MSGUTs: Into the Pleroma

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Plan

- Clues and Targets for Unification
- SO(10) Minimal GUTs Structure
- Soluble SSB, GUT scale spectra
- Threshold effects and Unification
  - Predicting S-spectra.
  - Resolving Susy $d = 5$ B violation problem
- RG Flow beyond M_X
- Pleromal Condensation and GUT SSB
- Aarti Girdhar(2002-2005), Sumit Garg (2005-2009), Ila Garg, Charanjit Kaur (2010-2014).

Target for any GUT

- Data for GUT To Explain : Measured(18) :

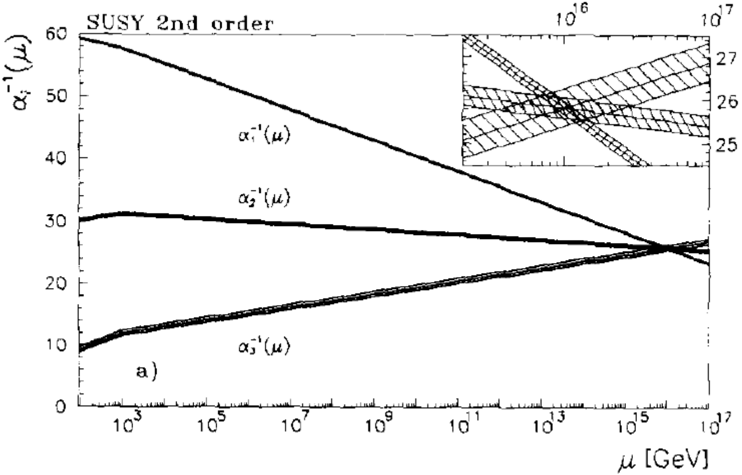
$$m_{q,l} : 10^{-4} - 10^{2}\, GeV \quad ; \quad \sin\theta_i^{CKM} \sim .003 - .22$$
$$\delta^{CKM} \sim \pi/3 \quad ; \quad \Delta m_\nu^2 \sim (10\, meV)^2$$
$$\theta_{12,23}^{PMNS} \sim \pi/4 \quad ; \quad \theta_{13}^{PMNS} \sim 8^\circ \pm 4^\circ$$

- **Awaited (4) :** $M_\nu, \delta^{PMNS}, \alpha_{1,2}^{PMNS}$
- Exotic processes and contributions : Baryon, Lepton, number and Flavour violation, muon g-2, severe constraints :

$$\tau_P > 10^{34} yrs \quad ; \quad a_\mu \sim 3 \times 10^{-9}$$
$$B.R.(B_s \to \mu\gamma) \sim 3 \times 10^{-4} \ldots.$$

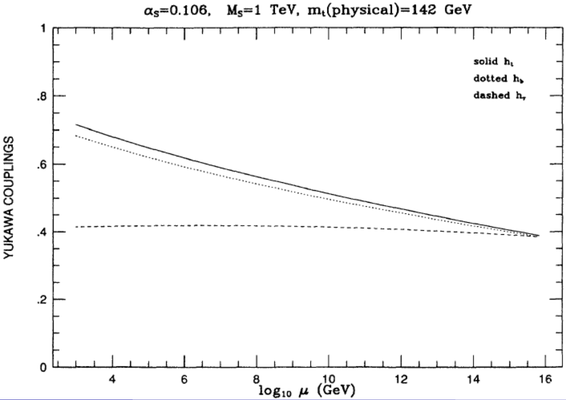
Hints of (Supersymmetric !) Unification

- MSSM Gauge Unification at $M_X^0 \sim 10^{16.25}$ GeV
- $y_t \simeq y_b \simeq y_\tau(M_X)$ *for* $\tan\beta > 40 - 60 \Rightarrow$ same GUT irrep !
- $10^{1-2} meV = M_{\nu_L} \sim \frac{m_{top}^2}{(10^{-3} M_X^0)} \Rightarrow\Rightarrow M_{\nu_L^c} \sim 10^{-3} M_X^0$ (Type I Seesaw)


SUSY 2nd order
$\alpha_i^{-1}(\mu)$
60
50
40
30
20
10
0
$\alpha_1^{-1}(\mu)$
$\alpha_2^{-1}(\mu)$
$\alpha_3^{-1}(\mu)$
a)
10^3
10^5
10^7
10^9
10^{11}
10^{13}
10^{15}
10^{17}
μ [GeV]
10^{16}
10^{17}
27
26
25

Top-Bottom-Tau Unification in SO(10)

- Ananthnarayan, Lazarides and Shafi(1991) : SO(10) 10-plet ($= 5 + \bar{5}$) Higgs as sole source of 3-generation masses , large $\tan\beta \sim m_t/m_b$ and large $m_t > 140$ GeV implies third generation Yukawa Unification !



Cosmological Motivations

- About 25% of cosmic density is Dark matter(DM) : Low energy effective theory of GUT should contain DM candidate :
- Neutral, long lived, quasi stable particle with very low interaction cross section with SM matter.
- Favourite candidate of many : Supersymmetric WIMP. Relic density constraint satisfied only for certain S-spectra not generically !
- Inflationary paradigm well established. Inflation scale may even be $M_X^0 \simeq 2 \times 10^{16} GeV$ (BICEP2, March 2014)!!
- Successful GUT should contain viable inflationary model with low energy theory well coupled to inflaton to allow reheating after inflation.

Spin(10) Reminder

- Rank 5 Orthogonal Group dimension 45.
- 16 dimensional Chiral Spinor irreps.
- Antisymmetric m index irreps, :

$$m = 1..4 \qquad d(m) \;=\; {}^{10}C_m = 10, 45, 120, 210$$
$$MSGUTUSESALLONCE!$$
$$m = 5 \qquad Self - Dual \qquad d(5) \;=\; 126 \qquad (Complex!)$$

- Gauge Anomaly Free !!

VIRTUES OF SO(10) GUTs

- $\{(Q_L, L_L, u_L^c, d_L^c, l_L^c) \oplus \nu_{\mathbf{L}}\} \equiv 16$: Tight and complete

- Simple Tri-band FM Higgs Channel Spectrum

$$\begin{aligned} 16 \otimes 16 &= 10 \oplus 120 \oplus 126 \Rightarrow (10 + 120 + \overline{126})_H \\ \overline{126} &= (15,2,2) + \Delta_R(10,1,3) + \Delta_L(\overline{10},3,1) + (6,1,1) \end{aligned}$$

- $(-)^{3(B-L)} \equiv M_p \subset U(1)_{B-L} \subset G_{LR} \subset G_{PS} \subset SO(10)$

- Only Even B-L vevs $< \Delta_{L,R} > \Rightarrow \Rightarrow R_p \surd \surd \Rightarrow \Rightarrow$ Stable LSP

House with Two Seesaws

- NATURAL HOME TO BOTH SEESAWS :

  $\overline{\mathbf{126}} \supset \Delta_R(10,1,3)_{TypeI} \oplus \Delta(\overline{10},3,1)_{TypeII} \oplus \Phi(15,2,2) \oplus +....$

  SuperK $\Rightarrow M_{\not B} \simeq M_{\not L} L$:

- Type I : Right handed neutrino mass from Δ_R

$$M_{B-L} \sim < \vec{\Delta}_R >_{SM=0} \Rightarrow M_{\nu^c} \Rightarrow M^I_\nu \sim \frac{v_W^2}{M_{B-L}}$$

- Type II : Tadpole in Δ_L v_{EW} $\Rightarrow\Rightarrow$ small neutrino Majorana mass

$$< \vec{\Delta}_L >_{Y=2,T_{3L}=-1} \Rightarrow\Rightarrow M^{II}_\nu \sim \frac{v_W^2}{M_{\Delta_L}}$$

TWO SCHOOLS OF SO(10)

Renormalizable SO(10)	NON-REN GUTS
Renormalizable couplings	Non Renorm. couplings
No ad-hoc discrete symmetries	Ad-hoc discrete necessary
Large(126,210,..) few (AS)	Small (10,16,45,54) irreps (AF)
# Parameter minimal	Unlimited # parameters
No Higgs duplication	Duplicates Higgs
$M_p \subset SO(10)$	"string motivated" Z_2
Higgs-Matter distinct	Higgs-Matter mix
Only B-L even vevs	R_p broken
UNSTRUNG !!	STRING INSPIRED !!
a) $210 \oplus 126 \oplus \overline{126}$	$16^n_H \oplus 10^m \oplus 45^l$ plethora
b)$54 \oplus 45 \oplus 126 \oplus \overline{126}$	

New Minimal Supersymmetric Grand Unified Theory

- $3 \times 16_F,\ 10_H,\ \overline{126}_H,\ 126_H,\ 210_H, 120_H,\ 45_V$ [1]
- **AM Higgs** : $< 210,\ \overline{126},\ 126 > \ \Rightarrow$ Susy $SO(10) \longrightarrow MSSM$
- **MSGUT(No 120) Superpotential**

$$
\begin{aligned}
W &= m\ 210^2 + \lambda\ 210^3 + M\ 126 \cdot \overline{126} + \eta\ 210 \cdot 126 \cdot \overline{126} \\
&+ 10 \cdot 210(\gamma\ 126 + \bar{\gamma}\ \overline{126}) \\
&+ M_H\ 10^2 + h_{AB}\ 16_A \cdot 16_B + f'_{AB}\ 16_A 16_B
\end{aligned}
$$

Superpotential Parameters : $((\mathbf{2} \times \mathbf{7} - \mathbf{4}) + \mathbf{3} + \mathbf{2} \times \mathbf{6} = \mathbf{25})$

Minimal [2] New Minimal($\equiv$ Old !!) [3]

[1] CSA, Mohapatra(1982), Clark, Kuo and Nakagawa (1982)
[2] CSA, Bajc,Melfo,Senjanovic, Vissani(2003)
[3] CSA, Garg(2006)

NMSGUT-SSB

- **GUT scale VEVS** : $SO(10) \to MSSM$

$$\langle(15,1,1)\rangle_{210} : a \qquad \langle(15,1,3)\rangle_{210} : \omega \qquad \langle(1,1,1)\rangle_{210} : p$$
$$\langle(10,1,3)\rangle_{\overline{126}} : \bar{\sigma} \quad ; \quad \langle(\overline{10},1,3)\rangle_{126} : \sigma$$

- D Terms conditions, preserve SUSY : $|\sigma| = |\bar{\sigma}|$
- F Terms

$$\begin{aligned}
F_a &= 0 = 2(m+\lambda a)a + 4\lambda\omega^2 + \eta\sigma\bar{\sigma} \\
F_p &= 0 = 2mp + 6\lambda\omega^2 + \eta\sigma\bar{\sigma} \\
F_\omega &= 0 = 2(m+\lambda p) + 4a\omega - \eta\sigma\bar{\sigma} \\
F_\sigma &= 0 = (M + \eta(p + 3a - 6\omega))^{(\bar{\sigma})}
\end{aligned}$$

These 4 coupled cubic equations (together with the D term condition) are analytically soluble !

NMSGUT-SSB

- SSB completely analyzable 4 eqns $\Rightarrow$ Units : $\frac{m}{\lambda}$
  $\tilde{a} = \frac{(x^2+2x-1)}{(1-x)}$; $\tilde{p} = \frac{x(5x^2-1)}{(1-x)^2}$; $\tilde{\sigma}\tilde{\overline{\sigma}} = \frac{2}{\eta}\frac{\lambda x(1-3x)(1+x^2)}{(1-x)^2}$

  EOM reduce to single Cubic in $x = -\lambda\omega/m : \xi = \frac{\lambda M}{\eta m}$.
  $8x^3 - 15x^2 + 14x - 3 = -\xi(1-x)^2$
- 592 Higgs Chiral and 33 Majorana gauge supermultiplets occur in 22 complex (pairs) and 4 real MSSM representation types. Explicit solution of SSB allows explicit determination of their mass matrices and eigenvalues !
  (CSA,Girdhar,Bajc,Melfo,Senjanovic,Vissani,Fukuyama,Ilakovac,Kikuchi, Melajnac,Okada)
- Explicit superheavy spectra allow computation of superheavy one loop threshold effects on gauge unification and allow constructive demonstration that SO(10)realistic gauge unification is NOT futile. (CSA, Girdhar 2005)

Gauge Threshold corrections

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_G(M_X)} + 8\pi b_i \ln\frac{M_X}{M_Z} + 4\pi\sum_j \frac{b_{ij}}{b_j}\ln X_j - 4\pi\lambda_i(M_X) + ...$$

$$X_j = 1 + 8\pi b_j \alpha_G(M_X^0) ln \frac{M_X^0}{M_Z}$$

- Superheavy thresholds.

$$\lambda_i(\mu) = -\frac{2}{21}(b_{iV} + b_{iGB}) + 2(b_{iV} + b_{iGB})\ln\frac{M_V}{\mu} + 2b_{iS}\ln\frac{M_V}{\mu} + 2b_{iF}\ln\frac{M_F}{\mu}$$

- Corrections depend upon the ratios of masses: independent of m (mass of **210**-plet),the overall mass scale parameter. The spread of mass eigenvalues allows cancellation among threshold corrections and a sensible result.

- The threshold corrections in $M_X, \alpha_3(M_Z), \alpha_G^{-1}$: :

$$\Delta(Log_{10} M_X) = 0.222 + \frac{5(\bar{b}_1' - \bar{b}_2')}{56\pi} \sum_{M'} Log_{10} \frac{M'}{M_X}$$

$$\Delta(\alpha_3) = .000311667 \sum_{M'} (5\bar{b}_1' - 12\bar{b}_2' + 7\bar{b}_3') \ln \frac{M'}{M_X}$$

$$\Delta(\alpha_G^{-1}) = -1.27 + \frac{1}{56\pi} \sum_{M'} (33\bar{b}_2' - 5\bar{b}_1') \ln \frac{M'}{M_X}$$

- Fixation of overall scale parameter m :

$$|m| = M_X^0 10^{+\Delta_X} \frac{|\lambda|}{g\sqrt{4|\tilde{a}+\tilde{w}|^2 + 2|\tilde{p}+\tilde{\omega}|^2}} \text{GeV}$$

$g = \sqrt{4\pi(25.6 + \Delta_G)^{-1}}$ is the threshold corrected SO(10) gauge coupling.

Opening the Higgs Portal

- 6 pairs of doublets from $\{\mathbf{10}, \overline{\mathbf{126}}, \mathbf{126_H}, \mathbf{210_H}, \mathbf{120}\}_H$ mix into the single pair of MSSM doublets $H, \overline{H}$:
- PORTAL into guts of UV completion. Novel NMSGUT insights ALL flow from a focus on the implications of this crucial fact !!!
- Consistency Condition(a.k.a Fine tuning) : $Det\,\mathcal{H} = 0$
- Bi-Unitary transformation $\Rightarrow \bar{U}^T \mathcal{H} U$ is diagonal.

$$\alpha_i = U_{i1} \quad ; \quad \bar{\alpha}_i = \overline{U}_{i1}$$

$$H = \sum_i \alpha_i^* h_i \quad ; \quad \overline{H} = \sum_i \bar{\alpha}_i^* \bar{h}_i$$

$$L_{eff} : h_i \to \alpha_i H \quad ; \bar{h}_i \to \bar{\alpha}_i \overline{H}$$

- Matter Yukawas, Masses determined by Higgs fractions :

$$\Psi_A.(h_{AB} H + f_{AB} \Sigma + g_{AB} \Theta)\Psi_B \quad \Rightarrow \quad 3 + 12 + 6 = 21 \quad \text{parameters}$$

MSGUT(no 120) Contretemps

- 2003-2005 : Fermion fitting frenzy(ignoring quantum threshold effects) in SO(10) using generic SO(10) fermion mass formulae and assuming complete parameter freedom.
- Bloom2Doom : 2005. MSGUT mass formulae do not permit fit of both charged and neutrino masses in terms of TREE LEVEL parameters.
- As an alternative we proposed 10 , 120 fit charged fermion masses while very weakly coupled $\overline{126}$ responsible for enhanced neutrino masses via Type I seesaw.(CSA, Garg 2006)
- Constraints due to 10,120 combo (typically d,s quarks are too light) lifted by large $\tan\beta$ driven lowering of $(y_{d,s})_{SM}$ at M_S threshold.
- At tree level 10-120 implies $b - \tau = s - \mu$.

Achievements of MSGUTs : I

- Consistent threshold corrected gauge unification.
- Realistic fit of all fermion mass mixing data using Quantum corrected Mass Formulae
  C.S.A., S. K. Garg NPB 2008
- Susy Thresshold corrections at $M_S \Rightarrow$ Prediction of distinctive MSSM spectra(2008)
  - Normal s-hierarchy ($m_{\tilde{q}_3,\tilde{l}_3} >> m_{\tilde{q}_{1,2},\tilde{l}_{1,2}}$)
  - A_0 and $\mu > 10$ TeV required for $y_{d,s}$ fit!(2008)
  - Large A_0 now (2012) *necessary* for $M_H^{Susy} \simeq 126$ GeV
  - Light smuon (muon g-2 and CDM co-annihilation) possible

Achievements of MSGUTs : II

- Generic mechanism raises $d = 5$ operator mediated proton lifetime from $\tau_p \sim 10^{27}$yrs to $\tau_p > 10^{34}$yrs .
(C.S.A(2011), C. S.A., I. Garg, C. K. Khosa, NPB882 (2014))
- (New !) Corrections at M_X can also invalidate tree level $10 + 120$ constraints and give less distinctive Susy spectra ! Besides repairing τ_p can lift $m_{d,s}$ hugely !
- Programmatic shift : Quantum threshold corrections crucial at large N!

M_S Threshold

- NMSGUT success is Quantum found / *not tree level engineered* : Quantum corrections to Light-Heavy matching resolve conundra of unification.
- Fermion masses : $\overline{\mathbf{126}}$ couplings suppressed to fit $M_\nu \Rightarrow\Rightarrow$
- $\mathbf{10} \oplus \mathbf{120}$ only fits charged fermion masses $y_t \simeq y_b \simeq y_\tau(M_X)$ and $\tan\beta \simeq 50$ IF, MSSM radiative corrections raise $Y_{d,s}^{GUT}$ by 3-4 times while Y_b^{GUT} lowered by 5%.

M_S threshold corrections(contd.)

- Large $\tan\beta$ driven (H-Hbar mixing) threshold corrections to down type fermion yukawa masses. (α_s(gluino) and ($A_t y_t^2$ loops for 3d gen)) Also 10-15% gluino corrections for m_{top}.

$$y_i^{MSSM}(M_S)\cos\beta = \frac{y_i^{SM}(M_S)}{1+\epsilon_i(m_{\tilde{f}}, M_i, \mu, A_t)\ \tan\beta}$$

- Dominant corrections for quarks:

$$\epsilon_i^G = -\frac{2\alpha_S}{3\pi}\frac{\mu}{M_3}H_2(u_{\tilde{Q}_i}, u_{\tilde{d}_i}) \qquad \epsilon^t = -\frac{y_t^2}{16\pi^2}\frac{A_t^0}{\mu}H_2(v_{\tilde{Q}_3}, v_{\tilde{u}_3})$$

M_S Threshold corrections (contd.)

- $H_2 < 0 \ \Rightarrow$ lowering $y_{d,s}^{SGUT} \Rightarrow \ \mu, -A_t \sim 10^2\,TeV >> M_{\tilde{f}} \sim 10\,TeV >> M_\lambda \sim 1\,TeV$ with cancellation/6% enhancement for y_b.
- Normal S-Hierarchy : Third gen sfermions heavier than first two.Right Smuon often lightest charged scalar close to the LSP ! Distinct region of Susy parameter space, class of spectra, LHC signatures
- Precisely at large $\tan\beta$ gluino and chargino loops modify down type quarks sufficiently *provided*
  - Light gauginos : $\sim .1 - 1.5$ TeV
  - $M_S > 5$ TeV
  - $\mu, A_0 \sim 5 - 100$ TeV
  - $\tilde{f}^c = \tilde{\mu}, u\tilde{u}^c$ often lightest NLSP.($\Rightarrow$ co-annihilation of LSP)
  - Normal s-hierarchy $m_{\tilde{3}} >> m_{\tilde{1},\tilde{2}}$

M_X^0 threshold : Quantum Naturopathy for $d = 5$ disease

- MSSM Higgs blend of 6 pairs from NMSGUT Higgs $\Rightarrow\Rightarrow$ $\sim 10^3$ heavy fields renormalize light Higgs : *Generically* drive it to "Higgs dissolution edge" :

$$Z_{H,\bar{H}} \simeq 0$$

- $\Rightarrow\Rightarrow$

$$Y_{GUT} \sim \sqrt{Z_H} Y^{MSSM}(M_X) << Y^{MSSM}(M_X) < 1$$

- But $\mathcal{A}(\Delta B \neq 0, d = 5) \sim \frac{Y_{GUT}^2}{M_X}$!! $\Rightarrow\Rightarrow$
  $\tau_p >> 10^{28}$ yrs (generic) $\longrightarrow\longrightarrow$ $\tau_p > 10^{34}$ yrs !

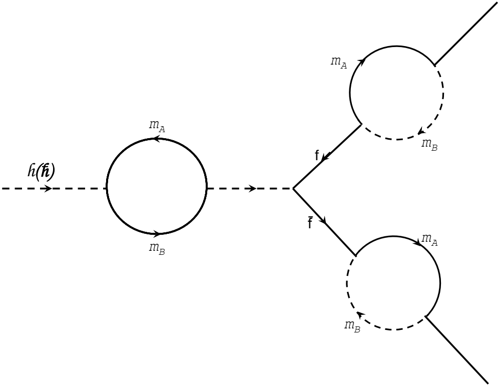


Figure: Loop corrections to fermion, antifermion and Higgs line

$$\mathcal{L} = \left[\sum_{A,B} (\bar{f}_A^\dagger (Z_{\bar{f}})_A^B \bar{f}_B + f_A^\dagger (Z_f)_A^B f_B) + H^\dagger Z_H H + \overline{H}^\dagger Z_{\overline{H}} \overline{H} \right]_D + ..$$

- Generic form of correction factor for any chiral field Φ_i is $(Z = 1 - \mathcal{K})$:

$$\mathcal{K}_i^j = -\frac{g_{10}^2}{8\pi^2} \sum_\alpha Q_{ik}^{\alpha *} Q_{kj}^\alpha F(m_\alpha, m_k) + \frac{1}{32\pi^2} \sum_{kl} Y_{ikl} Y_{jkl}^* F(m_k, m_l)$$

- F : Passarino-Veltman 1-loop function.

$$F_{12}(M_A, M_B, Q) = \frac{1}{(M_A^2 - M_B^2)} (M_A^2 \ln \frac{M_A^2}{Q^2} - M_B^2 \ln \frac{M_B^2}{Q^2}) - 1$$

- There are precisely 26 different combinations of the 26 MSSM representation types that occur in the (N)MSGUT multiplets which can run in the loops on the Higgs lines in the MSSM matter fermion Yukawa vertices.

$$
\begin{aligned}
(16\pi^2)\mathcal{K}_H \quad = \quad & 3K_{J\bar{D}} + 8K_{R\bar{C}} + 9K_{X\bar{P}} + K_{VF} + 3K_{E\bar{J}} \\
& + 9K_{P\bar{E}} + 6K_{B\bar{M}} + 3K_{X\bar{T}} + 3K_{D\bar{I}} + 24K_{Q\bar{C}} + 3K_{T\bar{E}} \\
& + 6K_{Y\bar{L}} + 18K_{W\bar{B}} + 8K_{C\bar{Z}} + 9K_{E\bar{U}} + 9K_{U\bar{D}} + 3K_{HO} \\
& + K_{\bar{V}\bar{A}} + 3K_{K\bar{X}} + K_{H\bar{F}} + 6K_{N\bar{Y}} + 18K_{Y\bar{W}} + 3K_{V\bar{O}} \\
& + 6K_{L\bar{B}} + 3K_{S\bar{H}} + K_{G\bar{H}}
\end{aligned}
$$

- To illustrate the complexity : one of the simpler corrections (from the $J\bar{D}$ channel) on Higgs line is :

$$\sum_{a=1}^{d(J)} \sum_{a'=1}^{d(D)} \Bigg| \Bigg(\gamma V^J_{2a} U^D_{1a'} - \frac{\gamma}{\sqrt{2}} V^J_{3a} U^D_{1a'} \bar{\gamma} V^J_{2a} U^D_{2a'} + \frac{\bar{\gamma}}{\sqrt{2}} V^J_{3a} U^D_{2a'}$$
$$- \frac{ik}{\sqrt{2}} V^J_{3a} U^D_{3a'} \Bigg) V^H_{11} + \Bigg(\frac{2\eta}{\sqrt{3}} V^J_{2a} U^D_{1a'} - \sqrt{6}\eta V^J_{3a} U^D_{1a'} - \frac{2i\bar{\zeta}}{\sqrt{3}} V^J_{2a} U^D_{3a'}$$
$$+ \sqrt{\frac{3}{2}} i\bar{\zeta} V^J_{3a} U^D_{3a'} \Bigg) V^H_{21} + \Bigg(\frac{-i}{\sqrt{6}} \zeta V^J_{3a} U^D_{3a'} - \frac{2i\zeta}{\sqrt{3}} V^J_{2a} U^D_{3a'} + \frac{2\eta}{\sqrt{3}} V^J_{2a} U^D_{2a'}$$
$$- \sqrt{\frac{2}{3}} \eta V^J_{3a} U^D_{2a'} \Bigg) V^H_{31} - \Bigg(\frac{i\rho}{3} V^J_{5a} U^D_{3a'} + 4\eta V^J_{1a} U^D_{1a'} 2i\bar{\zeta} V^J_{1a} U^D_{3a'}$$
$$+ 2\bar{\zeta} V^J_{5a} U^D_{2a'} \Bigg) V^H_{41}$$

$+$THIRTEEN MORE TERMS

Threshold Effects On $\Gamma^{\Delta B \neq 0}_{d=5}$

$$
\begin{aligned}
W^{\Delta B} &= \quad L_{ABCD} Q_A Q_B Q_C L_D + R_{ABCD} \bar{U}_A \bar{U}_B \bar{D}_C \bar{L}_D \\
(L,R)_{ABCD} &\sim \sum \frac{(h/f/g)_{AB}(h/f/g)_{CD}}{M_X}
\end{aligned}
$$

- Canonical kinetic terms require rescaling by wavefunction renormalization matrices. Coefficients L_{ABCD}, R_{ABCD} of d=5, $\Delta B = \pm 1$ decay operators reduced by factors $\sim Z_H$
- *Unitarity and perturbativity via $Z > 0$ imply couplings are small but $|Z_{H,\bar{H}}| \approx 0$. Therefore $1/\sqrt{Z_{H,\bar{H}}}$ lowers the magnitude of the SO(10) Yukawas required to match MSSM data. d=5 operators have no external Higgs line so lowered SO(10) couplings will suppress decay rate mediated by d=5 operators strongly.*

- MSSM μ and B parameters larger by the factor of $(Z_H Z_{\bar{H}})^{-1/2}$. Scalar soft masses and soft Higgs masses modified by a factor of Z_f^{-1} and $Z_{H/\bar{H}}^{-1}$ respectively. A_0 same. Y_ν and Higgs field redefinition modify the Type I seesaw formula.
- We constrained the B decay rates while searching :

$$\text{Max}(L'_{ABCD}, R'_{ABCD}) < 10^{-22}\,\text{GeV}^{-1}$$

  to get proton life time above 10^{34} Yrs. This constraint forces the search towards the regions of parameter space which produce $Z_{H,\bar{H}} \ll 1$
- RG weighted average M_{susy} over Susy particles is used in Susy corrections to $\alpha_s(M_S)$.Typically $M_{susy} \sim 2 - 10$ TeV with our spectra.

$$\Delta_{\alpha_s}^{\text{Susy}} \approx \frac{-19\alpha_s^2}{28\pi} \ln \frac{M_{\text{Susy}}}{M_Z}$$

$$M_{\text{Susy}} = \prod_i m_i^{-\frac{5}{38}(4b_i^1 - 9.6b_i^2 + 5.6b_i^3)}$$

Effects on $Y_{d,s}(M_X)$

- Recent searches for "single throw at M_X" fits give much larger values of MSSM $Y_{d,s}(M_X)$ than ever possible before with $10 + 120$ tree level fits !
- Thus very large A_0, μ no longer required, though still large.
- S-Hierarchy still normal but not so glaring.

Field	Mass(GeV)
$M_{\tilde{G}}$	1000.14
$M_{\chi^{\pm}}$	569.81, 125591.22
M_{χ^0}	210.10_{LSP}, 569.81, 125591.20, 125591.20
$M_{\tilde{\nu}}$	15308.069, 15258.322, 21320.059
$M_{\tilde{e}}$	1761.89, 15308.29, $211.57_{\tilde{\mu}}$, 15258.60, 20674.72, 21419.56
$M_{\tilde{u}}$	11271.80, 14446.76, 11270.63, 14445.80, 24607.51, 40275.87
$M_{\tilde{d}}$	8402.99, 11272.10, 8401.48, 11270.95, 40269.19, 51845.93
M_A	377025.29
$M_{H^{\pm}}$	377025.30
M_{H^0}	377025.28
M_{h^0}	124.00_{h^0}

Table: Large $\mu, B, A_0 \Rightarrow\Rightarrow$ LSP $\simeq \tilde{B}, \tilde{\chi}^{\pm}\tilde{W}_{\pm})$. Light gauginos,Normal Shierarchy $\Rightarrow$ Higgs h^0 as found ,Light smuon ! Other sfermions multi-TeV : Decoupled & Mini-split, large μ, A_0

Beta functions for SO(10) couplings

CSA, Ila Garg, Charanjit Kaur Phys.Rev. D98 (2018) no.7, 075006 , arXiv 1509.00422 : Beta functions for all couplings (soft and hard) of the NMSGUT calculated up to 2- loops .

- SO(10) gauge beta functions of MSGUT irreps are HUGE :

$$\beta_g^{(1)} \equiv b_0 g^3 = g^3(S(R) - 3C_2(G)))$$

$D(R)(S_2(R))$: , 45(8), 10(1), 16(2), 120(28), 126(35), 210(56) .

$NMSGUT$: $b_0 = 137!!$ $MSGUT$: $b_0 = 109$

Yukawa beta functions also HUGE

$$\beta_\lambda^{(1)} = 3\lambda(4|k|^2 + 180|\lambda|^2 + 2|\rho|^2 + 240|\eta|^2 + 6(|\gamma|^2 + |\bar{\gamma}|^2) + 60(|\zeta|^2 + |\bar{\zeta}|^2) - 24g_{10}^2)$$

- Gauge and Yukawa couplings DIVERGE in the UV : Landau pole very close above M_X
- NO a fixed points/sub-manifolds of perturbative RG flow of couplings/ ratio to the gauge coupling (Pendleton-Ross FP/S) above M_X possible
- Strong flow can justify negative soft masses-squared from positive ones for Higgs scalars etc (required by successful fits at M_X) !!

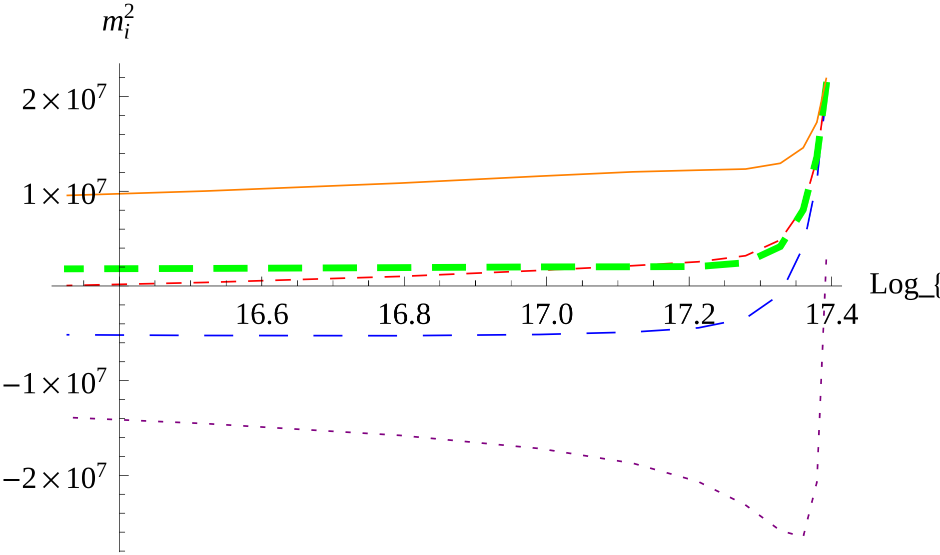


Figure: Soft masses from Planck to GUT scale. m^2_Φ, m^2_H, m^2_Θ, m^2_Σ and $m^2_{\bar\Sigma}$

1982 : Gluino Condensation : VENEZIANO -YANKEILOWICZ effective potential

: W_{VY} describes Gluino condensation in Susy SU(N) YM via Glueball field S and obeys anomaly requirements etc.:

$$S = -\frac{1}{32\pi^2} Tr(W^\alpha W_\alpha) = \frac{1}{16\pi^2} Tr(\lambda^\alpha \lambda_\alpha + \ldots).$$

$$\Lambda = \mu e^{\frac{-8\pi^2}{b_0 g^2(\mu)}} \qquad b_0 = 3N$$

$$W_{VY} = NS\left(1 - \log\frac{S}{\Lambda^3}\right) \Rightarrow < S > = \Lambda^3 \qquad !!$$

1983-1995 : The Power of Holomorphy : Seiberg-istics

- Superpotential not renormalized perturbatively. Non-perurbative renormalization can occur but still restricted by Holomorphy.
- 1993 Seiberg -Polchinski (String Theory) : Holomorphic couplings λ_i promoted to background (dummy) chiral fields with vevs(spurions)
- Enhanced Symmetry G ($U(1)_R$ symmetry) at $\lambda_i = 0$ spontaneously broken by $\lambda_i \neq 0 \Rightarrow L_{eff}^{Wilson}(\Phi_I, \lambda_i)$ G invariant !
- Asymptotic freedom (Λ dependence), Weak coupling analysis combined with localisation/symmetry and Holomorphicity lead to many results exact non-perturbative W_{eff} and thus phase structure (moduli dependence) of Strongly Coupled YM theories.

1994: Seiberg-Witten : Confinement "proof" in $\mathcal{N} = 1$ perurbate of $\mathcal{N} = 2$ Supersymmetric YM Model.

- Seminal solution of $\mathcal{N} = 2$ Susy YM (gauge plus Adjoint chiral) perturbed to $\mathcal{N} = 1 \Rightarrow$ effective theory exhibits particle/monopole duality and confinement !.
- Quantum Moduli space definition mapped to theory of algebraic curve $(y^2 = (x^2 - 1)(x - u))$ and Riemann Surface of genus 1.
- Realization of 't Hooft mechanism for gauge electric flux confinement by dual Meissner effect by monopole condensate .
  Condensate/confinement $\mathcal{N} = 2$ y broken to $N = 1$ by a superpotential perturbation.

Chiral Rings

- N.B. Ring : Group with also an addition operation. E.g. the ring of integers $\mathcal{Z}$.
- Chiral superfield : $\bar{D}_{\dot{\alpha}}\Phi = 0$. Lowest component $\phi(x)$ annihilated by $\bar{\mathcal{Q}}_{\dot{\alpha}}$: $[\bar{\mathcal{Q}}_{\dot{\alpha}}, \phi(x)] = 0$
- Chiral operators(CO) : *gauge invariant* $\mathcal{O}(x)$ annihilated by $\bar{\mathcal{Q}}_{\dot{\alpha}}$.

Chiral Ring -2

- VEV $< \{\bar{\mathcal{Q}}_{\dot{\alpha}}, ...\} >= 0 \quad \Rightarrow$ Equivalence relation : $\mathcal{O}_1(x) \simeq \mathcal{O}_2(x)$ with *gauge invariant* operator $X_{\dot{\alpha}}(x)$

$$\mathcal{O}_1(x) = \mathcal{O}_2(x) + \{\bar{\mathcal{Q}}^{\dot{\alpha}}, X_{\dot{\alpha}}(x)\},$$

- Then equivalence classes of Chiral operators form a Ring !

- Vev of product of chiral operators is constant and factorizes :

$$\langle 0| T\Big(\mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\Big)|0\rangle = \langle\mathcal{O}_1\rangle\ldots\langle\mathcal{O}_n\rangle$$

Chiral Ring-3

- $U(N)$ gauge theory Φ in the adjoint , $Q, \tilde{Q}$ fundamental and anti-fundamental

- Generators of chiral ring are just

$$Tr\Phi^k, \quad TrW_\alpha\Phi^k, \quad TrW^\alpha W_\alpha\Phi^k, \quad \tilde{Q}\Phi^k Q.$$

- For U(N) with Adjoint define generating functions for entire set of Ring operators :

$$\begin{aligned} T(z) &= \sum_{k\geq 0} z^{-1-k} Tr\Phi^k = Tr\frac{1}{z-\Phi}; \\ w_\alpha(z) &= \frac{1}{4\pi} TrW_\alpha \frac{1}{z-\Phi}; \\ R(z) &= -\frac{1}{32\pi^2} Tr W_\alpha W^\alpha \frac{1}{z-\Phi} \end{aligned}$$

Konishi Anomaly

- Φ_r generic irrep r chiral field , $F_r(W_\alpha, \Phi)$ also representation r. ClassicalEOM imply *classical* relation

$$\frac{\partial W(\Phi)}{\partial \Phi_r} F_r(W_\alpha, \Phi) = 0 \qquad \text{in the classical chiral ring.} \tag{1}$$

- PI Measure Non invariance under $\Phi \to \Phi + \eta F(\Phi, W_\alpha) \Rightarrow$ I generalized Konishi anomalies each field and F_r !

$$\bar{D}^2 \left(\bar{\Phi}^q (e^V)_q^r F_r \right) = -\frac{\partial W}{\partial \Phi_r} F_r - \frac{1}{32\pi^2} W^{\alpha s}{}_r W_{\alpha}{}^r{}_q \frac{\partial F_s}{\partial \Phi_q}.$$

LHS=Chirally exact $= 0 \Rightarrow$ for Quantum Chiral ring and in susy Vacuum :

$$\frac{\partial W}{\partial \Phi_r} F_r = -\frac{1}{32\pi^2} W^{\alpha s}{}_r W_{\alpha}{}^r{}_q \frac{\partial F_s}{\partial \Phi_q}. \quad \text{in the quantum chiral ring}$$

Glueball Superpotential and Chiral vevs from basic Konsihi Anomaly

- Holomorphic (superpotential) Couplings are sources for their invariants

$$\frac{\partial}{\partial \lambda_k} W_{\mathrm{eff}} = \langle X_k \rangle. \tag{2}$$

- If KA $\Rightarrow \langle X_k \rangle(\lambda_j, S)$ $W_{\mathrm{eff}(\lambda,S)}$ by integration ! $W_{VY}(S)$ is integration constant !
- U(N) SQCD with one flavour $Q(N), \tilde{Q}(\bar{N})$. Meson $M = \tilde{Q}Q \Rightarrow W_{tree} = mM + \lambda M^2$
- Using factorization $\langle M^2 \rangle = \langle M \rangle^2$

$$m\langle M \rangle + 2\lambda \langle M^2 \rangle = S.$$

$$\langle M \rangle = -\frac{m}{4\lambda} \pm \sqrt{\frac{m^2}{16\lambda^2} + \frac{S}{2\lambda}}.$$

Glueball Superpotential and Chiral vevs from basic Konsihi Anomaly-2

- Classical limit :$S \to 0$: $+$ sign $\Rightarrow \langle M \rangle = 0$: U(N) unbroken, $-$ sign $\Rightarrow \langle M \rangle \neq 0$: U(N-1) unbroken.
  $\partial W_{eff} \partial m =< M(S, m, \lambda) >$; $\partial W_{eff} \partial \lambda =< M(S, m, \lambda) >^2$

$$W_{\text{eff}} = -\frac{m^2}{8\lambda} \pm \frac{m^2}{8\lambda}\sqrt{1 + \frac{8\lambda}{m^2}S} + S\log\frac{m}{\Lambda} + S\log\left(1 \pm \sqrt{1 + \frac{8\lambda}{m^2}S}\right) +$$

- C(S) determined by matching $\lambda \to 0$ limit (SQCD with massive flavor) to its known effective superpotential W_{VY} (matching QCD scale $\tilde{\Lambda} = (\Lambda)(m/Lambda)^{\frac{1}{3N}}$ to one flavour scale Λ)

$$W_{\text{eff}} = -\frac{m^2}{8\lambda} \pm \frac{m^2}{8\lambda}\sqrt{1 + \frac{8\lambda}{m^2}S} + S\log\frac{m}{\Lambda} + S\log\left(1 \pm \sqrt{1 + \frac{8\lambda}{m^2}S}\right) +$$

- Extremizing w.r.t S determines $\langle M(m, \lambda) \rangle, \langle S(m, \lambda) \rangle$
- Also reproduced by a Vector matrix model calculation

Dijkgraaf-Vafa : Non perturbative Susy from perturbative bosonic Matrix Model !

- String Theory originated conjecture found justified in $\mathcal{N}=1$ Susy YM -Higgs models.
- Planar diagrams matrix model, even though no large N limit taken !.
- U(N) YM theory $\mathcal{N}=1$ Susy with Adjoint and superpotential

$$
\begin{aligned}
W_{\rm tree} &= \sum_{k=0}^{n} \frac{g_k}{k+1} tr\Phi^{k+1} \\
W'_{\rm tree}(z) &\equiv g_n \prod_{i=1}^{n} (z - a_i) \\
\Phi_{classical} &= Diag(a_1 \mathcal{I}_{N_1}, a_2 \mathcal{I}_{N_2}, ...) \Rightarrow U(N) \rightarrow \Pi_i U(N_i)
\end{aligned}
$$

- Chiral fields massive $L_{eff}(S_i, w_{i\alpha} \equiv TrW_{i\alpha})$ computable by perturbative calculation in auxiliary bosonic matrix model !

Seminal : 2003 : Cachazo, Douglas,Seiberg, Witten : DV justified by Generalized Konishi Anomaly !

- GKA to show generating functions for U(N) with adjoint satisfy

$$\begin{aligned} R^2(z) &= W'(z)R(z) + \frac{1}{4}f(z), \\ 2R(z)w_\alpha(z) &= W'(z)w_\alpha(z) + \frac{1}{4}\rho_\alpha(z), \\ 2R(z)T(z) + w_\alpha(z)w^\alpha(z) &= W'(z)T(z) + \frac{1}{4}c(z) \end{aligned}$$

- $f(z), \rho(z)_\alpha, c(z)$ are polynomials of degree $n-1$ where $n+1$ is the superpotential degree.
- $R(z)$ single valued on Riemann surface genus $n-a$ branched over the z plane due to the splitting of classical critical points of $W(z)$. $w_\alpha(z)$, $T(z)$ are derived from $R(z)$

- Coefficients of $f(z), c(z)$ related to Glueballs S, S_i of U(N) and $U(N_i)$ and ranks N, N_i by contour integrals around the critical points z_i of $W(z)$
- Some z_i split into branch points linked by branch cuts defining a higher genus ($g \geq 1$)) Riemann surface

$$
\begin{aligned}
f_{n-1} &= -4g_n S \qquad ; \qquad c_{n-1} = -4g_n N \\
S_i &= -\frac{1}{4\pi i}\oint_{C_i} dz \sqrt{W'(z)^2 + f(z)} \\
N_i &= -\frac{1}{8\pi i}\oint_{C_i} dz \frac{c(z)}{\sqrt{W'(z)^2 + f(z)}}
\end{aligned}
$$

- For renormalizable i.e. cubic Superpotential $n = 2$ so $g = 1$ and the lower coefficients f_0 is determined by the splitting of a zero in the factorization of $W'(z)^2 + f(z)$..

O(3) Symmetric Traceless Toy Model

- $O(3) \simeq SU(2)$ Susy YM with Symmetric traceless 3×3 matrix $\Phi = \Phi^T$; $Tr\Phi = 0$ $(j = 2, d(j) = 5, S_2(5) = 10)$ $b_0 = 3 \times 2 - 10 = -4$: NOT Asymptotically free !!
- $V_D \sim Tr[\phi, \phi^\dagger]^2 = 0$ for classical vacua $\Phi_{Class} = Diag(a, b, -(a + b))$ and the Classical moduli space of vacua is two(complex) dimensional.
- CSA, " Taming Asymptotic Strength " , hep-ph0210337 ,Oct. 2002 : Toy model for scenario of "Pleromal Unification"
- Gluino condensate in UV due to Asymptotically Strong gauge coupling coupling !! : Assumption still described by S , W_{VY} ??!!
- Then drives development/modification of chiral condensates via Konishi Anomaly connection. (GKA not then used !!).
- Analyzed using Seiberg-istics and Konishi Anomaly : $W_{tree} = -mTr\Phi^2/2 + \lambda Tr\Phi^3/3 \equiv -mX/2 + \lambda Y/3$

- Quadratic (X) and cubic (Y) moduli saturate anomalies : proper high energy degrees of freedom ?
- Low energy gauge group is either completely broken, partially broken $O(2)$ or unbroken due to decoupling of whole massive symmetric multiplet when $\lambda = 0$
- Large variety of solutions found in terms holomorphic $G(X^3/6Y^2)$ (assumed lowest order in Instanton expansion, ignoring m, λ dependent higher terms : Now understood to be justified by linearity principle !!)

$$W_{eff} = W_{dyn} + W_{tree} = \Lambda^3((X/\lambda)^{5/4} G(X^3/6Y^2) - mX/2 + \lambda X/3$$

- Equivalently $W_{dyn}(X, Y, S)$ was be defined.
- CDSW technology developed in 2003 actually permits complete solution of condensates !!

Alday Cirafici Solution -1

- O(3)-5-plet model solved by Alday and Cirafici (2003)

$$\begin{aligned} R^2(z) &= \left(W'(z) - \frac{1}{N}W'(\Phi)\right) R(z) + \frac{1}{4} f(z) \\ T(z) &= -\frac{1}{4}\frac{c(z)}{\sqrt{\left(W'(z) - \frac{1}{N}W'(\Phi)\right)^2 + f(z)}} \\ &- 2\frac{\frac{d}{dz}\left(\left(W'(z) - \frac{1}{N}W'(\Phi)\right) - \sqrt{\left(W'(z) - \frac{1}{N}W'(\Phi)\right)^2 + f(z)}\right.}{\sqrt{\left(W'(z) - \frac{1}{N}W'(\Phi)\right)^2 + f(z)}} \end{aligned}$$

$$W'(z) = mz + gz^2 \quad ; \quad W'(\Phi) = gTr\Phi^2 \quad ; \quad \because Tr\Phi = 0 \tag{4}$$

Alday Cirafici Solution -2

- Imposing factorization :
$(W'(z) - W'(\Phi)/3)^2 + (f_0 + f_1 z) \equiv g^2(z-k)^2(z^2 + az + b)$ and
$R \to S/z, T \to N/z \quad as(1/z) \to 0$ and $Tr\Phi = 0$

$$f_1 = -2gS \quad ; \quad c_1 = -12g; \quad c_0 = -12m$$
$$k = a - m/g \quad ; \quad b = 3a^2/4 - m/g - 2S/(3g)$$

- a then satisfies a cubic resulting in complicated equations. However a series solution for $W_{eff}(S)$by eliminating the invariants X, Y is given by them for any N (here $\epsilon = -1$) up to $O(S^6)$
-

$$W_{eff} = (N - 2\epsilon)\frac{S}{2}\log m + \frac{g^2(-\epsilon N + 4)S^2}{2Nm^3}$$
$$+ \frac{g^4(160\,\epsilon - 24N - N^2\,\epsilon)S^3}{12m^6N^2} + \ldots$$

Dynamical Generation of Toy GUT mass scale

- $m = 0$ limit of AC solution very singular. Resolve : Impose $m = 0$ and factorization

$$
\begin{aligned}
f_1 &= -2gS \quad ; \quad c_1 = -12g; \quad c_0 = 0m \\
k &= a; \quad b = 3a^2/4 - 2S/(3g) \\
Tr\Phi^3 &= 5S \quad !!!!
\end{aligned}
$$

- a then satisfies a cubic resulting in complicated equations. However a series solution for $W_{eff}(S)$ should be possible, inprogress
- Explicit Dynamical Generation of GUT scale directly from gaugino Condensate !!

Interpretation of Condensates

- If O(3) is completely broken all fields are massive and the low energy theory is tempty.
- If O(2) is unbroken there is a massless vector multiplet after decoupling of the massive charged $W^{\pm}$ and $\Phi_{\pm 2,0}$. Since O(2) gauginos are free and cannot condense in the IR it must be that the O(3)condensate is due to the $\lambda^{+}\lambda^{-}$ condensation !!
- YM little group like $SU(3) \in H$ condense with $\lambda_d \sim GeV^3$ must be H singlet G_{GUT}/H coset gaugino condensates that give $S_{G_{GUT}} \sim \Lambda_{UV}^3$.

- In GUT case it is thus the leptoquark gauginos that will condense in the UV Not the MSSM gauginos !!
- Complicated ASGUTs $\Rightarrow$ complex system Generalized KA equations constraining Quantum Gauge Chiral SO(10) singlet condensates of $210, 126, ...$.
- Either classical part (210, 126)which have SM singlets
- Or be purely quantum to avoid breaking SM symmetries at GUT scales !! (10,120 have NO SM singlets !!)

OUTLOOK

- Analysis of Chiral condensate system in Asysmptotically Strong GUTs using Generalized Konsishi anomaly called for !
- Dual, weakly coupled description of UV strong systems highly desirable to cross check consclusions based upon anlaogy with AF theories.
- Phenomenological effects of light fields participating in superlarge Quantum condensates need to be clarified !
- The PLEROMA is the Limit !!