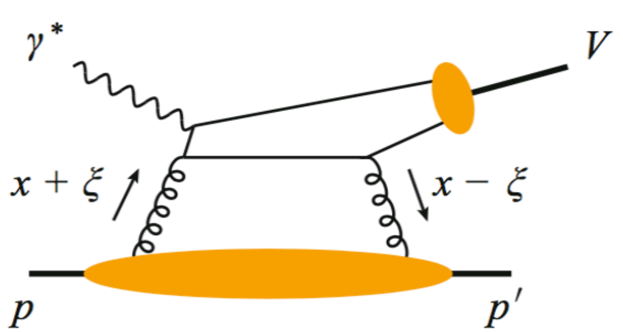
$$\Delta G = \lim_{x_{\min}\to 0} \int_{x_{\min}}^{1} \Delta g(x)\, dx$$

For ΔG ≈ 0.33 (2/3 of proton spin) there would be little room for orbital angular momentum L_z

Importance of L_z remains to be seen

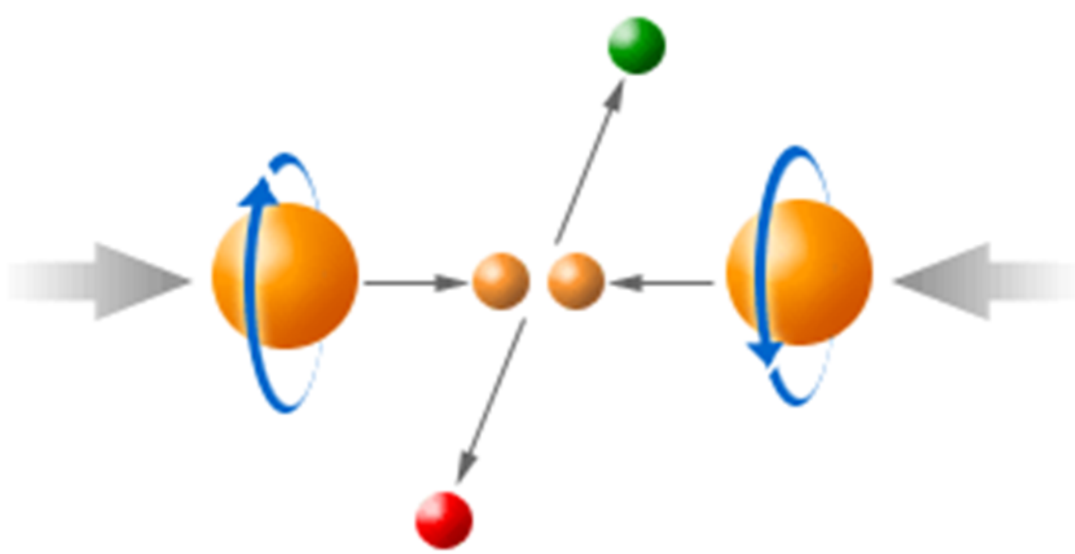
Independent estimates of L_z can be obtained from DVCS (also at EIC)

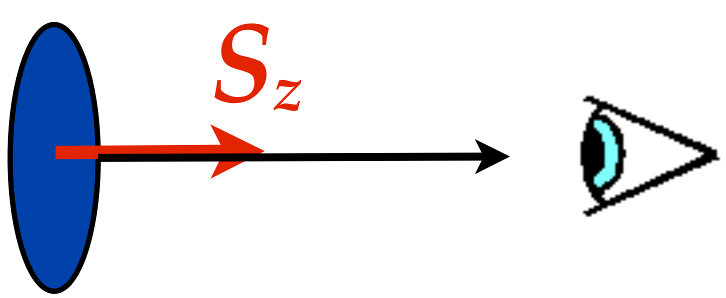


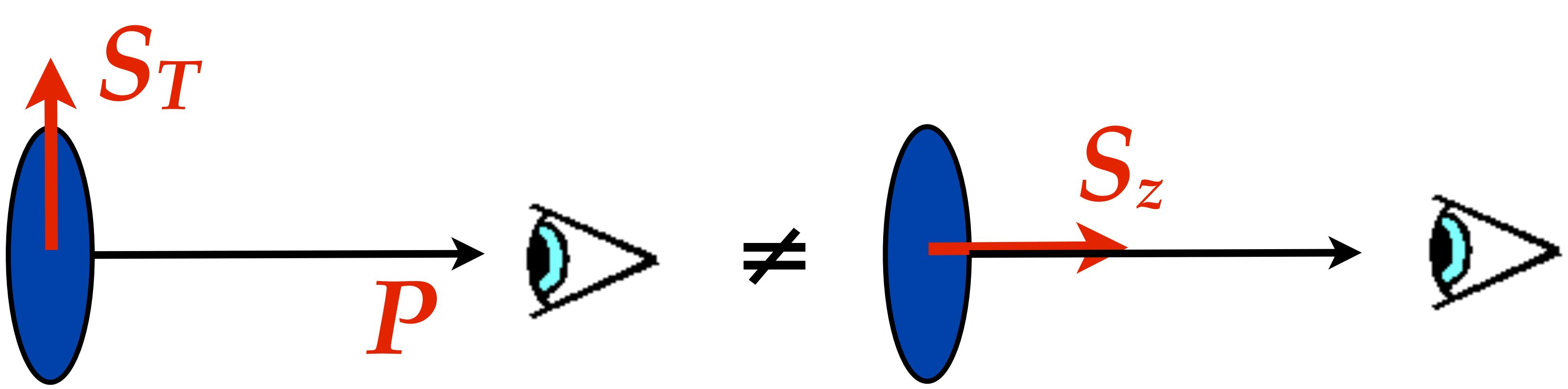


Transverse spin structure

The proton spin decomposition refers to spin along the momentum direction:







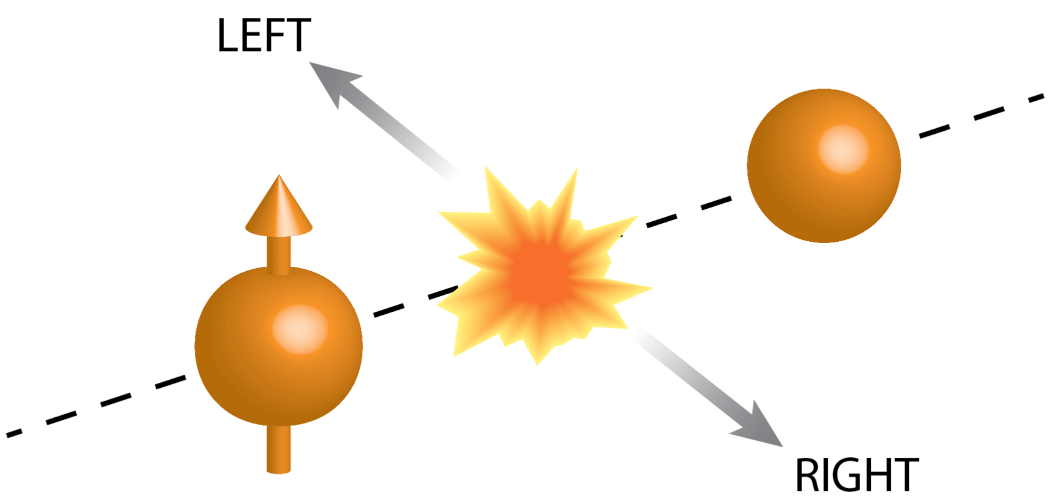
What one sees to the left and right of the plane spanned by P & S_T may differ

A left-right asymmetry is called the Sivers effect

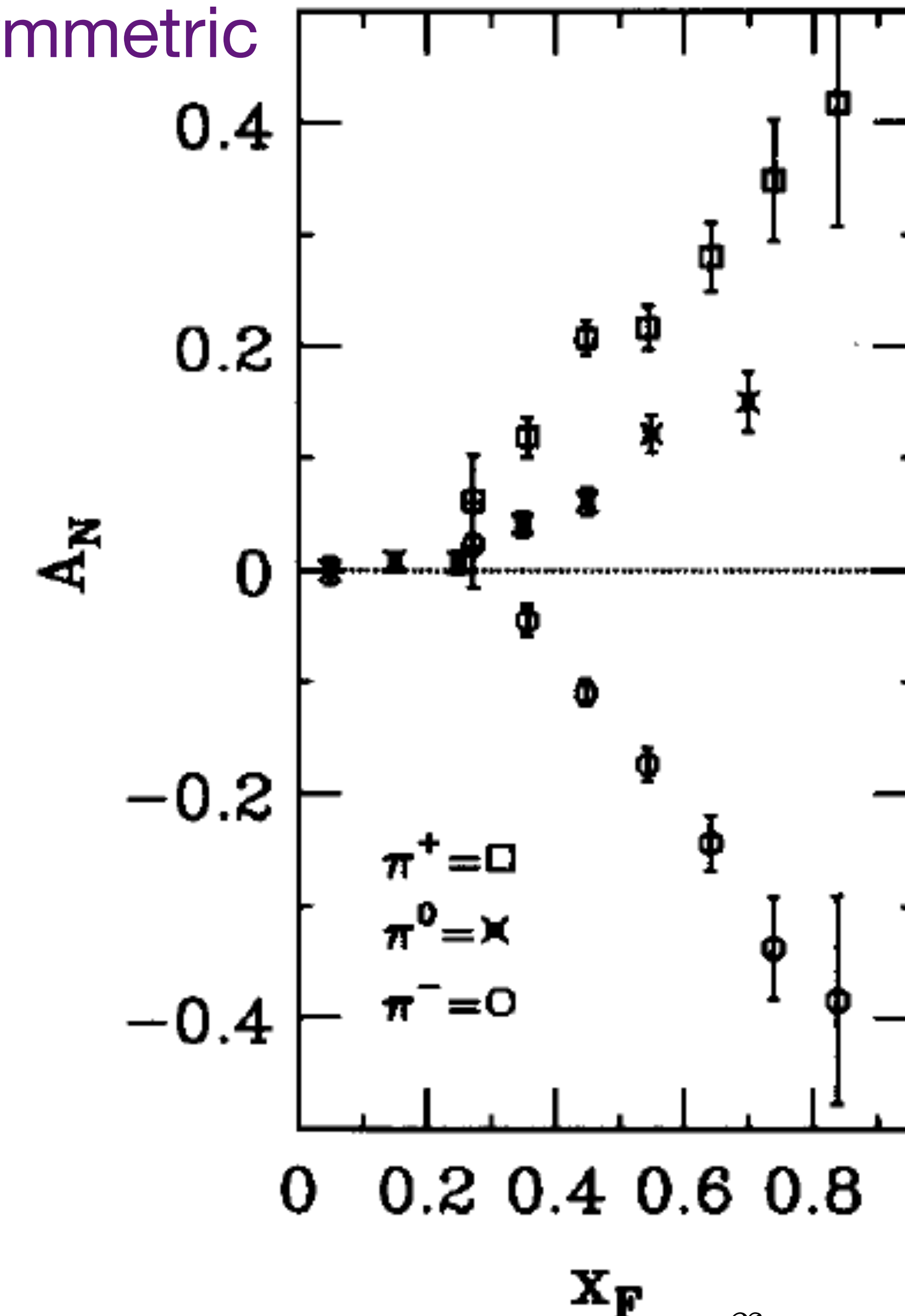
Sivers, 1989/90

Transverse spin asymmetries

Distribution of produced particles highly asymmetric



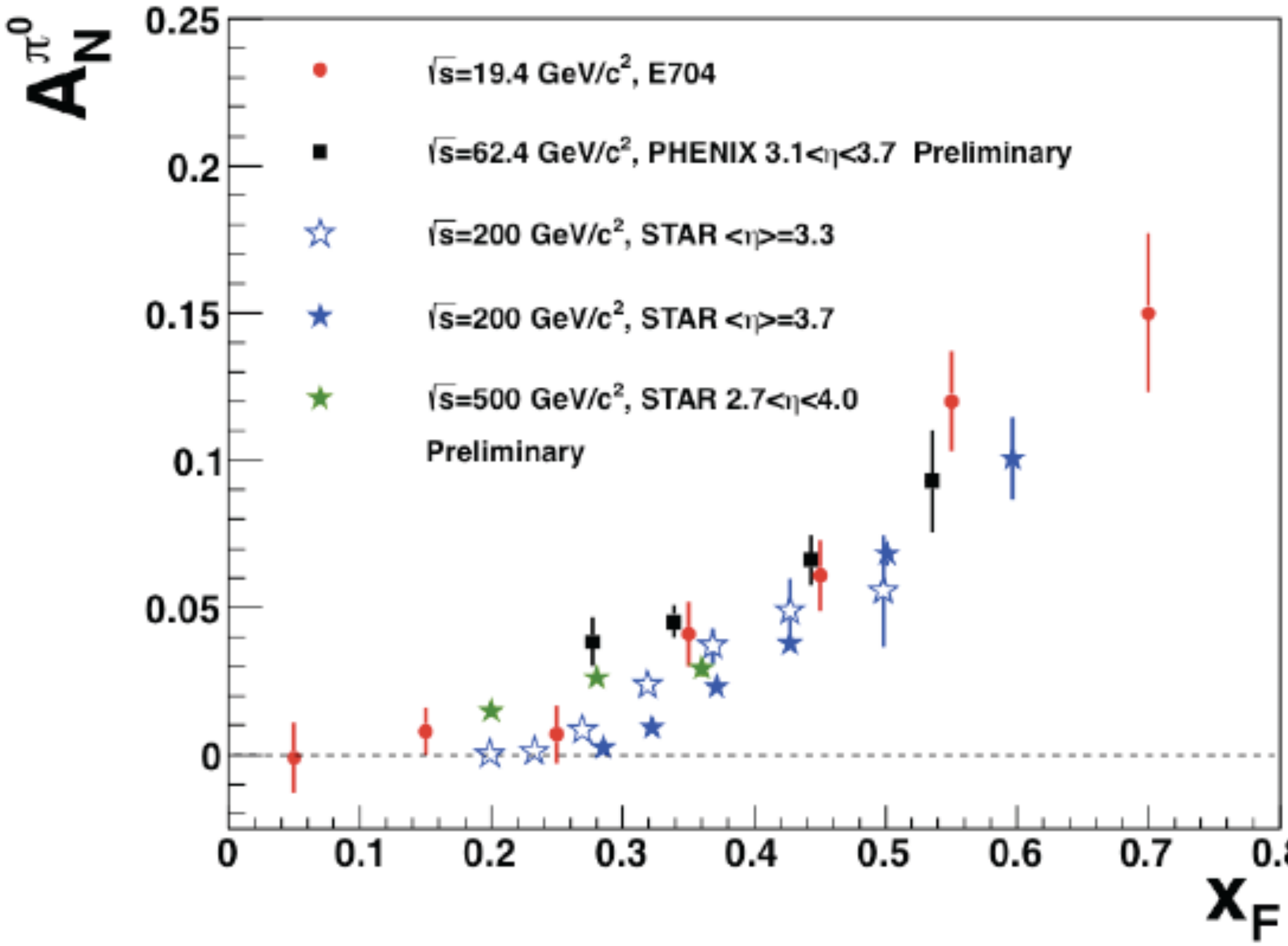
$$A_N = \frac{\sigma(p^\uparrow p \to \pi X) - \sigma(p^\downarrow p \to \pi X)}{\sigma(p^\uparrow p \to \pi X) + \sigma(p^\downarrow p \to \pi X)}$$

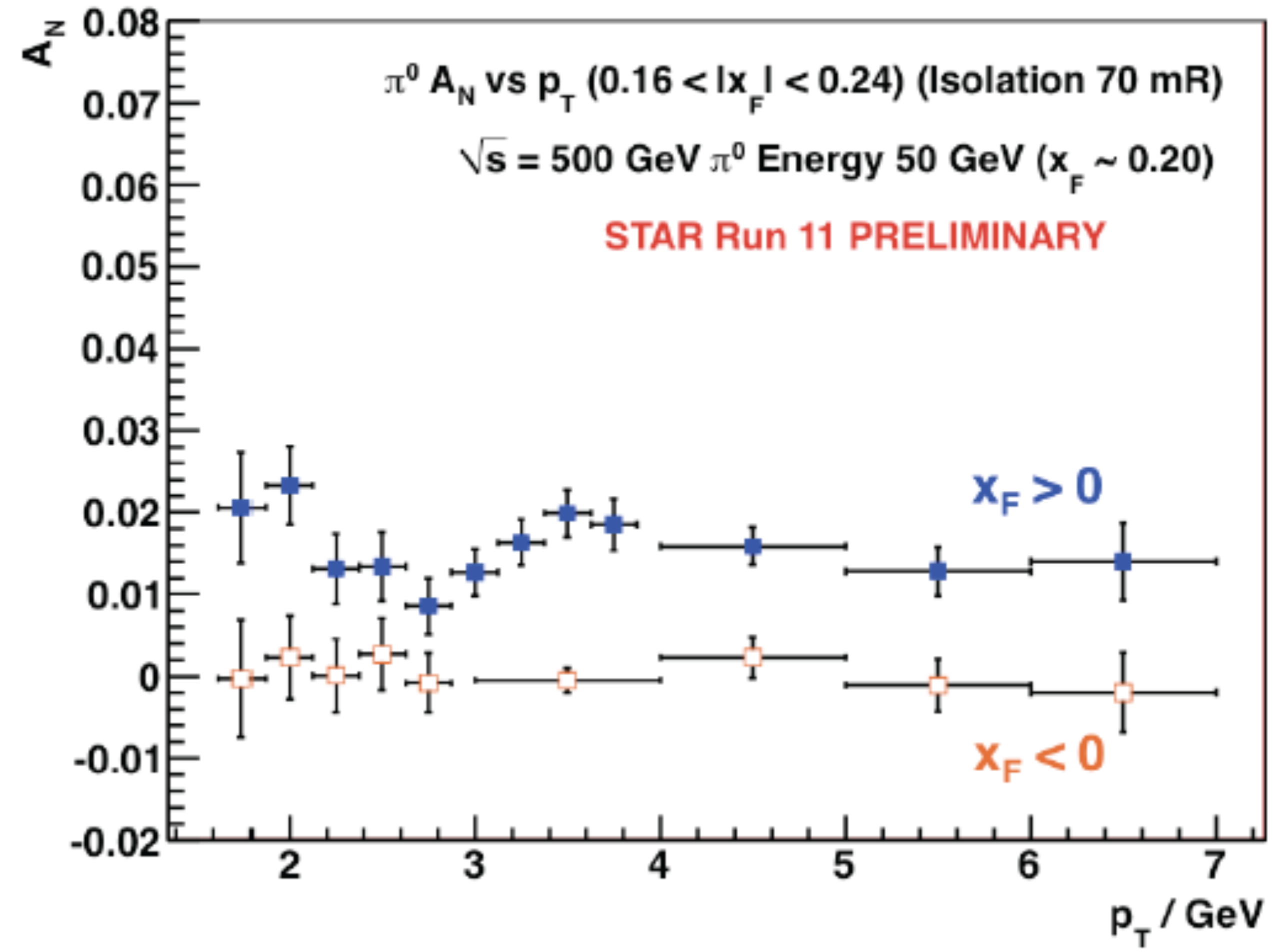


$$x_F = \frac{2p_z}{\sqrt{s}}$$

[Fermilab: E704 ('91) & BNL: AGS ('99); STAR ('02); BRAHMS ('05); PHENIX ('13)]

Left-right asymmetries

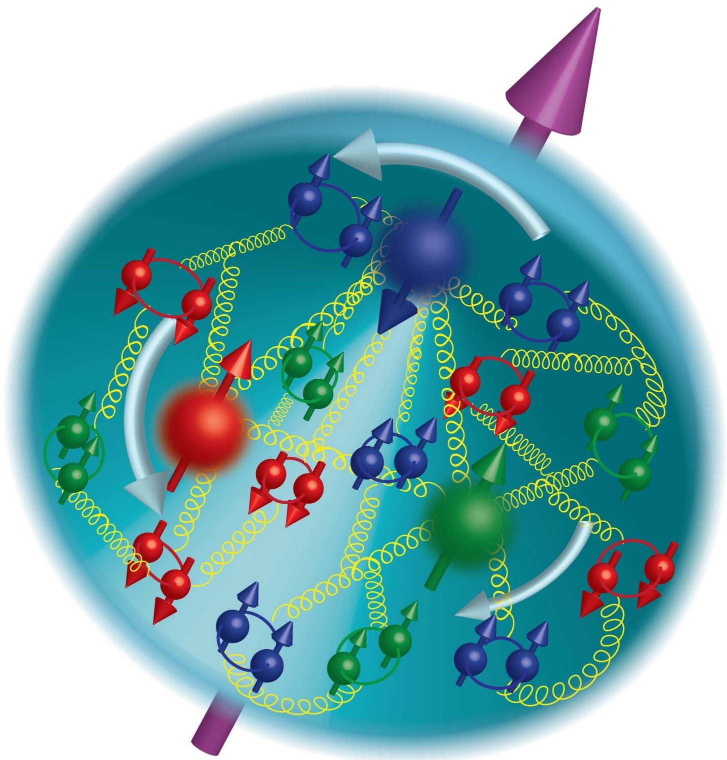




The asymmetries remain as the center of mass energy of the collision increases (even at 0.5 TeV and for pion p_T up to 7 GeV (≈ 50 m_π)

High energy means short distance, i.e. very local interactions

Left-right asymmetries



Feynman & Bjorken (1969): high energy scattering off a proton is to good approximation scattering off free quarks

(plus small corrections)

At energies up to 500 GeV the standard perturbative description ought to work (like for the cross section)

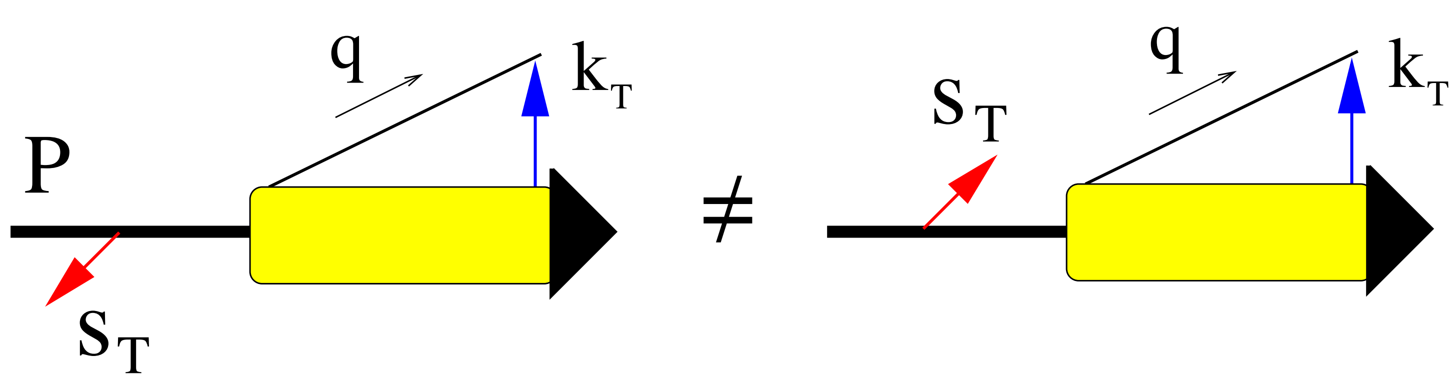
However, if the asymmetry is generated entirely perturbatively, then it is tiny

$$A_N \propto \alpha_s m_q / \sqrt{\hat{s}} \ll 10^{-4}$$

Transverse spin asymmetries

Clearly a spin-orbit coupling that persists to high center of mass energies

What is the explanation on the quark-gluon level?



D. Sivers ('89/'90)

$$k_T \times S_T$$

Introducing such a quantity allows to describe the data

[Anselmino, Boglione, D'Alesio, Murgia, ... ('95-...)]

Theoretical definition as a k_T-dependent parton distribution (a TMD) is not straightforward and has been modified/improved over the years

[Soper, 1977; Collins, 1993 & 2002; Belitsky, Ji & Yuan, 2003; Ji, Ma & Yuan, 2005; Collins 2011]

Gluon TMDs

Gluon TMDs

Transverse momentum dependent distributions involve more than just:

$$g(x, Q^2) \to g(x, k_T, Q^2)$$

The transverse momentum has a direction, which can be correlated with the transverse spin

Formally scattering processes involve the gluon correlator:

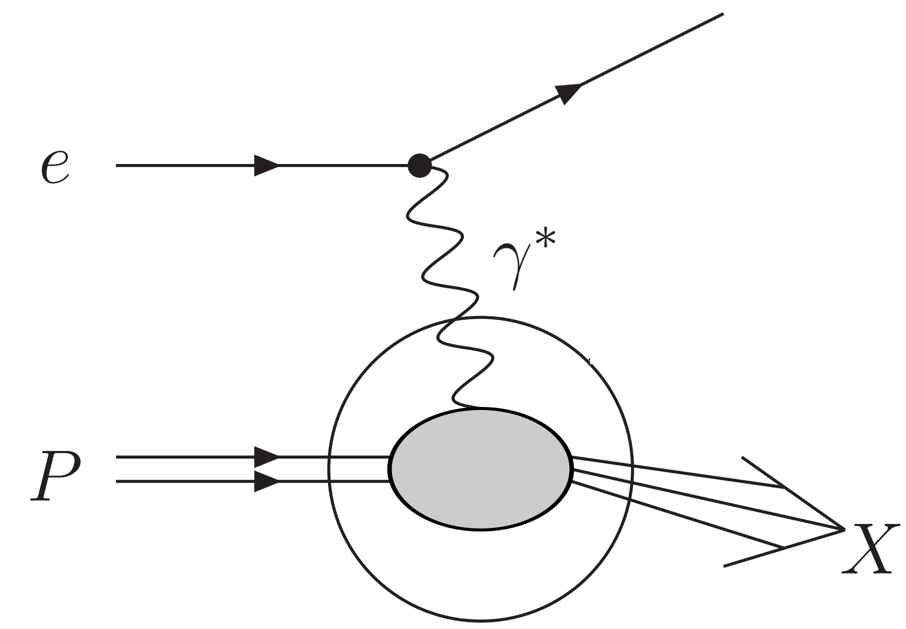
$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x, k_T) \equiv \mathrm{F.T.}\langle P|\mathrm{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

This is a nonlocal operator matrix element

High energy scattering

No matter how high the energy and how small the coupling constant(s), there will always be a nonperturbative contribution from large distances

The idea is to use the high energy to make an expansion and factorize the scattering into long and short distance parts

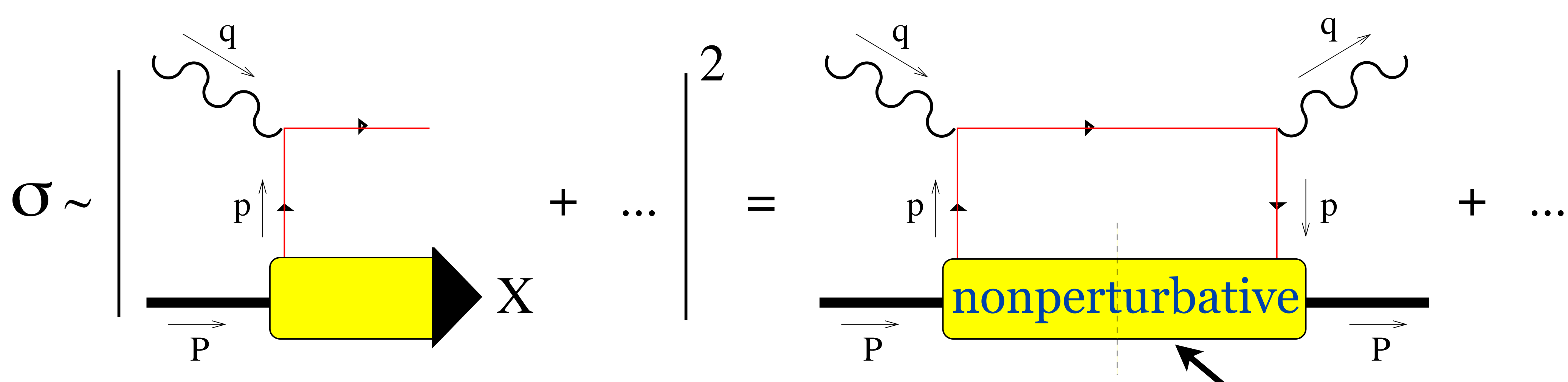


Sometimes (rarely!) this can be done by the Operator Product Expansion in terms of local operators, but often one needs nonlocal operators

$$T\, J(x) J(0) = \sum_{i,n} C_n^{(i)}(x^2)\, x^{\mu_1} \ldots x^{\mu_n}\, O^{(i)}_{\mu_1 \ldots \mu_n}(0)$$

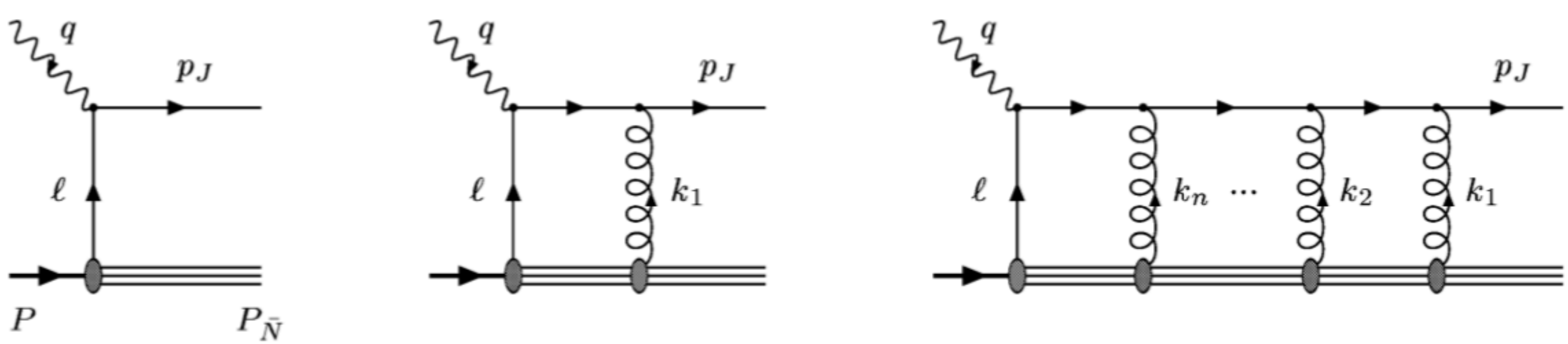
Scattering processes

$\gamma^* p$ scattering (DIS) in lowest order: parton scattering times parton density



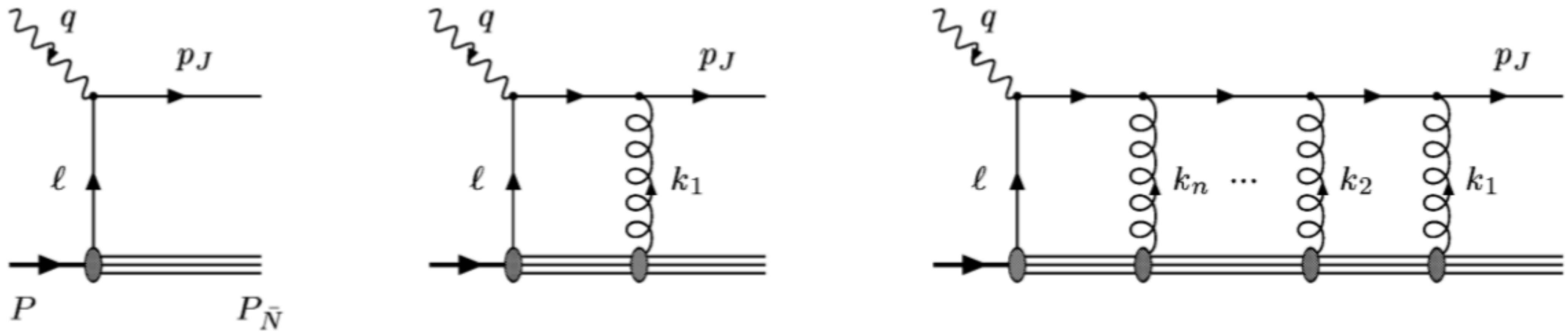
⟨ hadron state | operator | hadron state ⟩ → $\Phi \propto \langle P|\overline{\psi}(0)\psi(\xi)|P\rangle$

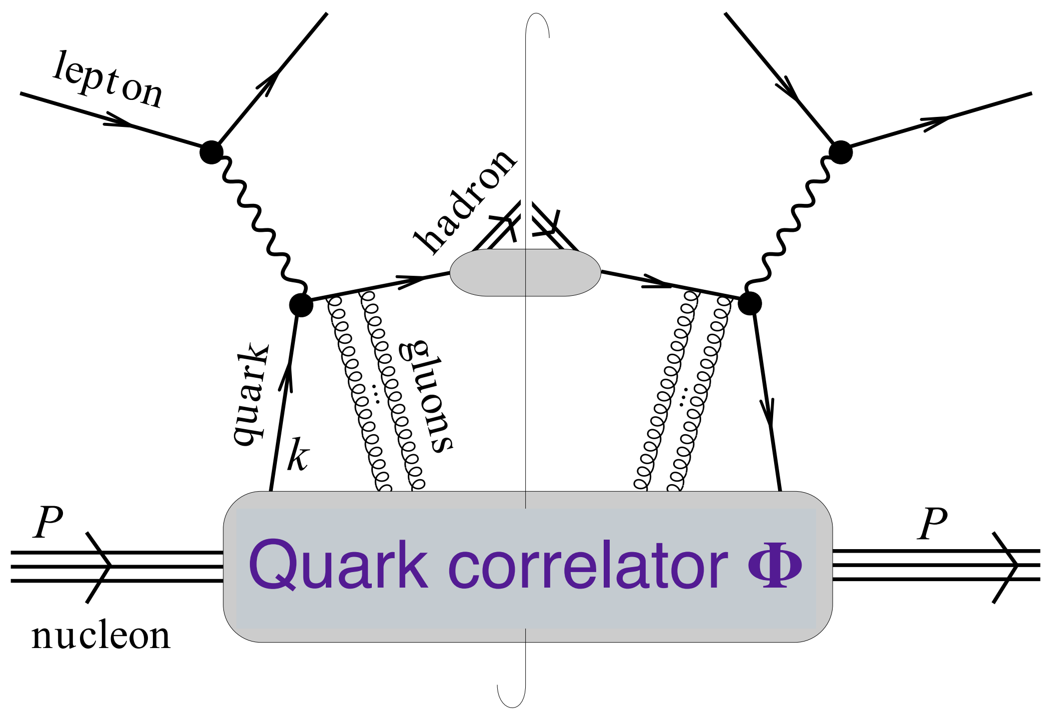
The operator involved becomes gauge invariant only after inclusion of infinitely many gluon exchanges



Gauge invariance of correlators

Summation of all gluon exchanges leads to gauge links (path-ordered exponentials) in the operators





$$\Phi \propto \langle P|\overline{\psi}(0)\mathcal{U}_{\mathcal{C}}[0,\xi]\psi(\xi)|P\rangle$$

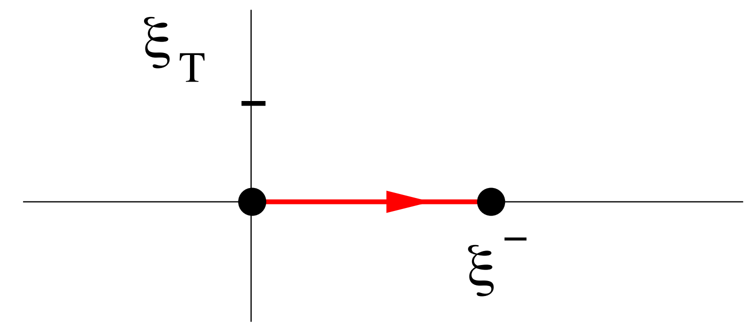
$$\mathcal{U}_{\mathcal{C}}[0,\xi] = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s)\right)$$

Efremov & Radyushkin, Theor. Math. Phys. 44 ('81) 774

Gauge invariance of correlators

The path C depends on whether the color interactions are with an incoming or outgoing color charge, yielding different paths for different processes

[Collins & Soper, 1983; Boer & Mulders, 2000; …]



The integration path in DIS is a straight gauge link along the lightcone

Picking the lightcone gauge A⁺=0 yields $\mathcal{U}_C[0,\xi]=1$

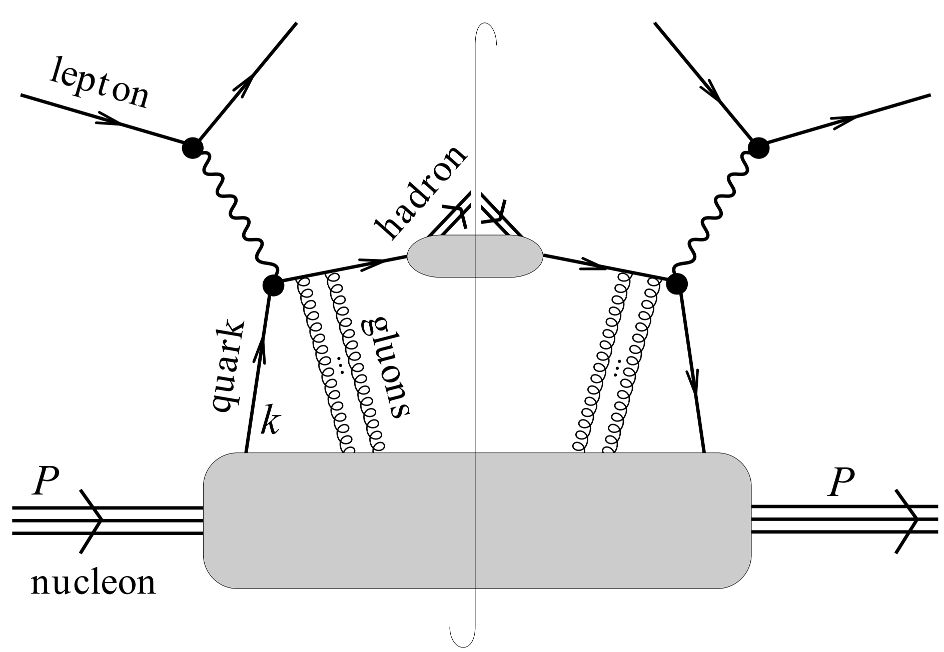
$$x^{\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^3)$$

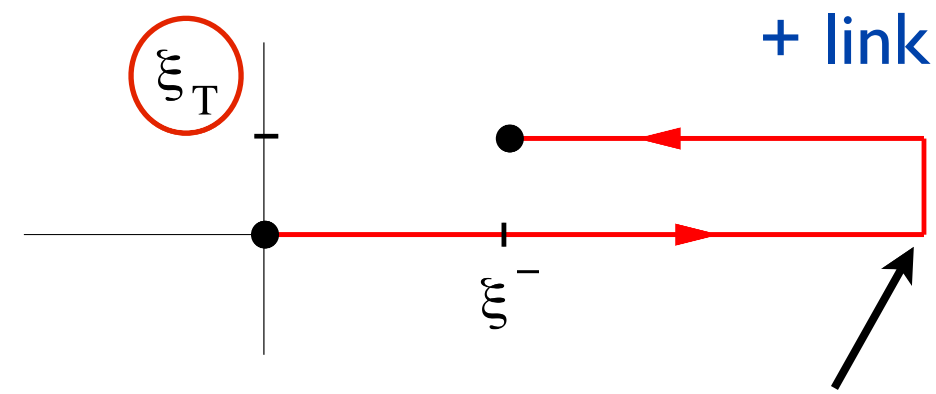
semi-inclusive DIS

$$e\,p \rightarrow e'\,h\,X$$

$$k \approx xP + k_T$$

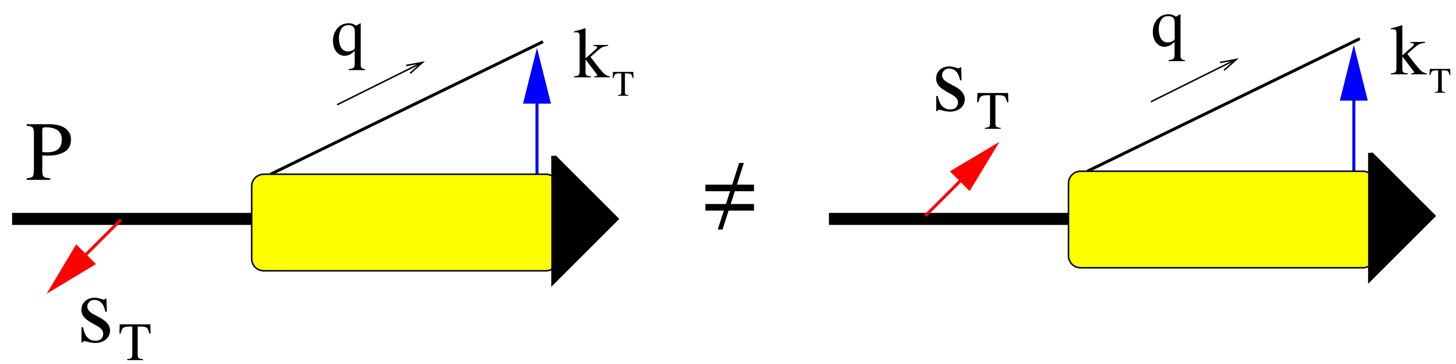
$$P^\mu \approx P^+$$





lightcone infinity ∞⁻

Sivers TMD definition



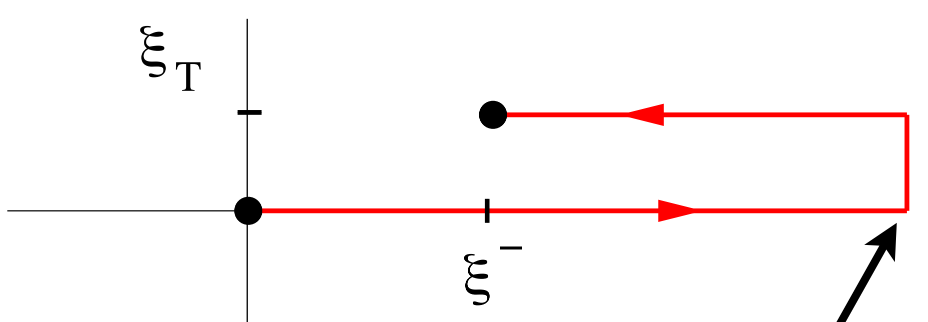
D. Sivers ('89/'90)

Sivers effect as a TMD:

$$P\cdot(\mathbf{k}_T \times \mathbf{S}_T)\, f_{1T}^{\perp[\mathcal{C}]}(x,\mathbf{k}_T^2) \propto \text{ F.T. } \langle P, S_T | \bar{\psi}(0)\, \mathcal{L}_{\mathcal{C}[0,\xi]}\, \gamma^+\, \psi(\xi) | P, S_T\rangle \big|_{\xi=(\xi^-,0^+,\xi_T)}$$

$$\mathcal{L}_{\mathcal{C}}[0,\xi] = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s)\right)$$

semi-inclusive DIS
$e\,p \to e'\,h\,X$

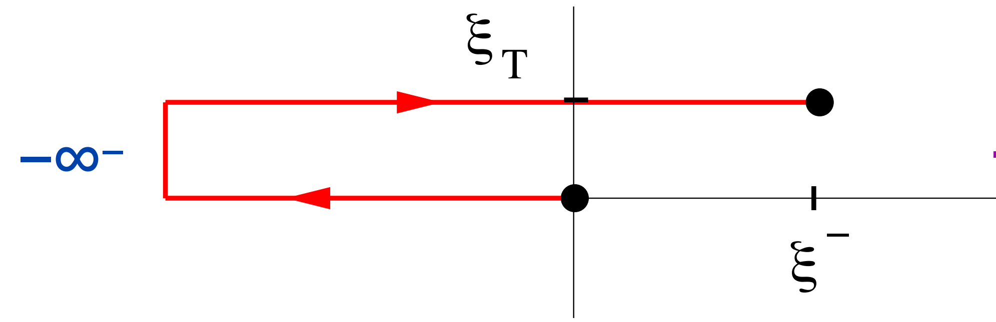


+ link

lightcone infinity ∞^-

Effects of gauge links

$pp \to \gamma^* X$ Drell-Yan:



− link

Different gauge links does not automatically imply that they affect observables, but it turns out that they do in certain cases

Gauge links can affect observables sensitive to transverse momentum

This was first pointed out for spin asymmetries

Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, Ji & Yuan, 2003

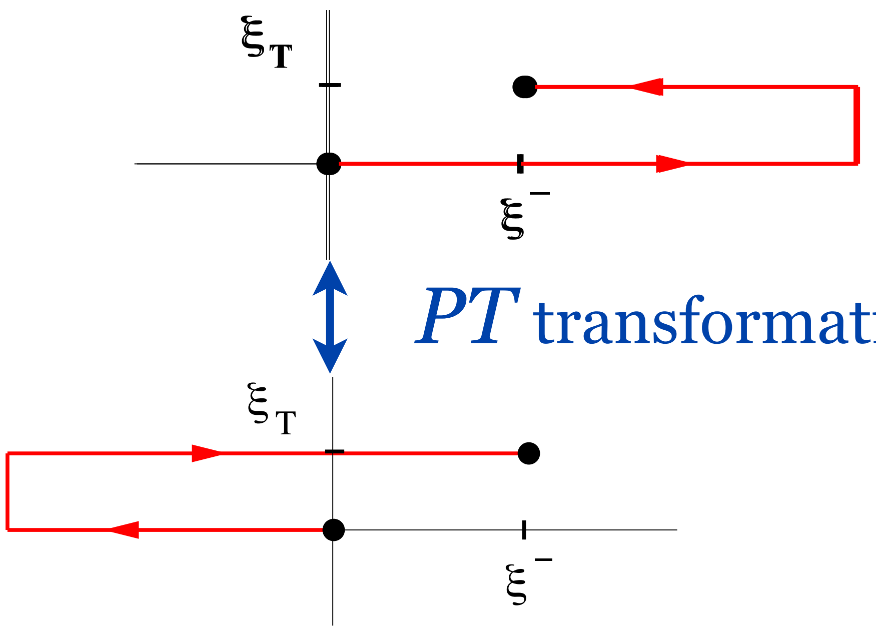
Gauge links affect even unpolarized gluon TMDs

Dominguez, Marquet, Xiao, Yuan, 2011

Process dependence of Sivers TMD

Initially it was thought that the path of the gauge link is irrelevant, because a gauge can always be chosen such that it is unity

Lightcone gauge ($A^+=0$) with advanced boundary condition ($A_T(\infty^-,\xi_T)=0$):



$= 1$

Sivers effect is <u>odd</u> under + ↔ −

PT transformation: $f_{1T}^{\perp[+]}(x,p_T^2) = -f_{1T}^{\perp[-]}(x,p_T^2)$

$$x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^3) \xrightarrow{T} -x^\mp \xrightarrow{P} -x^\pm$$

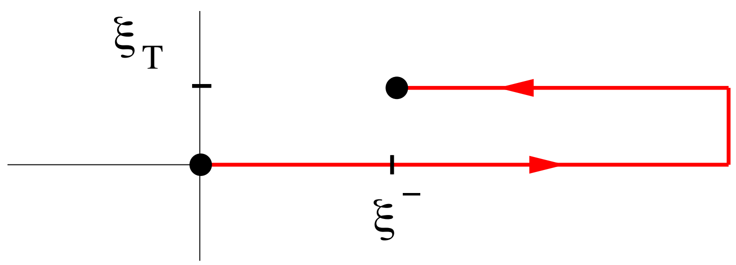
[Collins '02]

If the gauge link does not matter, then the Sivers effect must vanish

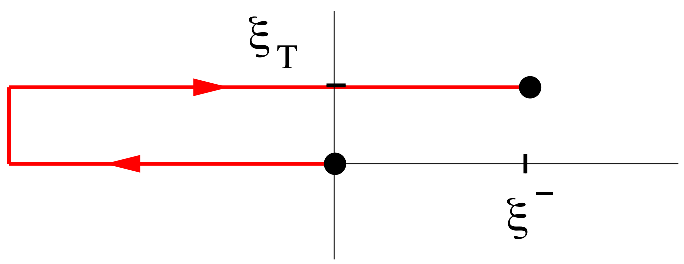
However, under a *PT* transformation the advanced boundary condition becomes a retarded one and one cannot impose both simultaneously!

Contour gauge

Imposing lightcone gauge ($A^+=0$) with advanced *and* retarded boundary condition ($A_T(\pm\infty^-,\xi_T)=0$) is not allowed, as it overfixes the gauge

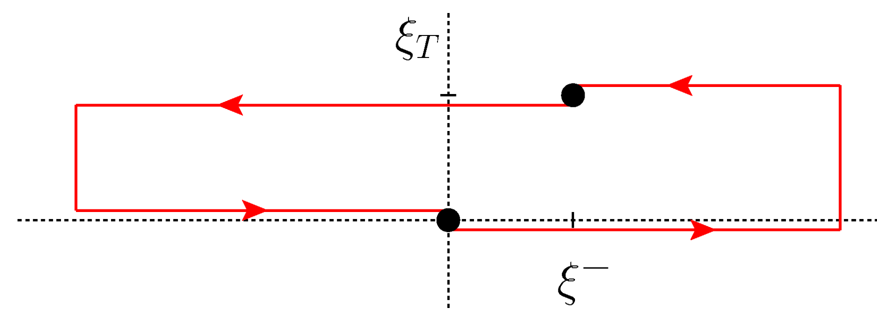


$= 1$



$= 1$

$\Big\}\Longrightarrow$



$= 1$

Not allowed since gauge loop is gauge invariant

It measures the flux of $F^{\mu\nu}$ through the loop

Contour gauge:

$$P\exp\left(ig_s\int_{x_0}^{x}ds_\mu A^\mu(s)\right)=1$$

S.V. Ivanov, G.P. Korchemsky & A.V. Radyushkin, Sov. J. Nucl. Phys. 44 (1986) 145

Contour gauge can be imposed for each *x* for just one non-self-intersecting path from x_0 to x

Sign change relation of the Sivers TMD

Consequence: a *calculable* process dependence

$$f_{1T}^{\perp[\text{SIDIS}]} = -f_{1T}^{\perp[\text{DY}]}$$

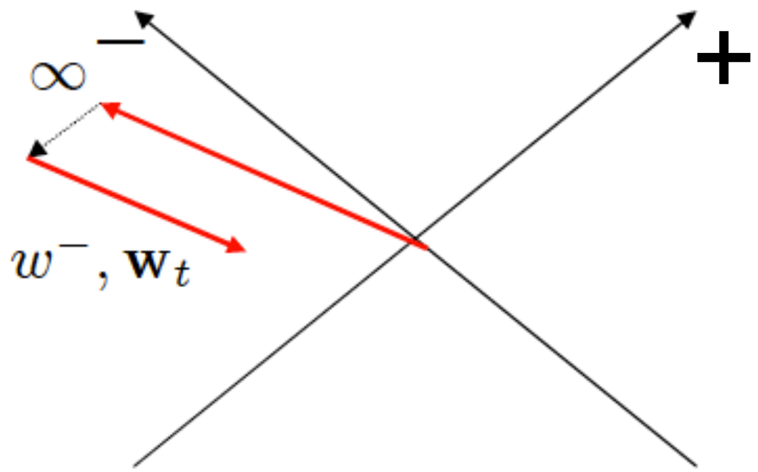
[Collins '02]

Experimental tests ongoing at CERN (COMPASS), Fermilab (SeaQuest), RHIC

Not just a test of this relation, but of TMD factorization formalism in essence

QCD corrections will also attach to the gauge link, which yields divergences when the gauge link is lightlike

As a regularization the path is taken off the lightcone

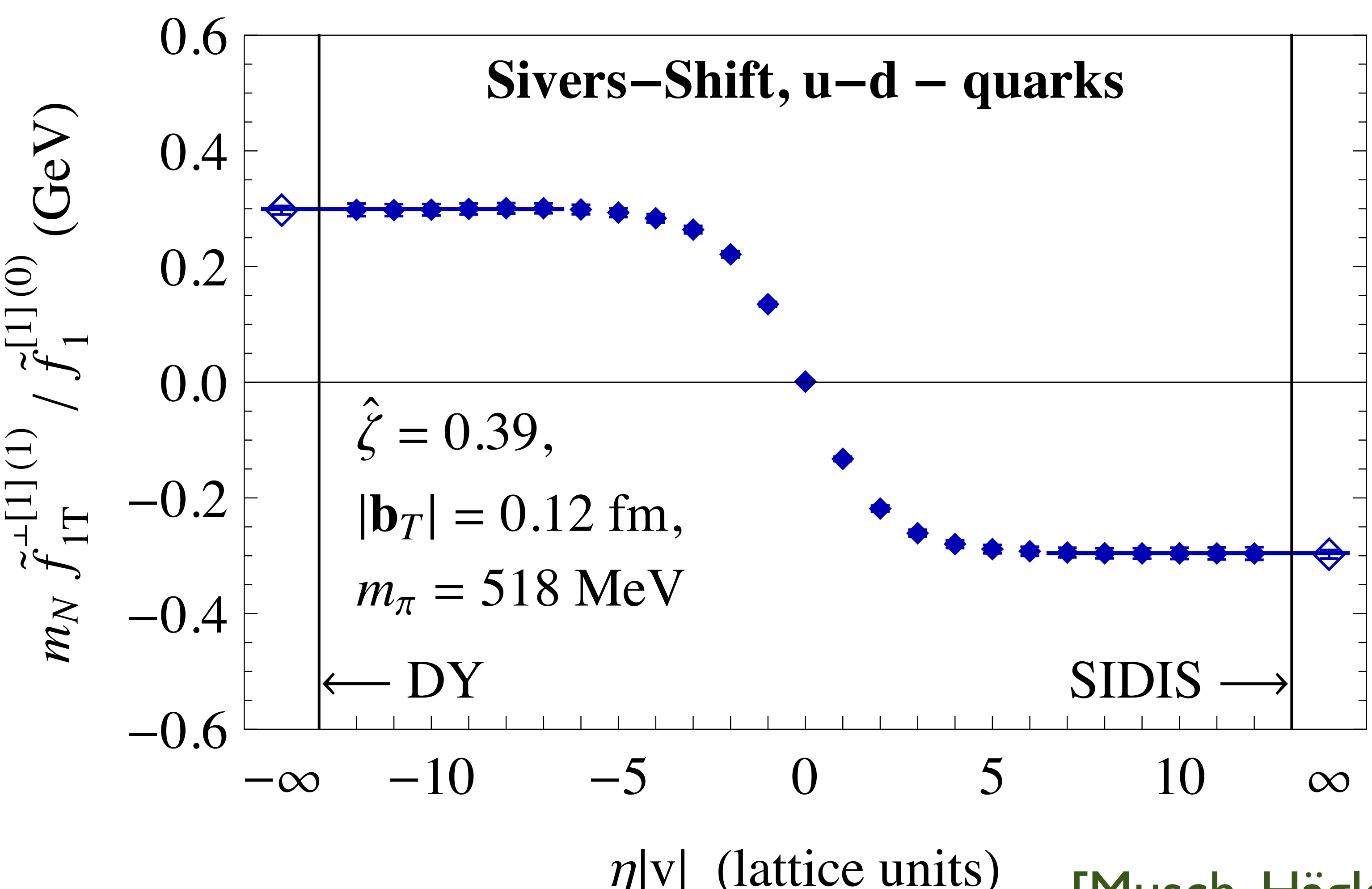


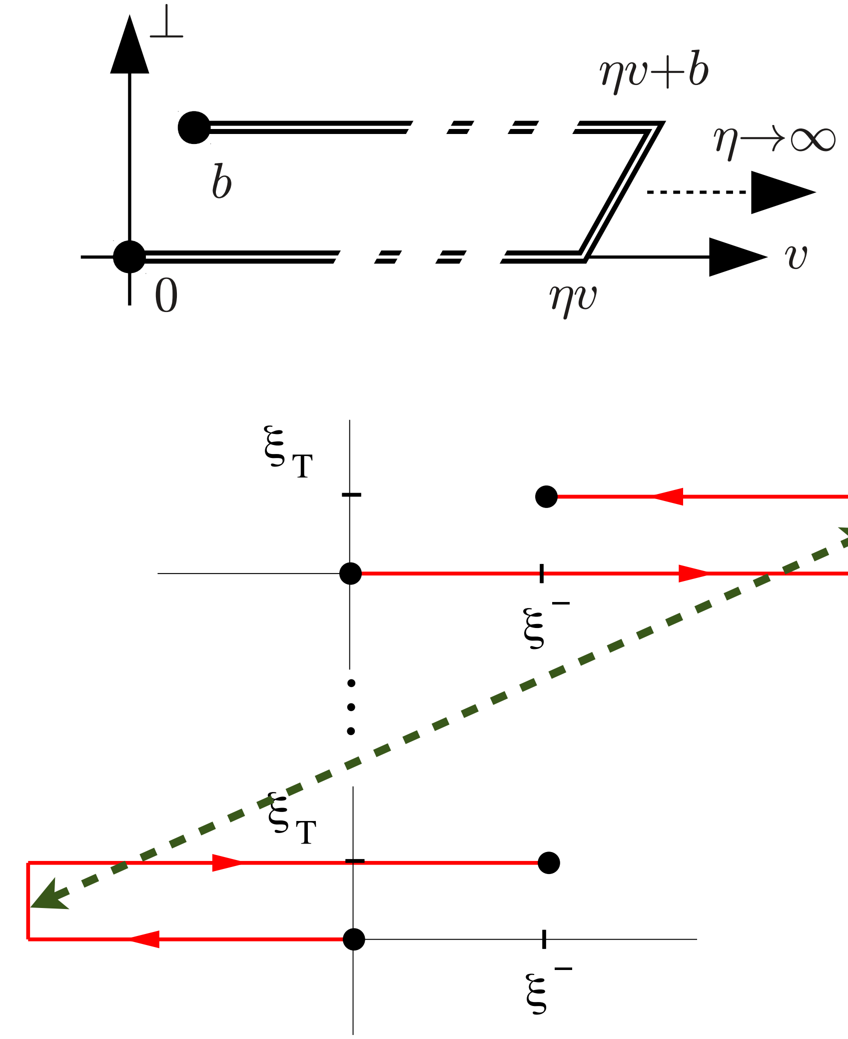
Allows for calculation of the Sivers effect on the lattice

Sivers effect on the lattice

The "Sivers shift" <$k_T \times S_T$> (the average transverse momentum shift orthogonal to transverse spin S_T) can be calculated on the lattice

Boer, Gamberg, Musch, Prokudin, 2011

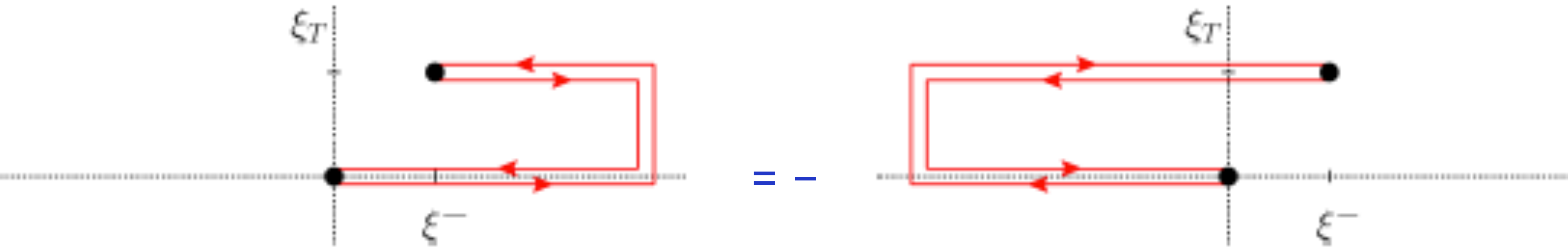




[Musch, Hägler, Engelhardt, Negele & Schäfer, 2012]

This is the first first-principle demonstration in QCD that the Sivers function is nonzero for staple-like links. It clearly displays the sign change relation

Sign change relation for gluon Sivers TMD

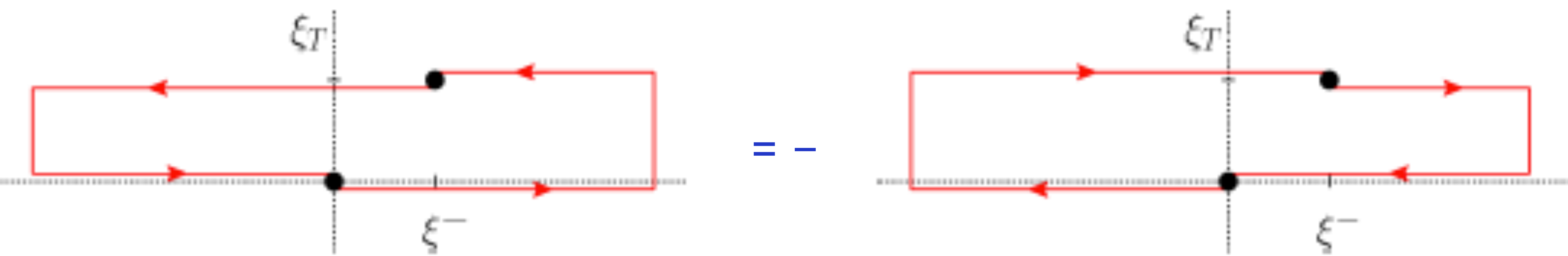


$$f_{1T}^{\perp\, g\,[e\,p^\uparrow \to e'\,Q\,\overline{Q}\,X]}(x, p_T^2) = -f_{1T}^{\perp\, g\,[p^\uparrow\,p \to \gamma\,\gamma\,X]}(x, p_T^2)$$

EIC

RHIC

D.B., Mulders, Pisano, J. Zhou, 2016



There is a distinct, *independent* gluon Sivers function with [+,−] links

Gluon Sivers TMDs for [+,+] & [+,−] are related to the f^{abc} & d^{abc} color structures

Bomhof, Mulders, 2007; Buffing, Mukherjee, Mulders, 2013

Color gauge loop effects at small x

Dipole versus WW distributions

For most processes of interest there are 2 relevant unpolarized gluon TMDs

$$xG^{(1)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle$$ [+,+]

$$xG^{(2)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle$$ [+,−]

For unpolarized gluons [+,+] = [−,−] and [+,−] = [−,+]

At small x the two correspond to the Weizsäcker-Williams (WW) and dipole (DP) distributions, which are generally different in magnitude and width:

$$xG^{(1)}(x,k_\perp) = -\frac{2}{\alpha_S}\int \frac{d^2v}{(2\pi)^2}\frac{d^2v'}{(2\pi)^2}\, e^{-ik_\perp\cdot(v-v')}\left\langle \mathrm{Tr}\left[\partial_i U(v)\right]U^\dagger(v')\left[\partial_i U(v')\right]U^\dagger(v)\right\rangle_{x_g}$$ WW

$$xG^{(2)}(x,q_\perp) = \frac{q_\perp^2 N_c}{2\pi^2\alpha_s} S_\perp \int \frac{d^2r_\perp}{(2\pi)^2} e^{-iq_\perp\cdot r_\perp}\frac{1}{N_c}\left\langle \mathrm{Tr}U(0)U^\dagger(r_\perp)\right\rangle_{x_g}$$ DP

Dominguez, Marquet, Xiao, Yuan, 2011

Gauge loop correlator

The [+,–] gluon TMD correlator becomes in the small-x limit:

$$\Gamma^{[+,-]\,ij}(x,\boldsymbol{k}_T) \xrightarrow{x\to 0} \frac{k_T^i k_T^j}{2\pi L}\Gamma_0^{[\Box]}(\boldsymbol{k}_T)$$

a single gauge loop matrix element

$$U^{[\Box]} = U^{[+]}_{[0,y]}U^{[-]}_{[y,0]}$$

The d-type gluon Sivers function $f_{1T}^{\perp\, g\,[+,-]}$ at small x is part of:

$$\left(\Gamma^{[+,-]} - \Gamma^{[-,+]}\right) \propto \mathrm{F.T.}\,\langle P,S_T|\mathrm{Tr}\left[U^{[\Box]}(0_T,y_T) - U^{[\Box]\dagger}(0_T,y_T)\right]|P,S_T\rangle$$

D.B., Echevarria, Mulders, J. Zhou, PRL 2016

At small x the single spin asymmetry probes the imaginary part of the loop

This can be identified with the *spin-dependent odderon* [J. Zhou, 2013]

Gluons TMDs

The gluon correlator:

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \mathrm{F.T.}\langle P|\mathrm{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

For unpolarized protons:

$$\Gamma_U^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{x}{2}\left\{-g_T^{\mu\nu}f_1^g(x,\boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right)h_1^{\perp\,g}(x,\boldsymbol{p}_T^2)\right\}$$

unpolarized gluon TMD

linearly polarized gluon TMD

[Mulders, Rodrigues '01]

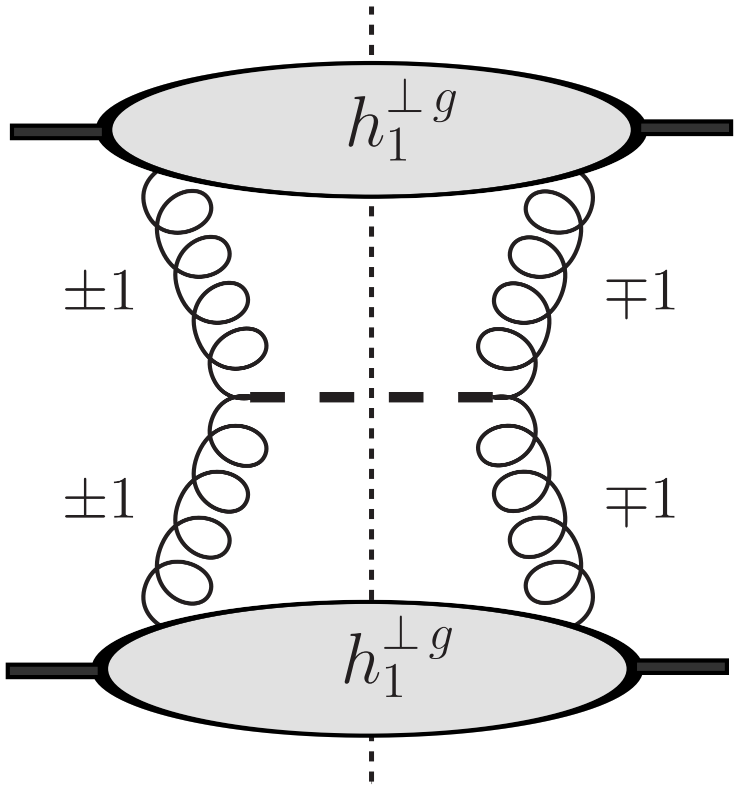
Gluons inside *unpolarized* protons can be polarized!

For transversely polarized protons:

gluon Sivers TMD

$$\Gamma_T^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{x}{2}\left\{g_T^{\mu\nu}\frac{\epsilon_T^{\rho\sigma}p_{T\rho}S_{T\sigma}}{M_p}f_{1T}^{\perp\,g}(x,\boldsymbol{p}_T^2) + \ldots\right\}$$

Linear gluon polarization at EIC



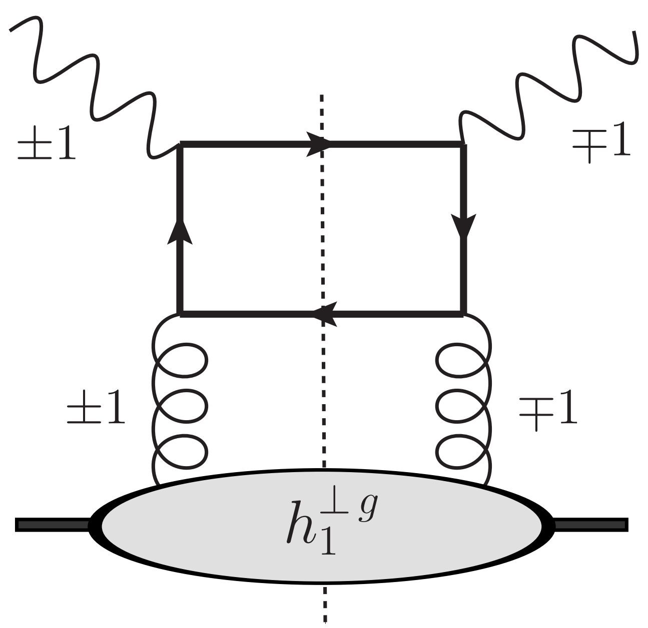
It affects Higgs production at the LHC

D.B., den Dunnen, Pisano, Schlegel, Vogelsang, 2011
Sun, Xiao, Yuan, 2011

It remains to be seen whether this can be exploited

At EIC it certainly can!

$$ep \to e' Q\bar{Q} X$$



$\cos 2(\phi_T - \phi_\perp)$ angular distribution

$\phi_{T/\perp}$ are the angles of $K_\perp^Q \pm K_\perp^{\bar{Q}}$

D.B., Brodsky, Mulders & Pisano, 2010

Linear gluon polarization at small x

The small-x limit of the DP correlator (at leading twist):

$$\Gamma^{[+,-]\,ij}(x,\boldsymbol{k}_T) \xrightarrow{x\to 0} \frac{k_T^i k_T^j}{2\pi L}\Gamma_0^{[\Box]}(\boldsymbol{k}_T) \qquad U^{[\Box]} = U_{[0,y]}^{[+]}U_{[y,0]}^{[-]}$$

$$\Gamma_U^{ij}(x,\boldsymbol{k}_T) = \frac{x}{2}\left[-g_T^{ij}\, f_1(x,\boldsymbol{k}_T^2) + \frac{k_T^{ij}}{M^2}\, h_1^\perp(x,\boldsymbol{k}_T^2)\right] \xrightarrow{x\to 0} \frac{k_T^i k_T^j}{2M^2}\, e(\boldsymbol{k}_T^2)$$

$$\lim_{x\to 0} x f_1(x,\boldsymbol{k}_T^2) = \frac{\boldsymbol{k}_T^2}{2M^2}\lim_{x\to 0} x h_1^\perp(x,\boldsymbol{k}_T^2) = \frac{\boldsymbol{k}_T^2}{2M^2}\, e(\boldsymbol{k}_T^2)$$

In the TMD formalism the DP $h_1^{\perp g}$ becomes maximal when x → 0

Boer, Cotogno, van Daal, Mulders, Signori & Ya-Jin Zhou, 2016

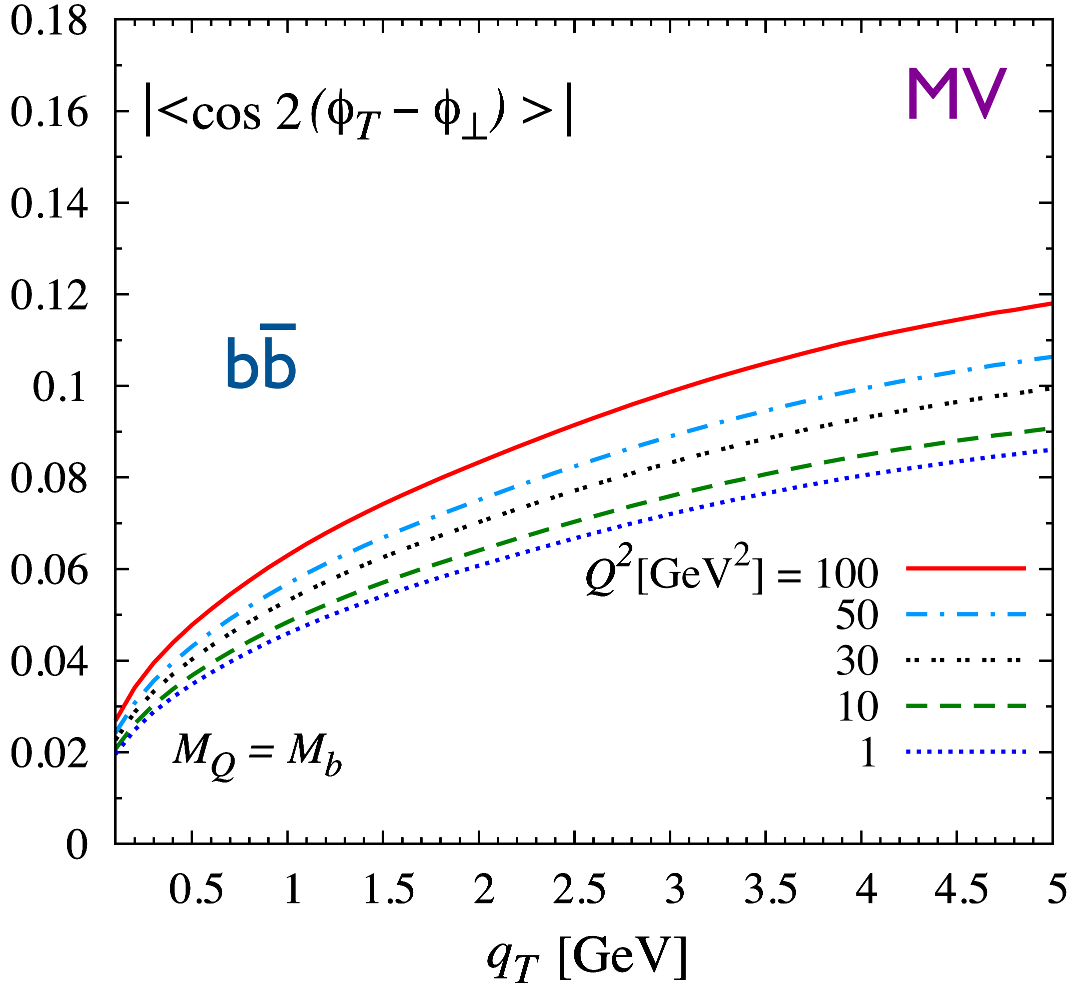
In line with MV model calculation:

$$x h_{1,DP}^{\perp g}(x,k_\perp) = 2x f_{1,DP}^{g}(x,k_\perp)$$

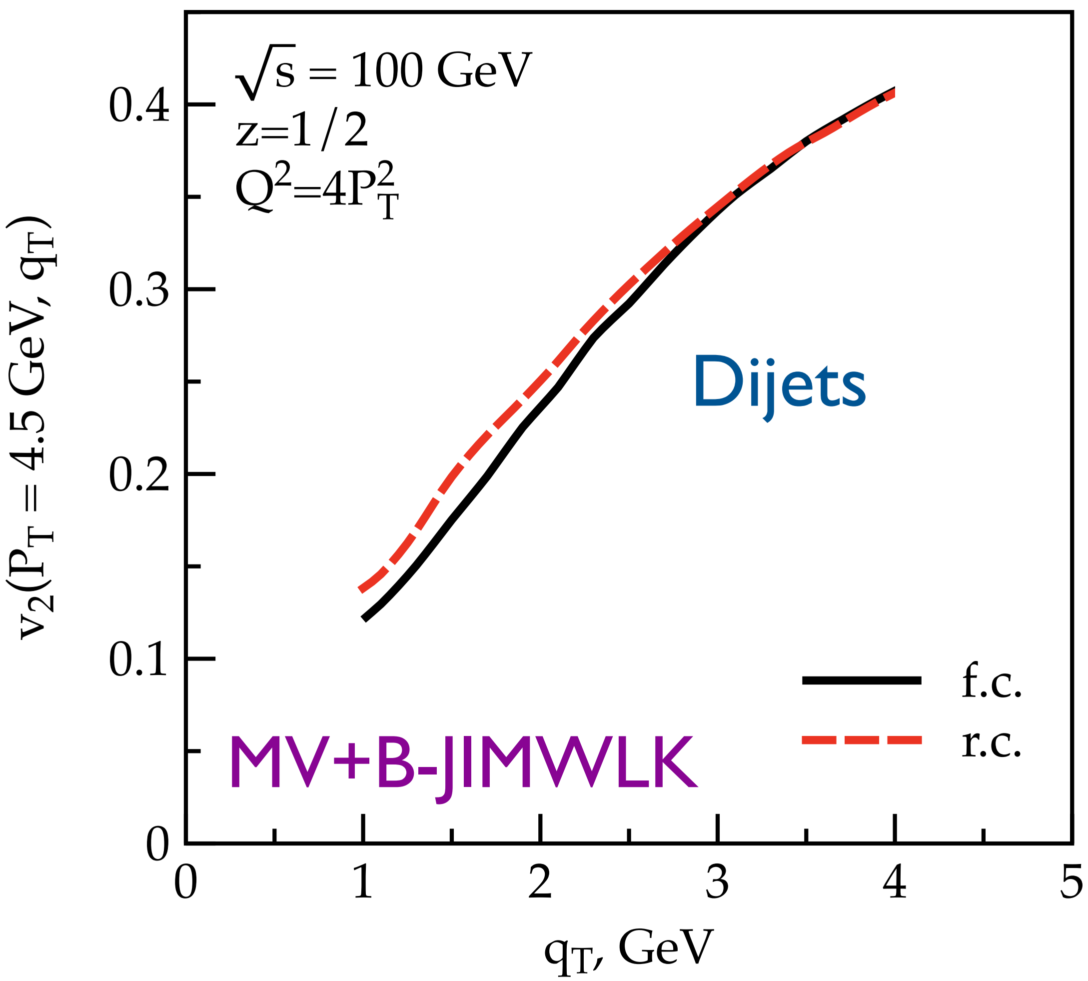
Metz, Zhou, 2011

Linear gluon polarization at EIC

$h_1^{\perp g}$ is expected to keep up with the growth of the unpolarized gluons as $x \to 0$



D.B., Pisano, Mulders, Zhou, 2016



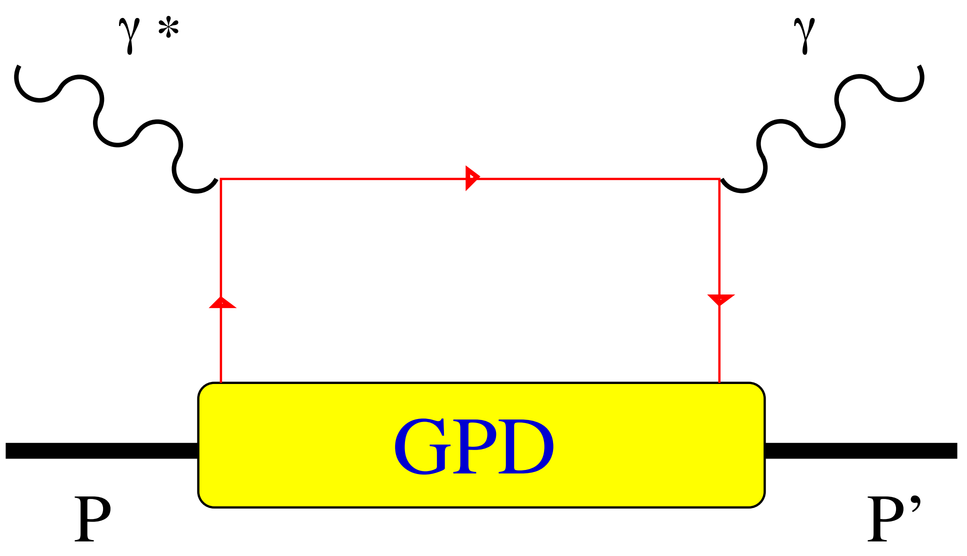
Dumitru, Lappi, Skokov, 2015

CGC gluons are linearly polarized, the size of the effects depends on the process

Gluon GPDs

GPDs

Deeply Virtual Compton Scattering (DVCS):



Theoretical description involves Generalized Parton Distributions (GPDs)

GPDs are off-forward matrix elements (P' ≠ P)

This describes the spatial distribution of quarks inside nucleons

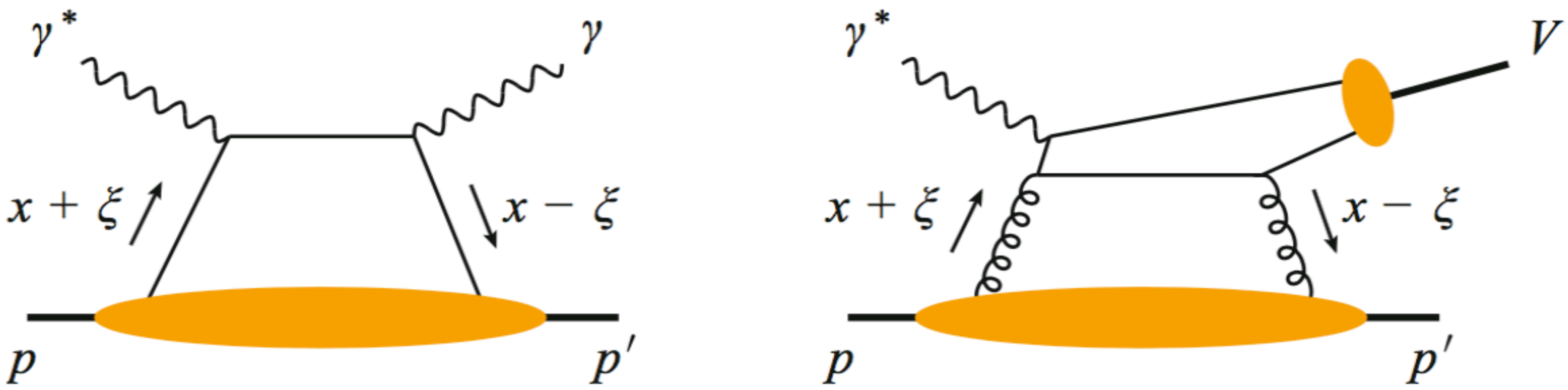
b_T is *not* the Fourier conjugate of k_T

$b_\perp$ = transverse spatial distance w.r.t. the "center" of the proton

The transverse center of longitudinal momentum: $$\mathbf{R}_\perp^{CM} \equiv \sum_i x_i \mathbf{r}_{\perp i}$$

[Burkardt 2000; Soper 1977]

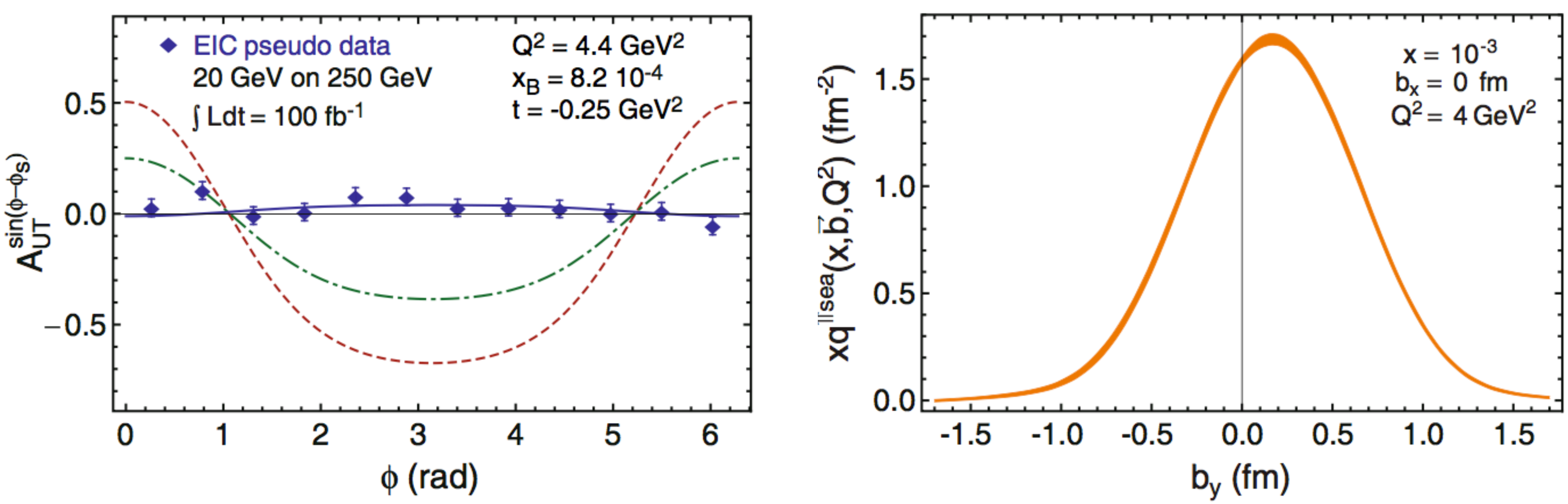
GPDs



At EIC GPDs will be extracted in order to study OAM

$$J^q = \frac{1}{2}\int \mathrm{d}x\, x\, [H^q(x,\xi,t=0) + E^q(x,\xi,t=0)]$$

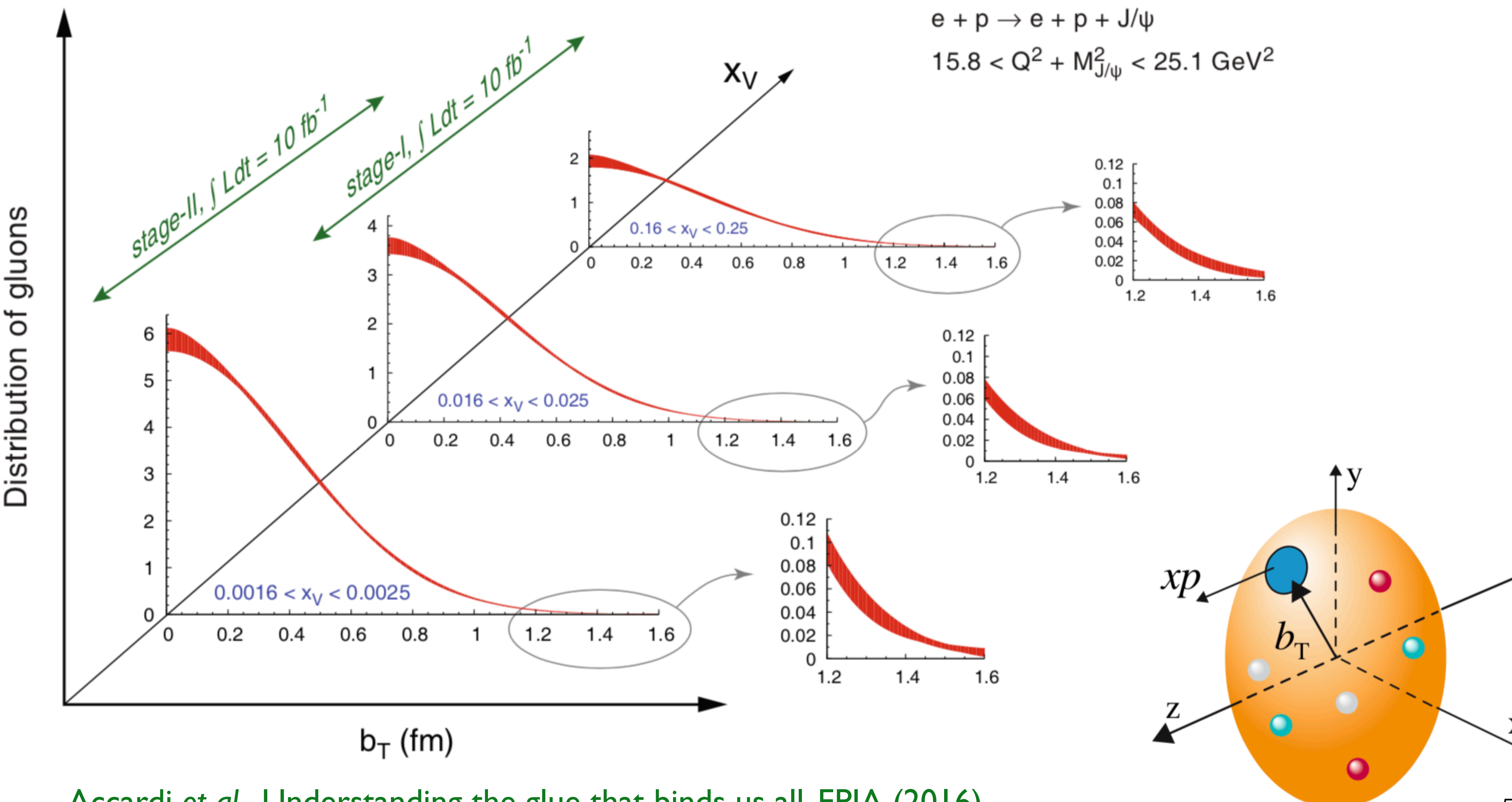
Sivers-like distortions ($b_T \times S_T$) give rise to transverse spin asymmetries



See Boer et al., arXiv:1108.1713; Accardi *et al.*, Understanding the glue that binds us all, EPJA (2016)

Gluon GPD from exclusive J/ψ production

Projected precision of the transverse spatial distribution of gluons



Accardi *et al.*, Understanding the glue that binds us all, EPJA (2016)

GTMDs

Nucleon tomography: spatial distributions

GPDs: off-forward PDFs (proton stays intact but gets a kick)

Give access to the transverse spatial distributions

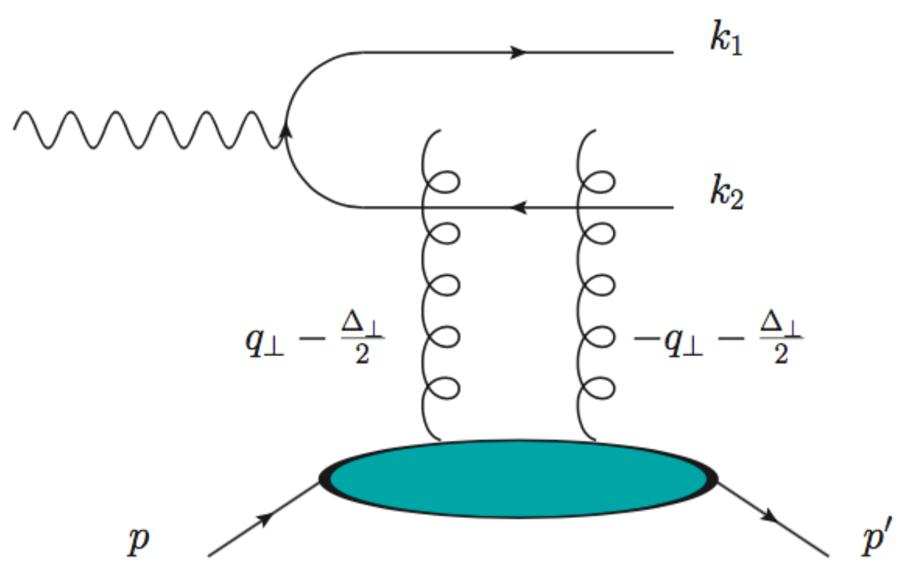
GTMD = off-forward TMD = Fourier transform of a Wigner distribution

$$G(x, \boldsymbol{k}_T, \boldsymbol{\Delta}_T) \overset{FT}{\longleftrightarrow} W(x, \boldsymbol{k}_T, \boldsymbol{b}_T)$$

Meißner, Metz, Schlegel, 2009 Ji, 2003; Belitsky, Ji & Yuan, 2004

Diffraction dijet production in eA at EIC can be used to probe gluon GTMDs

Altinoluk, Armesto, Beuf, Rezaeian, 2016;
Hatta, Xiao, Yuan, 2016



Gluon GTMDs for unpolarized protons

For unpolarized protons there are 2 (real valued) gluon TMDs:

$$\Gamma_U^{ij}(x, \boldsymbol{k}_T) = \frac{x}{2}\left[-g_T^{ij}\, f_1(x, \boldsymbol{k}_T^2) + \frac{k_T^{ij}}{M^2}\, h_1^{\perp}(x, \boldsymbol{k}_T^2)\right]$$

$$a_T^{ij} \equiv a_T^i a_T^j - \frac{1}{2} a_T^2 \delta_T^{ij}$$

Mulders, Rodrigues, 2001

For GTMDs one has one more vector so more anisotropic terms can arise

For unpolarized protons there are 4 (complex valued) gluon GTMDs

$$G^{[U,U']\,ij}(x, \boldsymbol{k}_T, \boldsymbol{\Delta}_T) = x\left(\delta_T^{ij}\, \mathcal{F}_1 + \frac{k_T^{ij}}{M^2}\, \mathcal{F}_2 + \frac{\Delta_T^{ij}}{M^2}\, \mathcal{F}_3 + \frac{k_T^{[i}\Delta_T^{j]}}{M^2}\, \mathcal{F}_4\right)$$

Boer, van Daal, Mulders, Petreska, 2018

Lorce, Pasquini, 2013; More, Mukherjee, Nair, 2018

Like for TMDs gauge links *[U,U']* will matter

Dipole gluon GTMDs

In the x→0 the dipole gluon GTMD becomes a single Wilson loop correlator:

$$G^{[+,-]}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) \equiv \frac{1}{2\pi g^2}\left[\boldsymbol{k}_\perp^2 - \frac{\boldsymbol{\Delta}_\perp^2}{4}\right] G^{[\Box]}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp)$$

$$G^{[\Box]}(\boldsymbol{k}, \boldsymbol{\Delta}) \equiv \int \frac{d^2\boldsymbol{x}\, d^2\boldsymbol{y}}{(2\pi)^4}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})+i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}}\, \frac{\langle p'|\, S^{[\Box]}(\boldsymbol{x}, \boldsymbol{y})\,|p\rangle\big|_{\mathrm{LF}}}{\langle P|P\rangle},$$

$$S^{[\Box]}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) \equiv \frac{1}{N_c}\mathrm{Tr}\left[U^{[\Box]}(\boldsymbol{y}_\perp, \boldsymbol{x}_\perp)\right] \qquad U^{[\Box]}(y,x) = U^{[+]}_{[x,y]} U^{[-]}_{[y,x]}$$

Boer, van Daal, Mulders, Petreska, 2018

The off-forward generalization of the dipole (DP) gluon TMD distribution:

$$xG^{(2)}(x, q_\perp) = \frac{q_\perp^2 N_c}{2\pi^2 \alpha_s} S_\perp \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-iq_\perp\cdot r_\perp} \frac{1}{N_c}\left\langle \mathrm{Tr} U(0) U^\dagger(r_\perp)\right\rangle_{x_g}$$

Dominguez, Marquet, Xiao, Yuan, 2011

This is for isotropic (δ_{ij}) term but the same happens for the anisotropic terms

Wigner distributions

GTMDs are the Fourier transforms of Wigner distributions:

$$W(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) \equiv \int \frac{d\lambda}{2\pi P^+} d^2\boldsymbol{r}_\perp e^{i\lambda x} e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{r}_\perp}$$

$$\times \langle P^+, \boldsymbol{R}_\perp = 0 | \, \overline{\psi}(-\frac{\lambda}{2}n + \boldsymbol{b}_\perp - \frac{\boldsymbol{r}_\perp}{2})\gamma^+ \mathcal{L}\psi(\frac{\lambda}{2}n + \boldsymbol{b}_\perp + \frac{\boldsymbol{r}_\perp}{2}) \, | P^+, \boldsymbol{R}_\perp = 0 \rangle$$

$\xi \to 0$ is needed for density interpretation [Burkardt 2000]

Similar expressions hold for gluons

$$xW(x, \boldsymbol{b}, \boldsymbol{k}) = x\mathcal{W}_0(x, \boldsymbol{b}^2, \boldsymbol{k}^2) + 2\cos(\phi_b - \phi_k)\, x\mathcal{W}_1(x, \boldsymbol{b}^2, \boldsymbol{k}^2)$$
$$+ 2\cos 2(\phi_b - \phi_k)\, x\mathcal{W}_2(x, \boldsymbol{b}^2, \boldsymbol{k}^2) + \dots$$

The cos $2(\phi_b - \phi_k)$ part is called the elliptic Wigner distribution

Hatta, Xiao, Yuan, 2016; J. Zhou, 2016; Mäntysaari, Mueller, Schenke, 2019; Salazar, Schenke, 2019

There can be such an elliptic piece in any Wigner distribution

Elliptic Wigner distributions

$$xW(x,\boldsymbol{b},\boldsymbol{k}) = x\mathcal{W}_0(x,\boldsymbol{b}^2,\boldsymbol{k}^2) + 2\cos(\phi_b - \phi_k)\, x\mathcal{W}_1(x,\boldsymbol{b}^2,\boldsymbol{k}^2) + 2\cos 2(\phi_b - \phi_k)\, x\mathcal{W}_2(x,\boldsymbol{b}^2,\boldsymbol{k}^2) + \ldots$$

The cos 2(ϕ_b-ϕ_k) part is called the elliptic Wigner distribution

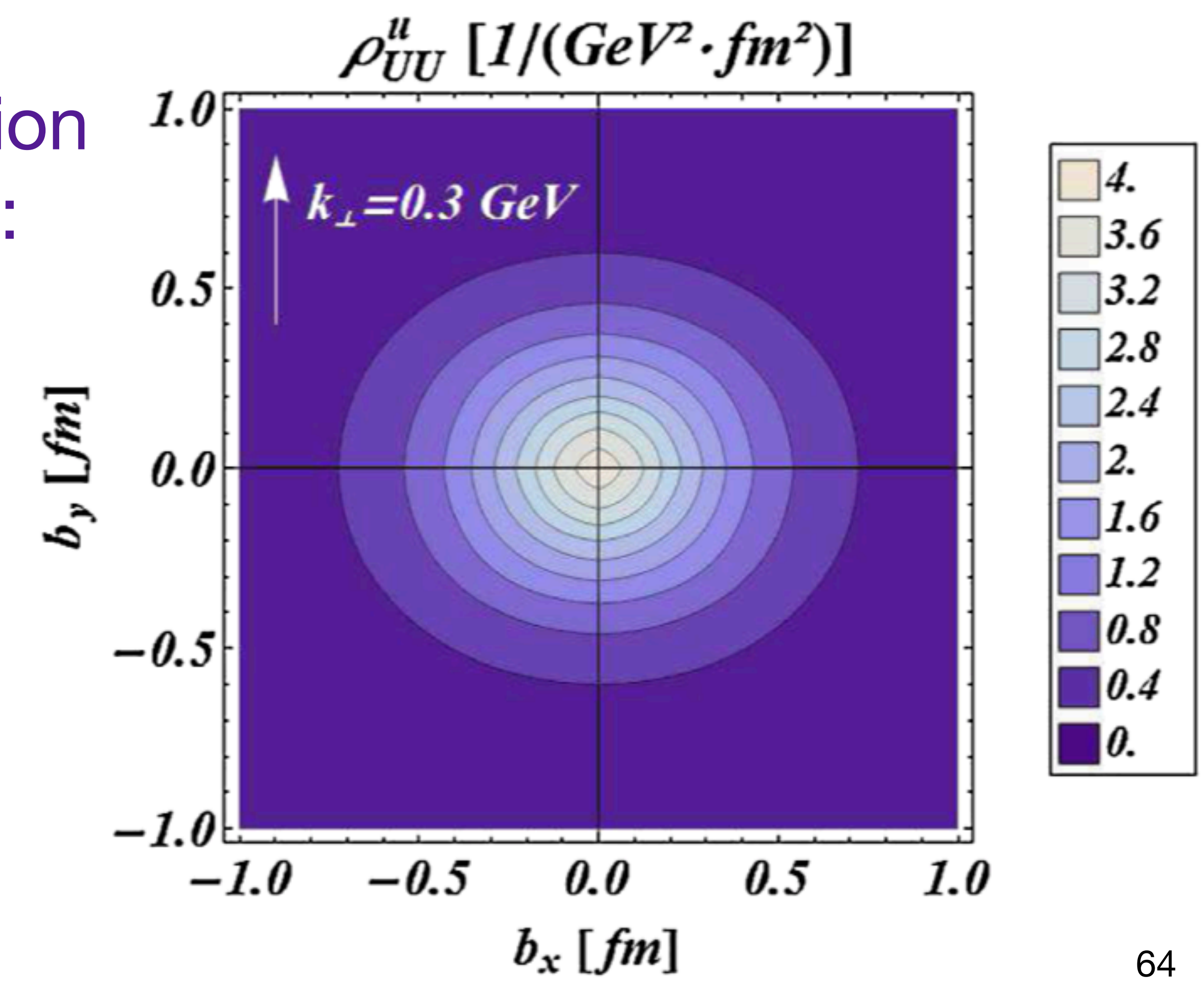
Hatta, Xiao, Yuan, 2016; J. Zhou, 2016; Mäntysaari, Mueller, Schenke, 2019; Salazar, Schenke, 2019

A nonzero elliptic quark Wigner distribution in the lightcone constituent quark model:

Lorce, Pasquini, 2011

Due to quark orbital angular momentum

Lorce, Pasquini, 2011; Hatta, 2011



Conclusions

Conclusions

- Even 50 years after gluons were first considered, the physics of gluons is still largely unexplored, especially regarding nuclear and spin effects
- The U.S.-based Electron-Ion Collider aims to measure such gluon effects through extraction of many different gluon distributions
- This will yield lots of new and unique information ranging from collective effects to spin effects
- There is lots of synergy with pp, pA & AA studies at the LHC in a similar x range but in a less clean environment (and without polarization)

Back-up slides

Odderons

Odderon GTMDs

S[□] can also have an imaginary part:

$$S^{[\Box]}(\boldsymbol{x},\boldsymbol{y}) = \mathcal{P}(\boldsymbol{x},\boldsymbol{y}) + i\mathcal{O}(\boldsymbol{x},\boldsymbol{y})$$

$$\mathcal{P}(\boldsymbol{x},\boldsymbol{y}) \equiv \frac{1}{2N_c}\mathrm{Tr}\left(U^{[\Box]} + U^{[\Box]\dagger}\right) \qquad \mathcal{O}(\boldsymbol{x},\boldsymbol{y}) \equiv \frac{1}{2iN_c}\mathrm{Tr}\left(U^{[\Box]} - U^{[\Box]\dagger}\right)$$

This “odderon” operator is C-odd and T-odd

$$\begin{aligned} G_{(d)}^{(\text{T-odd})\,ij}(\boldsymbol{k},\boldsymbol{\Delta}) &\equiv \frac{1}{2}\left(G^{[+,-]\,ij}(\boldsymbol{k},\boldsymbol{\Delta}) - G^{[-,+]\,ij}(\boldsymbol{k},\boldsymbol{\Delta})\right) \\ &= \frac{N_c}{\alpha_s}\left[\frac{1}{2}\left(\boldsymbol{k}^2 - \frac{\boldsymbol{\Delta}^2}{4}\right)\delta_T^{ij} + k_T^{ij} - \frac{\Delta_T^{ij}}{4} - \frac{k_T^{[i}\Delta_T^{j]}}{2}\right] \\ &\quad \times \left(G^{[\Box]}(\boldsymbol{k},\boldsymbol{\Delta}) - G^{[\Box^\dagger]}(\boldsymbol{k},\boldsymbol{\Delta})\right) \end{aligned}$$

$$G^{[\Box]}(\boldsymbol{k},\boldsymbol{\Delta}) - G^{[\Box^\dagger]}(\boldsymbol{k},\boldsymbol{\Delta}) \propto \int \frac{d^2\boldsymbol{x}\, d^2\boldsymbol{y}}{(2\pi)^4}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y}) + i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}}\, \langle \mathcal{O}(\boldsymbol{x},\boldsymbol{y})\rangle$$

Odderon GTMDs

$$G^{[\Box]}(\boldsymbol{k},\boldsymbol{\Delta}) - G^{[\Box^\dagger]}(\boldsymbol{k},\boldsymbol{\Delta}) \propto \int \frac{d^2\boldsymbol{x}\, d^2\boldsymbol{y}}{(2\pi)^4}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})+i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}}\, \langle \mathcal{O}(\boldsymbol{x},\boldsymbol{y})\rangle$$

Hermiticity and PT constraints imply:

$$G^{[\Box]*}(\boldsymbol{k},\boldsymbol{\Delta}) = G^{[\Box]}(\boldsymbol{k},-\boldsymbol{\Delta}) \qquad G^{[\Box]*}(\boldsymbol{k},\boldsymbol{\Delta}) = G^{[\Box^\dagger]}(-\boldsymbol{k},-\boldsymbol{\Delta})$$

$$G^{[\Box]}(\boldsymbol{k},\boldsymbol{\Delta}) - G^{[\Box^\dagger]}(\boldsymbol{k},\boldsymbol{\Delta}) \text{ only depends on odd powers of } \boldsymbol{k}\cdot\boldsymbol{\Delta}$$

$\Longrightarrow$ the odderon contains only odd harmonics $\cos[(2n - 1)(\phi_K - \phi_\Delta)]$, with $n \geq 1$

$$xW(x,\boldsymbol{b},\boldsymbol{k}) = x\mathcal{W}_0(x,\boldsymbol{b}^2,\boldsymbol{k}^2) + 2\cos(\phi_b - \phi_k)\, x\mathcal{W}_1(x,\boldsymbol{b}^2,\boldsymbol{k}^2) + 2\cos 2(\phi_b - \phi_k)\, x\mathcal{W}_2(x,\boldsymbol{b}^2,\boldsymbol{k}^2) + \ldots$$

$\mathcal{W}_1$ does not lead to odd harmonics in $\phi_K - \phi_\Delta$ in diffractive dijet production in DIS or in UPCs in pp/pA (it only enters squared)

Boer, van Daal, Mulders, Petreska, 2018

Directed flow

$\mathcal{W}_1$ does lead to odd harmonics in dihadron production through double parton scattering in pA collisions

$$\frac{d\sigma^{pA\to h_1h_2X}_{\text{DPS}}}{dy_1dy_2\,d^2\boldsymbol{k}_1d^2\boldsymbol{k}_2} \propto \int d^2\boldsymbol{b}_1\,d^2\boldsymbol{b}_2\,F_p(x_1,x_2,\boldsymbol{b}_1-\boldsymbol{b}_2)\int\frac{d^2\boldsymbol{r}_1d^2\boldsymbol{r}_2}{(2\pi)^4}\,e^{-i\boldsymbol{k}_1\cdot\boldsymbol{r}_1-i\boldsymbol{k}_2\cdot\boldsymbol{r}_2}$$
$$\times\left\langle S\left(\boldsymbol{b}_1+\tfrac{\boldsymbol{r}_1}{2},\boldsymbol{b}_1-\tfrac{\boldsymbol{r}_1}{2}\right)\,S\left(\boldsymbol{b}_2+\tfrac{\boldsymbol{r}_2}{2},\boldsymbol{b}_2-\tfrac{\boldsymbol{r}_2}{2}\right)\right\rangle$$

$$\frac{d\sigma^{pA\to h_1h_2X}_{\text{DPS}}}{dy_1dy_2\,d^2\boldsymbol{k}_1d^2\boldsymbol{k}_2} \propto \frac{\pi}{8R_N^2}\,f_p(x_1,x_2)\int db_1^2\,db_2^2\,e^{-\frac{b_1^2+b_2^2}{4R_N^2}}$$
$$\times\left[2I_0\left(\frac{b_1b_2}{2R_N^2}\right)x\mathcal{W}_0(x,\boldsymbol{b}_1^2,\boldsymbol{k}_1^2)\,x\mathcal{W}_0(x,\boldsymbol{b}_2^2,\boldsymbol{k}_2^2)\right.$$
$$+\,4\cos(\phi_{k_1}-\phi_{k_2})\,I_1\left(\frac{b_1b_2}{2R_N^2}\right)x\mathcal{W}_1(x,\boldsymbol{b}_1^2,\boldsymbol{k}_1^2)\,x\mathcal{W}_1(x,\boldsymbol{b}_2^2,\boldsymbol{k}_2^2)$$
$$\left.+\,4\cos 2(\phi_{k_1}-\phi_{k_2})\,I_2\left(\frac{b_1b_2}{2R_N^2}\right)x\mathcal{W}_2(x,\boldsymbol{b}_1^2,\boldsymbol{k}_1^2)\,x\mathcal{W}_2(x,\boldsymbol{b}_2^2,\boldsymbol{k}_2^2)\right]$$
$$+\,\ldots$$

Lappi, Schenke, Schlichting, Venugopalan, 2016

Spin dependent odderon

$$G^{[\Box]}(\boldsymbol{k},\boldsymbol{\Delta}) - G^{[\Box^\dagger]}(\boldsymbol{k},\boldsymbol{\Delta}) \text{ only depends on odd powers of } \boldsymbol{k}\cdot\boldsymbol{\Delta}$$

Therefore, no odderon in the forward limit for unpolarized protons

For polarized protons the forward limit does not need to vanish however

The d-type gluon Sivers function $f_{1T}^{\perp\, g\,[+,-]}$ at small x is part of:

$$\left(\Gamma^{[+,-]} - \Gamma^{[-,+]}\right) \propto \text{F.T.}\,\langle P, S_T|\text{Tr}\left[U^{[\Box]}(0_T,y_T) - U^{[\Box]\dagger}(0_T,y_T)\right]|P,S_T\rangle$$

D.B., Echevarria, Mulders, J. Zhou, PRL 2016

This can be identified with the *spin-dependent odderon* [J. Zhou, 2013]