Is SMEFT Enough?

(What's the physical content of an EFT lagrangian?)

Based on 2008.08597, 2108.03240, 2110.02967 with I. Banta, T. Cohen, N. Craig and X. Lu

Dave Sutherland

University of Glasgow

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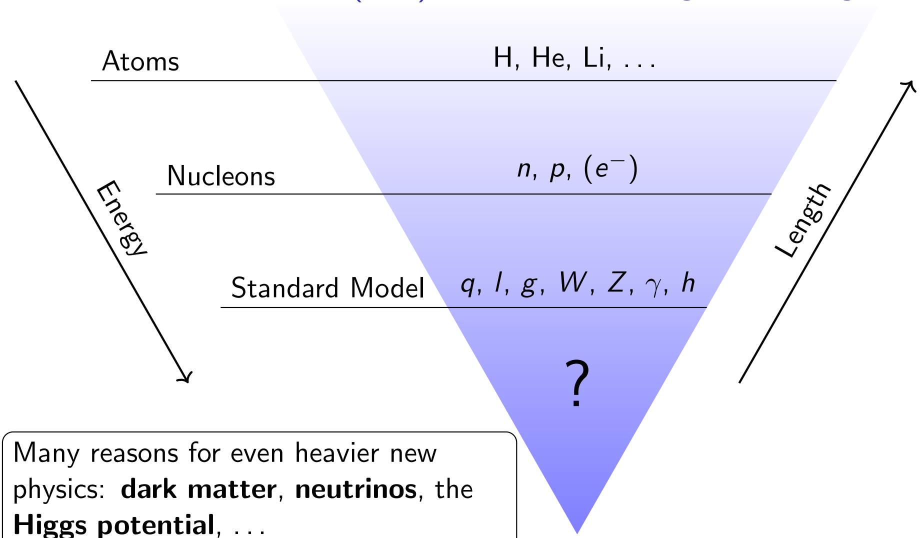
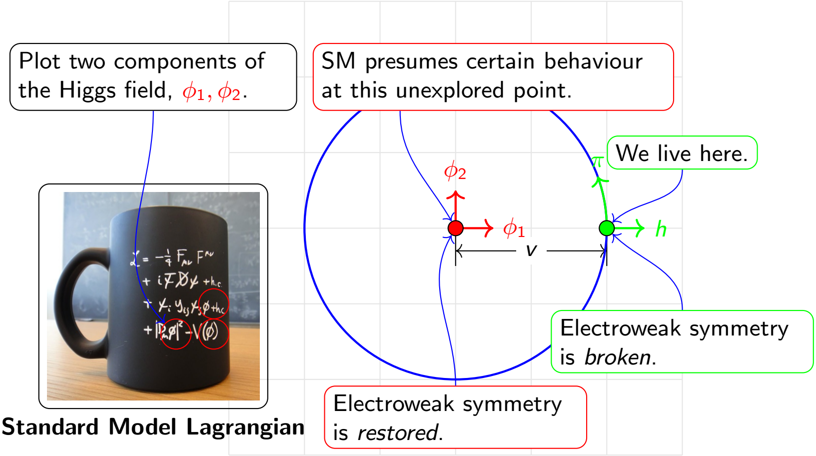
The Standard Model (SM) works at the highest energies



Many reasons for even heavier new physics: **dark matter**, **neutrinos**, the **Higgs potential**, ...

...but we haven't seen it.

The SM works *about a particular vacuum*



Standard Model Lagrangian

Could we have a theory that is Standard Model like at our vacuum, but wildly different at the symmetric point of the Standard Model?

Why study EFTs?

1) Because they parameterise all measurable effects

EFT: Given some field content (ψ, $\bar{\psi}$) and some symmetry assumptions (Poincaré, $\psi \to e^{i\alpha}\psi$, $\bar{\psi} \to \bar{\psi}e^{-i\alpha}$), write down all invariant local operators

$$\mathcal{L} = \ldots + c_1\,(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi) + \ldots + c_2\,(\bar{\psi}\gamma^\mu\psi)\Box(\bar{\psi}\gamma_\mu\psi) + \ldots$$

At the **amplitude** level, a *basis* of EFT operators spans all possible contact interactions among the known states

$$\mathcal{A}\left(\psi(1)\bar{\psi}(2)\psi(3)\bar{\psi}(4)\right) = \bar{v}(p_2)\gamma^\mu u(p_1)\bar{v}(p_4)\gamma_\mu u(p_3)\,(c_1 + c_2 s + \ldots) + \text{perms}$$

which can be joined together by light propagators to make the **most general perturbative amplitude consistent with locality**.

Why study EFTs?

2) Because they encode the low energy vestiges of all heavy new physics

Top down: *Matching* a UV theory with a heavy vector onto the EFT by Taylor expanding amplitudes in $\frac{1}{M}$

$$\begin{aligned}
&\mathcal{A}\left(\psi(1)\bar{\psi}(2)\psi(3)\bar{\psi}(4)\right)\\
&= \bar{v}(p_2)\gamma^\mu u(p_1)\frac{-e^2\, g_{\mu\nu}}{s-M^2}\bar{v}(p_4)\gamma^\nu u(p_3) + \text{perms}\\
&= \bar{v}(p_2)\gamma^\mu u(p_1)\bar{v}(p_4)\gamma_\mu u(p_3)\left(-\frac{e^2}{M^2}-\frac{e^2}{M^4}s+\ldots\right) + \text{perms}\\
&\overset{!}{=} \bar{v}(p_2)\gamma^\mu u(p_1)\bar{v}(p_4)\gamma_\mu u(p_3)\left(c_1 + c_2 s + \ldots\right) + \text{perms}
\end{aligned}$$

Bottom up: All EFT operators lead to unitarity violation at some energy E scale

$$\begin{aligned}
&\mathcal{A}\left(\psi(1)\bar{\psi}(2)\psi(3)\bar{\psi}(4)\right)\\
&= \bar{v}(p_2)\gamma^\mu u(p_1)\bar{v}(p_4)\gamma_\mu u(p_3)\left(c_1 + c_2 s + \ldots\right) + \text{perms}\\
&\sim \left(c_1 E^2 + c_2 E^4 + \ldots\right)
\end{aligned}$$

For particle physicists, EFTs can be cumbersome

A lot of this can be blamed on field redefinition redundancy

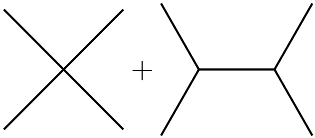
Replace a field for any local polynomial of fields and derivatives

$$\phi(x) = F[\eta, \partial] \overset{\text{e.g.}}{=} \eta(x) + \frac{1}{7\pi m}\eta(x)^2 + \ldots + \frac{3 \times 10^{10^{10^{10}}}}{m^{999}}\eta^{996}(x)\left(\partial_\mu \eta(x)\right)^4$$

and the S-matrix does not change.

This can make it difficult to:

- enumerate and agree on non-redundant operator bases
- calculate scattering amplitudes, RG, ...



Study the (simplified) EFT of the SM scalar sector

We have four scalar degrees of freedom: the Higgs boson and the three longitudinal components of the W^+, W^- and Z.

To describe their interactions, should we use:

- **SMEFT**: built about the electroweak preserving vacuum, out of fields $\vec{\phi}$ that linearly realise electroweak symmetry, or
- **HEFT**: built about our low energy vacuum, out of fields h, $\vec{\pi}$ that don't?

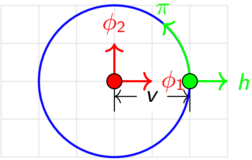
Answer

SMEFT expands about an assumed electroweak preserving vacuum, where it assumes the effects of new physics are small.

This is not necessarily true. There are viable models of heavy new physics which are poorly described by SMEFT.

SMEFT and HEFT fields

See (Alonso, Jenkins, and Manohar 2016b) for details



SMEFT uses four equivalent real scalars

$$\vec{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}, \qquad \vec{\phi} \to O\vec{\phi}, \qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_4 + i\phi_3 \end{pmatrix}$$

where $O \in O(4) \supset SU(2) \times U(1)$.

HEFT uses a real h and a unit vector $\vec{n}$ comprising 3 Goldstones π^i

$$h\,, \qquad \vec{n} = \begin{pmatrix} n_1 = \pi_1/v \\ n_2 = \pi_2/v \\ n_3 = \pi_3/v \\ n_4 = \sqrt{1 - n_1^2 - n_2^2 - n_3^2} \end{pmatrix}, \qquad \begin{cases} h & \to h \\ \vec{n} & \to O\vec{n} \end{cases}$$

SMEFT $\to$ HEFT: yes! HEFT $\to$ SMEFT: maybe?

(Alonso, Jenkins, and Manohar 2016b)

$$\mathcal{L}_{\text{SMEFT}} = \frac{1}{2}A(\vec{\phi}\cdot\vec{\phi})(\partial\vec{\phi}\cdot\partial\vec{\phi}) + \frac{1}{2}B\left(\vec{\phi}\cdot\vec{\phi}\right)(\vec{\phi}\cdot\partial\vec{\phi})^2 - V\left(\vec{\phi}\cdot\vec{\phi}\right)$$

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2}[K(h)]^2(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n}(\pi)\cdot\partial\vec{n}(\pi)) - V(h)$$

Using redefinitions

$$\vec{\phi} = (v_0 + h)\vec{n}(\pi); \qquad \begin{cases} (v_0 + h) = \sqrt{\vec{\phi}\cdot\vec{\phi}} \\ \vec{n} = \frac{\vec{\phi}}{\sqrt{\vec{\phi}\cdot\vec{\phi}}} \end{cases} \qquad (\vec{n}\cdot\vec{n} = 1)$$

$$\mathcal{L}_{\text{SMEFT}\to\text{HEFT}} = \frac{1}{2}\left[A + (v_0+h)^2 B\right](\partial h)^2 + \frac{1}{2}\left[(v_0+h)^2 A\right](\partial\vec{n})^2 - V$$

$$\mathcal{L}_{\text{HEFT}\to\text{SMEFT}} = \frac{1}{2}\frac{v^2F^2}{\vec{\phi}\cdot\vec{\phi}}(\partial\vec{\phi}\cdot\partial\vec{\phi}) + \frac{1}{2}\left(\frac{K^2}{\vec{\phi}\cdot\vec{\phi}} - \frac{v^2F^2}{(\vec{\phi}\cdot\vec{\phi})^2}\right)(\vec{\phi}\cdot\partial\vec{\phi})^2 - V$$

Note A, B and V even functions of $v_0 + h$.

This talk

Work 'geometrically' to identify:

- field redefinition invariant features of the Lagrangian that cannot be described by SMEFT
  (Cohen, Craig, Lu, and Sutherland 2021a)
- experimentally viable 'HEFTy' theories
  (Banta, Cohen, Craig, Lu, and Sutherland 2021)

Other work:

- how these features map onto amplitudes, and how 'HEFT' therefore does not decouple
  (Cohen, Craig, Lu, and Sutherland 2021b)

Geometric picture (often used in non-linear sigma models)

Fields ϕ^α are **coordinates** on space of field values (**target space**).

Field redefinitions — *without derivatives* — are **coordinate redefinitions** $\phi^\alpha = \phi^\alpha(\vec{\eta})$.

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The lagrangian, *up to two derivatives*, defines a **metric** and a **potential** on target space

$$\mathcal{L} = \frac{1}{2} g_{\alpha\beta}(\phi) \partial_\mu \phi^\alpha \partial^\mu \phi^\beta - V(\phi)$$

$$\overset{\text{Field redef.}}{=} \frac{1}{2} \left(g_{\alpha\beta}(\phi(\eta)) \frac{\partial \phi^\alpha}{\partial \eta^\gamma} \frac{\partial \phi^\beta}{\partial \eta^\delta} \right) \partial_\mu \eta^\gamma \partial^\mu \eta^\delta - V(\phi(\eta))$$

$$= \sum_n \frac{1}{n!} \phi^{\gamma_1} \ldots \phi^{\gamma_n} \left(\overline{g}_{\alpha\beta,\gamma_1\ldots\gamma_n} \frac{1}{2} \partial_\mu \phi^\alpha \partial^\mu \phi^\beta - \overline{V}_{,\gamma_1\ldots\gamma_n} \right)$$

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The amplitudes are built out of covariant quantities. At tree-level

$$\left(\prod_{i=1}^{n} \overline{g}^{1/2}_{\alpha_i\alpha_i} \right) \mathcal{A}_n = \overline{V}_{;(\alpha_1\dots\alpha_n)} + \sum_{1\le i<j\le n} s_{ij} \left(\tfrac{n-3}{n-1} \right) \left[\overline{R}_{\alpha_i(\alpha_1\alpha_2|\alpha_j;|\alpha_3\dots\hat{\alpha}_i\dots\hat{\alpha}_j\dots\alpha_n)} + \mathcal{O}\left(\overline{R}^2\right) \right]$$

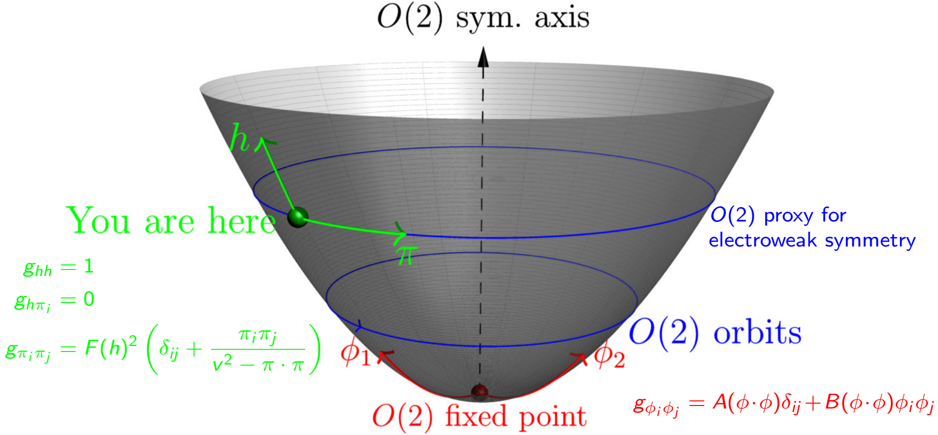
$$+ \text{factorizable pieces}\,,$$

where ‘,’=partial derivative, ‘;’=covariant derivative. A $\overline{\text{bar}}$ means evaluated at the vacuum $\phi = 0$.

Geometric picture of SMEFT & HEFT (custodial limit)

See (Alonso, Jenkins, and Manohar 2016b), (Helset, Martin, and Trott 2020)

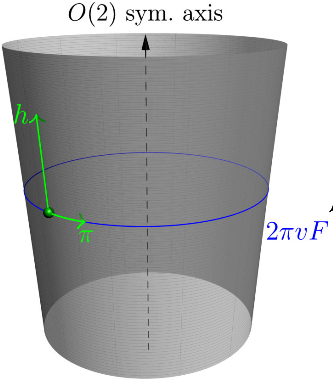
HEFT expands about our vacuum in h and Goldstones $\pi_1, \ldots$.



SMEFT expands about an electroweak preserving vacuum in components of the Higgs doublet $\phi_1, \phi_2, \ldots$

When is a HEFT not a SMEFT?

1) When it's a funnel (Alonso, Jenkins, and Manohar 2016b)



$$\mathcal{L}_{\mathrm{HEFT}} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h)$$

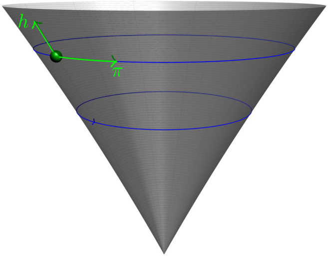
If the geodesic distance of closed $O(2)$ orbits are non-zero everywhere

$$F(h) \neq 0$$

then there's no fixed point about which to expand in SMEFT coordinates.

When is a HEFT not a SMEFT?

2) When it's a cone (Cohen, Craig, Lu, and Sutherland 2021a)



$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h)$$

Suppose $F(-v_0) = 0$ for some v_0. The HEFT chart is degenerate, and the HEFT lagrangian may hide non-analyticities.

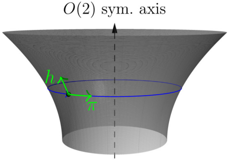
To diagnose non-analyticites, can use **curvature invariants**, e.g.

$$\mathcal{K}_h = -\frac{F''}{F}; \qquad \mathcal{K}_\pi = \frac{1}{v^2F^2}\left[1-(vF')^2\right];$$

$$\nabla^2 V = V'' + \frac{3V'}{F}$$

As $h \to -v_0$ and $F \to 0$, $\mathcal{K}_\pi \to \infty$ (a **conical singularity**) unless $F'(-v_0) = \frac{1}{v}$.

When is a HEFT not a SMEFT? (Examples)



1) When there are extra sources of EWSB, e.g., a triplet

$$\mathcal{L}_{\text{UV}} = |\partial H|^2 + \frac{1}{2}(\partial\Phi)^2 - \left(-\mu_H^2|H|^2 + \lambda_H|H|^4 + \frac{1}{2}m^2\Phi^2 - \frac{1}{2}\mu H^\dagger\sigma^a H\Phi_a + \kappa|H|^2\Phi^2 + \frac{1}{4}\lambda_\Phi\Phi^4\right)$$

Reparameterise as radial (r, f) and angular modes (π^a, β^i). [Note $f^2 = \Phi^a\Phi^a$.]

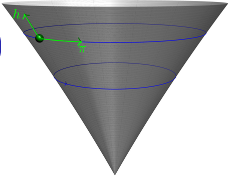
$$H = \frac{1}{\sqrt{2}} r \exp\left(i\frac{\pi^a}{v}\sigma^a\right)\begin{pmatrix}0\\1\end{pmatrix}; \quad \Phi_a = \frac{2f}{r^2}\exp\begin{pmatrix}0 & 0 & \beta_1\\ 0 & 0 & \beta_2\\ -\beta_1 & -\beta_2 & 0\end{pmatrix}\begin{pmatrix}H^\dagger\sigma^1 H\\ H^\dagger\sigma^2 H\\ H^\dagger\sigma^3 H\end{pmatrix}$$

to integrate out at tree-level (sub. in EOM solutions of f and β).

$$\mathcal{L}_{\text{EFT}} = \frac{1}{2}\left[1 + (f_c')^2 + \frac{8f_c^2}{r^2}\right](\partial r)^2 + \frac{1}{2}\left[\frac{r^2 + 4f_c^2}{v^2}\right]\left((\partial\pi_1)^2 + (\partial\pi_2)^2\right) + \frac{1}{2}\left[\frac{r^2}{v^2}\right](\partial\pi_3)^2 - V + \mathrm{O}(\partial^4, \pi^4)$$

Need $f_c \to 0$ as $r \to 0$ for SMEFT

When is a HEFT not a SMEFT? (Examples)



2) When turning off the Higgs vev gives massless BSM particles (Falkowski and Rattazzi 2019)

Extend the scalar sector with an EW singlet

$$\mathcal{L}_{\rm UV} = |\partial H|^2 + \frac{1}{2}(\partial S)^2 - \left(-\mu_H^2|H|^2 + \lambda_H|H|^4 + \frac{1}{2}\left(m^2 + \kappa|H|^2\right)S^2 + \frac{1}{4}\lambda_S S^4\right)$$

Match at tree-level: sub in the solution S^c to the EOM, assume $m^2, \kappa \leq 0$.

$$\frac{\delta S_{\rm UV}}{\delta S} = (\partial^2 + m^2 + \kappa|H|^2 + \lambda_S S^2)S = 0 \implies S^c = \sqrt{-\frac{m^2 + \kappa|H|^2}{\lambda_S}} + \mathrm{O}(\partial^2)$$

$$\mathcal{L}_{\rm EFT} = |\partial H|^2 - \frac{\kappa^2\left(\partial_\mu|H|^2\right)^2}{4\lambda_S\left(m^2 + \kappa|H|^2\right)} - \left(-\mu_H^2|H|^2 + \lambda_H|H|^4 - \frac{\left(m^2 + \kappa|H|^2\right)^2}{4\lambda_S}\right) + \mathrm{O}(\partial^4)$$

The lagrangian is non-analytic at $H = 0$ when $m^2 = 0$.

$\mathbb{Z}_2$ singlet example: loop level

$$
\begin{aligned}
\mathcal{L}_{\text{UV}} =& |\partial H|^2 + \frac{1}{2}(\partial S)^2 \\
& - \left(-\mu_H^2 |H|^2 + \lambda_H |H|^4 + \frac{1}{2}\left(m^2 + \kappa |H|^2 \right) S^2 + \frac{1}{4} \lambda_S S^4 \right)
\end{aligned}
$$

Choose $m^2, \kappa > 0$ such that we're on the trivial $S^c = 0$ branch

$$
\begin{aligned}
\mathcal{L}_{\text{EFT}} =& |\partial H|^2 + \frac{1}{384\pi^2} \frac{\kappa^2}{m^2 + \kappa |H|^2} \left(\partial |H|^2 \right)^2 \\
& + \mu_H^2 |H|^2 - \lambda_H |H|^4 + \frac{1}{64\pi^2} \left(m^2 + \kappa |H|^2 \right)^2 \left(\ln \frac{\mu^2}{m^2 + \kappa |H|^2} + \frac{3}{2} \right)
\end{aligned}
$$

The lagrangian is non-analytic at $H = 0$ when $m^2 = 0$.

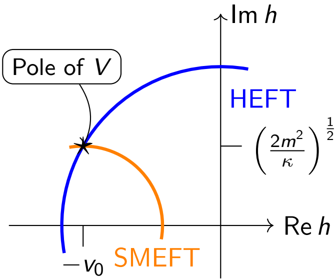
EFT convergence

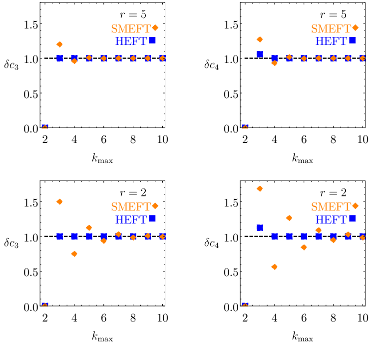
Expand $\Delta V = -\frac{1}{64\pi^2}\left(m^2+\kappa|H|^2\right)^2\left(\ln\frac{\mu^2}{m^2+\kappa|H|^2}+\frac{3}{2}\right)$ in powers of

$$X_{\text{SMEFT}} = \frac{\kappa|H|^2}{m^2} = \frac{\kappa v_0^2}{2m^2}\left(1+\frac{h}{v_0}\right)^2$$

$$X_{\text{HEFT}} = \frac{\kappa\left(|H|^2-\frac{1}{2}v_0^2\right)}{m^2+\frac{1}{2}\kappa v_0^2} = \frac{\kappa v_0^2}{2m^2+\kappa v_0^2}\left[2\frac{h}{v_0}+\left(\frac{h}{v_0}\right)^2\right]$$

and consider radius of convergence in terms of $r \equiv \frac{m^2}{\frac{1}{2}\kappa v_0^2}$.





Viable HEFTy models — the 'Loryons'[1]

(Banta, Cohen, Craig, Lu, and Sutherland 2021)

Should use HEFT when fraction of mass(-squared) from Higgs:

$$f_{\mathsf{max}} > \frac{1}{2}$$

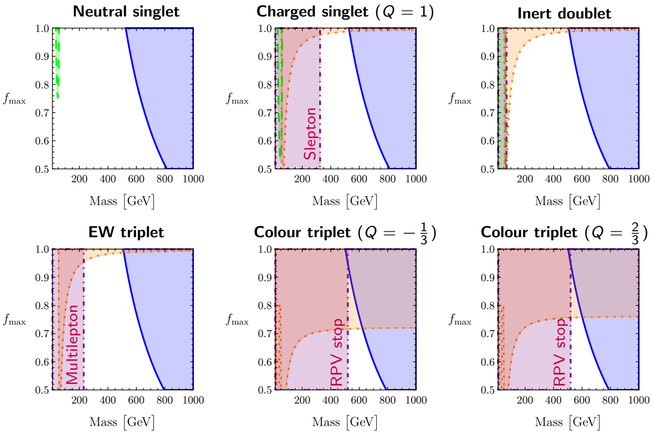
We study scalars and fermions in a variety of electroweak irreps, with approximate $\mathbb{Z}_2$ symmetry (like the loop-level singlet model)
Consider

- κ_γ, κ_g
- Perturbative unitarity constraints on coupling to Higgs (e.g $\lambda_{h\Phi}$ for scalars)
- Higgs decay
- Direct searches (charged components decay promptly via the least detectable of the lowest dimension operators)

[1]From *Finnegan's Wake*, "with Pa's new heft...see Loryon the comaleon."

These SMEFT-defying models are experimentally viable

(Banta, Cohen, Craig, Lu, and Sutherland 2021)



Disallowed regions in colour:

Orange, dotted: κ_γ or κ_g

Blue, solid: perturb. unitarity $\lambda_{h\Phi}$

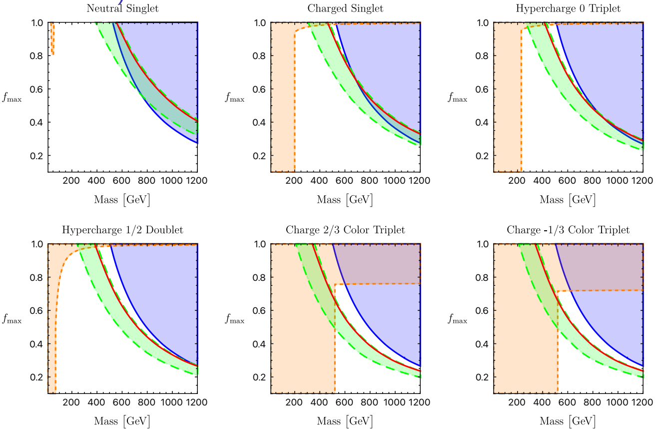
Green, dashed: Higgs decay

Purple, dot-dash: Direct search

Plot bounds: fraction of mass squared from Higgs ($f_{\max}$) *vs.* total mass.

These models can produce a strongly first order EWPT

(Banta 2022)



Orange, dotted: κ_γ or κ_g expt. constraints

Blue, solid: perturb. unitarity

Green, dashed: strongly first-order phase transition

Red, solid lower bound for stochastic gravitational wave background @ LISA

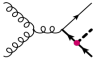
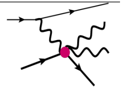
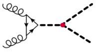
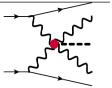
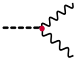
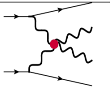
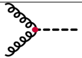
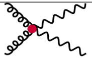
$$\frac{S_3}{T_n} \approx 140$$
$$\frac{v_n}{T_n} \gtrsim 1$$
$$T_n > 10 \text{ GeV}$$

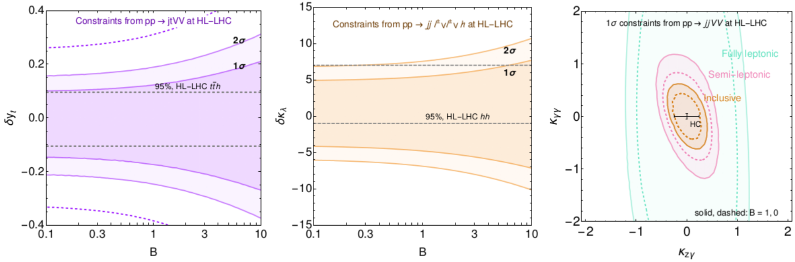
$$\alpha = \left(V_{\text{eff}} - \frac{T_n}{4} \frac{\mathrm{d} V_{\text{eff}}}{\mathrm{d} T} \right) \Bigg/ \frac{g_* \pi^2 T_n^4}{30} ,$$
$$\beta / H_* = \left. \frac{\mathrm{d} S_3}{\mathrm{d} T} \right|_{T_n} - \frac{S_3}{T_n} .$$
$$\log(\beta / H_*) \lesssim 1.2 \log \alpha + 8.8$$

Loryons may be poorly fit by SMEFT *at dimension 6*

At HL-LHC we may be able to probe the correlations of a single SMEFT operator across different Higgs multiplicities. (Henning, Lombardo, Riembau, and Riva 2019)

These are broken by Loryons.

		HC	HwH
κ_t	$\mathcal{O}_{y_t}$		
κ_λ	$\mathcal{O}_6$		
$\kappa_{Z\gamma}$ $\kappa_{\gamma\gamma}$ κ_V	$\mathcal{O}_{WW}$ $\mathcal{O}_{BB}$ $\mathcal{O}_r$		
κ_g	$\mathcal{O}_{gg}$		

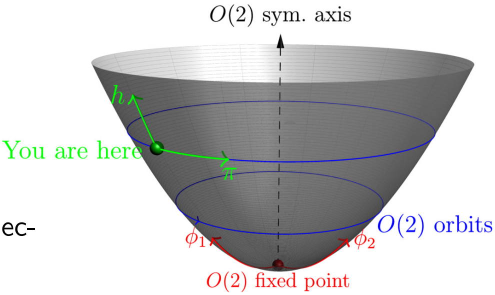


Summary

Dynamics encoded by a metric and potential.

HEFT expands about our vacuum.

SMEFT expands about the electroweak preserving vacuum.



A HEFT is poorly described by SMEFT when violence is done to the manifold between us and the EW preserving vacuum.

Simple UV completions of HEFT models remain currently viable.

EFT Lagrangians have redundancy. A useful way of extracting the physical content of an EFT is to think geometrically!

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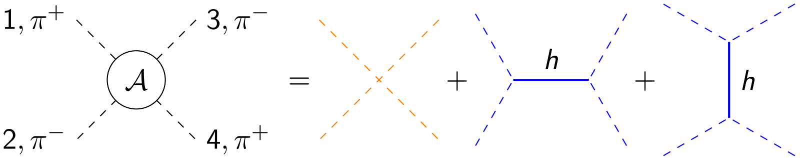
Backup

Example amplitude: $W_L^+ W_L^- \to W_L^+ W_L^-$

$$\mathcal{L} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}(vF(h))^2(\partial \vec{n})^2 - V(h) \supset + \frac{1}{2v^2}\left[\partial_\mu(\pi^+\pi^-)\right]^2 + 2\overline{F'}\, h\partial\pi^+\partial\pi^-$$



$$\mathcal{A} = -\frac{1}{v^2}(s+t) + \overline{F'}^2\left[\frac{s^2}{s-m_h^2} + \frac{t^2}{t-m_h^2}\right]$$

$$= \left(\overline{F'}^2 - \frac{1}{v^2}\right)(s+t) + \overline{F'}^2\left[2m_h^2 + \frac{m_h^4}{s-m_h^2} + \frac{m_h^4}{t-m_h^2}\right]$$

Note:

- we use vertices with different numbers of Higgses;
- the $\mathrm{O}(p^2)$ part has no kinematic pole.

(A $\overline{\text{bar}}$ means evaluated at our vacuum $h = \pi_i = 0$.)

$W_L^+ W_L^- \to W_L^+ W_L^-$ geometrically

Derivative key: ‘,’ partial, ‘;’ covariant (Nagai, Tanabashi, Tsumura, and Uchida 2019)

$$\mathcal{A} = -\left(\frac{1}{v^2} - \overline{F'}^2\right)(s+t) + 2m_h^2\overline{F'}^2 + m_h^4\overline{F'}^2\left[\frac{1}{s-m_h^2} + \frac{1}{t-m_h^2}\right]$$

$$\overline{R}_{+--+} = -\overline{g}_{+-,+-} + \overline{\Gamma}^h_{+-}\overline{\Gamma}_{-+h} = \frac{1}{v^2} - \overline{F'}^2$$

$$\overline{V}_{;(h+-)} = -\overline{V}_{,hh}\overline{\Gamma}^h_{+-} = -m_h^2\overline{F'}$$

$$\overline{V}_{;(++--)} = 2\overline{V}_{,hh}\overline{\Gamma}^h_{+-}\overline{\Gamma}^h_{+-} = 2m_h^2\overline{F'}^2$$

$$\mathcal{A} = -\overline{R}_{+--+}(s+t) + \overline{V}_{;(++--)} + \overline{V}_{;(h+-)}\overline{g}^{hh}\overline{V}_{;(h+-)}\left[\frac{1}{s-m_h^2} + \frac{1}{t-m_h^2}\right]$$

The components of R are **sectional curvatures**

$$\overline{R}_{+--+} \equiv \overline{\mathcal{K}_\pi}$$

(Alonso, Jenkins, and Manohar 2016a),(Cohen, Craig, Lu, and Sutherland 2021b)

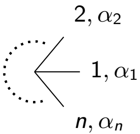
Feynman rules for scalar theories

Derivative key: ',' partial, ';' covariant. A $\overline{\text{bar}}$ means evaluated at the vacuum $\phi = 0$

Taylor expand the lagrangian

$$\begin{aligned}\mathcal{L} &= \frac{1}{2} g_{\alpha\beta}(\vec{\phi}) \partial_\mu \phi^\alpha \partial^\mu \phi^\beta - V(\vec{\phi}) \\ &= \sum_n \frac{1}{n!} \phi^{\gamma_1} \ldots \phi^{\gamma_n} \left(\overline{g}_{\alpha\beta,\gamma_1\ldots\gamma_n} \frac{1}{2} \partial_\mu \phi^\alpha \partial^\mu \phi^\beta - \overline{V}_{,\gamma_1\ldots\gamma_n} \right)\end{aligned}$$

to get a propagator: α ——— $\beta \; = \dfrac{i\overline{g}^{\alpha\beta}}{p^2 - m_\alpha^2}$ where $\overline{V}_{,\alpha\beta} \overline{g}^{\beta\gamma} = m_\alpha^2 \delta_\alpha^\gamma$,

and vertices: 

$$= -i\overline{V}_{,\alpha_1\ldots\alpha_n} - i \sum_{1\le i<j\le n} p_i \cdot p_j \, \overline{g}_{\alpha_i\alpha_j,\alpha_1\ldots\hat{\alpha}_i\ldots\hat{\alpha}_j\ldots\alpha_n}$$

$$= -i\overline{V}_{,\alpha_1\ldots\alpha_n} - i \sum_{1\le i<j\le n} s_{ij} \, \frac{1}{2} \overline{g}_{\alpha_i\alpha_j,\alpha_1\ldots\hat{\alpha}_i\ldots\hat{\alpha}_j\ldots\alpha_n}$$

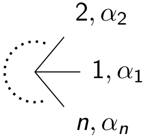
$$+ i \sum_{1\le i\le n} (n-1) m_i^2 \, \overline{g}_{\alpha_i(\alpha_1,\ldots\hat{\alpha}_i\ldots\alpha_n)} + i \sum_{1\le i\le n} (n-1) \left(p_i^2 - m_i^2 \right) \overline{g}_{\alpha_i(\alpha_1,\ldots\hat{\alpha}_i\ldots\alpha_n)} \, .$$

Feynman rules for scalar theories *in normal coordinates*

Derivative key: ',' partial, ';' covariant. A $\overline{\text{bar}}$ means evaluated at the vacuum $\phi = 0$

(aka) An inertial frame is an optimal set of coordinates — a special basis — which makes the lagrangian to amplitude map transparent.

$$\alpha \;\text{———}\; \beta \;=\; \frac{i\overline{g}^{\alpha\beta}}{p^2 - m_\alpha^2} \text{ where } V_{,\alpha\beta}g^{\beta\gamma} = m_\alpha^2 \delta_\alpha^\gamma\,,$$



$$= -i\overline{V}_{;\alpha_1\ldots\alpha_n} - i\sum_{1\le i<j\le n} s_{ij}\left(\tfrac{n-3}{n-1}\right)\left[\overline{R}_{\alpha_i(\alpha_1\alpha_2|\alpha_j;|\alpha_3\ldots\hat{\alpha}_i\ldots\hat{\alpha}_j\ldots\alpha_n)} + \mathrm{O}(R^2)\right]$$

$$+\, i\sum_{1\le i\le n}(n-1)m_i^2 \times 0 + i\sum_{1\le i\le n}(n-1)\left(p_i^2 - m_i^2\right)\times 0$$

These geometric tools can help us compute amplitudes efficiently, and understand the experimentally accessible content of EFTs!

Four legs good, more legs better

Following (Falkowski and Rattazzi 2019), (Abu-Ajamieh, Chang, Chen, and Luty 2020)

With the application of geometric/kinematic identities:

$$\mathcal{A}\left(\pi_i \pi_j h^{n-2}\right) = \overline{V}_{;(\pi_i \pi_j h \ldots h)} + \overline{R}_{\pi_i h h \pi_j; h \ldots h} \left(s_{12} - \frac{2 m_h^2}{n-1} \right) + \mathrm{O}\left(\overline{R}^2\right) + \text{factorizable pieces}$$

$$= \frac{1}{3}\, \delta_{ij}\, \overline{\partial_h^{n-2} \left(\nabla^2 V - \partial_h^2 V \right)} - \delta_{ij}\, \overline{\partial_h^{n-4} \mathcal{K}_h} \left(s_{12} - \frac{2 m_h^2}{n-1} \right) + \mathrm{O}\left(\overline{R}^2\right) + \text{factorizable pieces}\,.$$

The parts of the $n > 4$ amplitudes that grow with CoM energy E are *derivatives* of sectional curvatures.

$$\mathcal{A}\left(\pi_i \pi_j \to h^{n-2}\right) = -E^2\, \delta_{ij}\, \overline{\partial_h^{n-4} \mathcal{K}_h} + \mathrm{O}(E^0)$$

$\mathcal{K}_h$ is the sectional curvature in any $h - \pi_i$ direction (in the custodial limit). A $\overline{\text{bar}}$ means evaluated at our vacuum $h = \pi_i = 0$.

Unitarity bound for $\mathcal{A}(\pi_i\pi_j \to h^n)$

Unitarity bound

$$E \lesssim 4\pi \times \left|\frac{\overline{\partial_h^{n-2}\mathcal{K}_h}}{n!}\right|^{-\frac{1}{n}} (n!)^{\frac{1}{n}} \approx \begin{cases} 4\pi\,\left|\overline{\mathcal{K}_h}\right|^{-\frac{1}{2}} & n = 2 \\ 4\pi v_\star (n!)^{\frac{1}{n}} & n = \text{`a few'} \end{cases}$$

$2 \to 2$ and $2 \to n$ scattering access different **scales** in the theory.

$2 \to 2$ measures **how flat** the EFT is *at our vacuum*, $\left|\overline{\mathcal{K}_h}\right|^{-\frac{1}{2}}$.

$2 \to n$ measures (roughly) **the radius of convergence**, $v_\star$, of $\mathcal{K}_h$.

Many weakly coupled UV theories have a natural hierarchy between these scales, $v_\star^2|\overline{\mathcal{K}_h}| \ll 1$.

(See paper for analogous computation $\mathcal{A}(\pi_i\pi_j \to \pi_k\pi_l h^{n-4}) \propto -E^2\,\overline{\partial_h^{n-4}\mathcal{K}_\pi}$.)

Example: loop-level scalar singlet EFT

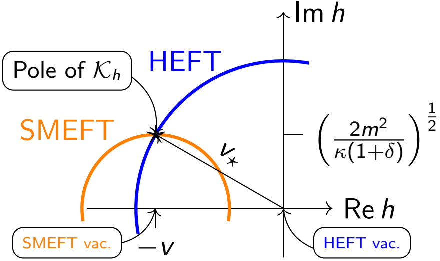
(Cohen, Craig, Lu, and Sutherland 2021a)

$$\mathcal{L}_{\text{UV}} = |\partial H|^2 + \mu_H^2 |H|^2 - \lambda_H |H|^4 + \frac{1}{2} S \left(-\partial^2 - m^2 - \kappa |H|^2\right) S \,.$$

Assume $m^2, \kappa > 0$. Let $\delta = \frac{\kappa}{96\pi^2}$. Integrate out S to get

Sectional curvature,

$$\mathcal{K}_h = \delta \frac{\kappa}{2} \frac{m^2}{\left(m^2 + \frac{1}{2}\kappa\,(1+\delta)\,(v+h)^2\right)^2}$$



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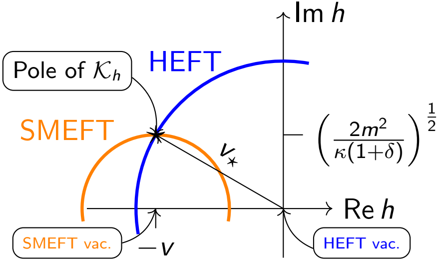


If m^2 small: S gets most of its mass from the Higgs; nearly flat at our vacuum; unitarity cutoff $4\pi v_\star \approx 4\pi v$; S does not decouple; EFT poorly described by SMEFT.

(Non-decoupling follows from position of pole: proximity to our vacuum gives TeV unitarity cutoff, proximity to EW preserving vacuum gives poor SMEFT expansion.)

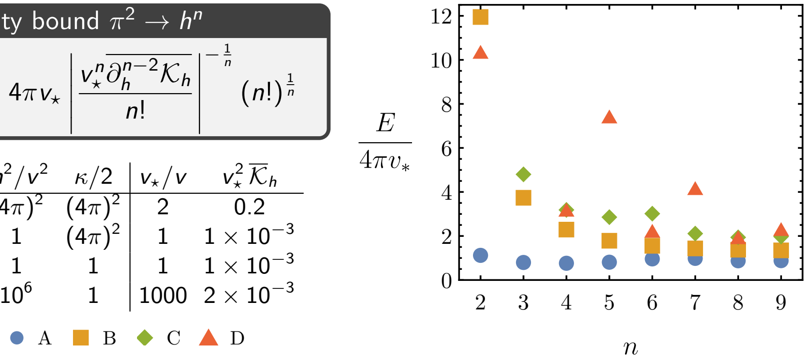
Example: loop-level scalar singlet unitarity cutoff

$$\mathcal{L}_{\text{UV}} = |\partial H|^2 + \mu_H^2 |H|^2 - \lambda_H |H|^4 + \frac{1}{2} S \left(-\partial^2 - m^2 - \kappa |H|^2\right) S \,.$$

Unitarity bound $\pi^2 \to h^n$

$$E = 4\pi v_\star \left| \frac{v_\star^n \overline{\partial_h^{n-2} \mathcal{K}_h}}{n!} \right|^{-\frac{1}{n}} (n!)^{\frac{1}{n}}$$

	m^2/v^2	$\kappa/2$	$v_\star/v$	$v_\star^2 \overline{\mathcal{K}}_h$
A	$(4\pi)^2$	$(4\pi)^2$	2	0.2
B	1	$(4\pi)^2$	1	1×10^{-3}
C	1	1	1	1×10^{-3}
D	10^6	1	1000	2×10^{-3}



Example: loop-level scalar singlet unitarity cutoff

$$\mathcal{L}_{\text{UV}} = |\partial H|^2 + \mu_H^2 |H|^2 - \lambda_H |H|^4 + \frac{1}{2} S \left(-\partial^2 - m^2 - \kappa |H|^2\right) S \,.$$

Unitarity bound $\pi^2 \to h^n$

$$E = 4\pi v_\star \left| \frac{v_\star^n \overline{\partial_h^{n-2} \mathcal{K}_h}}{n!} \right|^{-\frac{1}{n}} (n!)^{\frac{1}{n}}$$

	m^2/v^2	$\kappa/2$	$v_\star/v$	$v_\star^2\,\overline{\mathcal{K}}_h$
A	$(4\pi)^2$	$(4\pi)^2$	2	0.2
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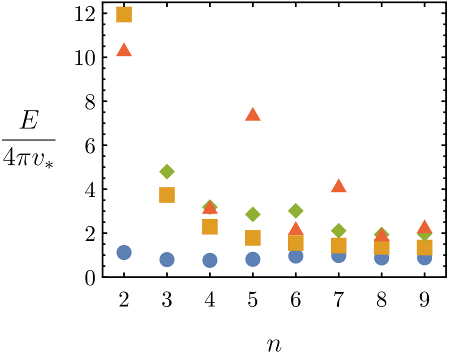


Models **B**, **C** and **D** have a lower cutoff than $2 \to 2$ scattering suggests.

Models **A**, **B** and **C** are non-decoupling ($v_\star \approx v$), with TeV scale cutoff. Model **D** is SMEFT.

(Many non-decoupling extensions of the SM scalar sector are still viable! (Banta, Cohen, Craig, Lu, and Sutherland 2021)(Banta 2022)

Amplitudes summary

Measurable quantities are geometric!

In a scalar EFT, *up to two derivatives*:

- Operator coeffs ≈ **partial** derivatives of metric and potential
- Amplitude coeffs ≈ **covariant** derivatives of Riemann curvature tensor and potential ≈ curvature invariants and their derivatives

---

In perturbatively matched models, $2 \to n$ amplitudes measure distance to pole in curvature invariant, $v_\star$.

This allows us to understand decoupling in scalar sector of SM, and distinctions in its SMEFT and HEFT descriptions.

For ideas on going beyond two derivatives, see (Cohen, Craig, Lu, and Sutherland 2022).

Why go beyond two derivatives?

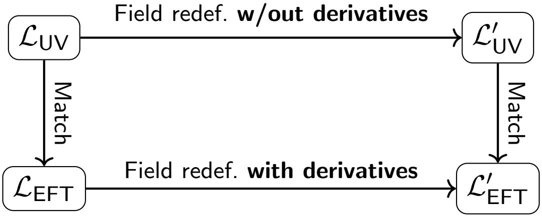
Field redefinitions with derivatives change, e.g., the metric and curvature

$$\mathcal{L} = -V(\tilde{\phi}) + \frac{1}{2} g_{\alpha\beta}(\tilde{\phi}) \partial\tilde{\phi}^\alpha \partial\tilde{\phi}^\beta + \mathrm{O}(\partial^4)$$

$$\tilde{\phi}^\alpha = \phi^\alpha + \frac{1}{2} h^\alpha_{\gamma_1\gamma_2}(\phi)(\partial\phi^{\gamma_1}\partial\phi^{\gamma_2})$$

$$\mathcal{L} = -V(\phi) + \frac{1}{2}\left(g_{\alpha\beta}(\phi) - V_{,\gamma}(\phi) h^\gamma_{\alpha\beta}(\phi)\right)\partial\phi^\alpha\partial\phi^\beta + \mathrm{O}(\partial^4)$$

It's possible to do this *inadvertently*, because



(See e.g. (Criado and Pérez-Victoria 2019))

The full covariance of correlators

(Cohen, Craig, Lu, and Sutherland 2022)

$$S[\phi] = \int \mathrm{d}^4 x \left(-V(\vec{\phi}) + \frac{1}{2} g_{\alpha\beta}(\vec{\phi}) \partial_\mu \phi^\alpha \partial^\mu \phi^\beta + \dots \right)$$

To deal with derivatives, we go off-shell, and go from looking at the fields at one spacetime point (**target space**) to fields at every spacetime point (**configuration space**).

Use DeWitt's condensed notation. Let x denote the ensemble of ϕs spacetime coordinates, flavour indices, Lorentz indices, ...

$$\phi^\alpha(x) \to \phi^x$$

$$\int \mathrm{d}^4 x \phi^\alpha(x) J_\alpha(x) \to \phi^x J_x$$

(See (**DeWitt:2003pm**).)

Amputated correlators $\mathcal{M}$ are halfway to amplitudes

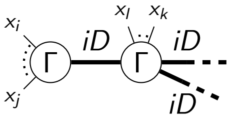
Recall the usual path integral setup

$$e^{i(\Gamma[\phi]+J_x\phi^x)} = e^{iW[J]} = \int \mathcal{D}\eta\, e^{i(S[\eta]+J_x\eta^x)}\,. \text{ Define } (-iD^{-1}_{xy}) = \frac{\delta^2(-\Gamma)}{\delta\phi^x\delta\phi^y}$$

We study the amputated correlator (the sum of tree graphs built of 1PI vertices, Γ, and propagators, iD)

$$\mathcal{M}_{x_1\cdots x_n} \equiv -\left(-iD^{-1}_{x_1y_1}\right)\cdots\left(-iD^{-1}_{x_ny_n}\right)\frac{\delta^n W[J]}{\delta J_{y_1}\cdots\delta J_{y_n}}\,.$$

$$= \sum_{\text{graphs}}$$



and its geometric properties.

To get amplitudes, set $\phi^x = J_x = 0$ and contract with wavefns $\prod_i(\epsilon_i^\mu)e^{ip_i\cdot x_i}$.

$$\left[\prod_{i=1}^{n}(\epsilon_i^\mu)e^{ip_ix_i}\right]\mathcal{M}_{x_1\cdots x_n}\Big|_{J=0} = -(2\pi)^4\delta^4\left(\sum_i p_i\right)Z_\eta^{-n/2}\mathcal{A}$$

These indexed quantities generalise target space tensors

Metric: Inverse propagator $iD^{-1}_{xy} = \dfrac{\delta^2(-\Gamma)}{\delta\phi^x\delta\phi^y}$

$$iD^{-1}_{xy}|_{J=0} = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} e^{ip(x-y)} \left\{ \overline{V}_{,\alpha\beta} - p^2 \overline{g}_{\alpha\beta} + \ldots \right\}.$$

Connection: 3-point vertex $G^y_{x_1x_2} = iD^{yz} \dfrac{\delta^3(-\Gamma)}{\delta\phi^z\delta\phi^{x_1}\delta\phi^{x_2}}$

$$G^y_{x_1x_2}|_{J=0} e^{ip_1x_1} e^{ip_2x_2} = e^{i(p_1+p_2)y} \left\{ -\frac{\overline{V}_{;\,\alpha_1\alpha_2}^{\;\beta}}{((p_1+p_2)^2 - m_c^2)} + \overline{\Gamma}^{\beta}_{\alpha_1\alpha_2} + \ldots \right\}.$$

Higher order tensors: Amputated correlators

$$\mathcal{M}_{x_1x_2x_3x_4} = \frac{\delta^4(-\Gamma)}{\delta\phi^{x_1}\delta\phi^{x_2}\delta\phi^{x_3}\delta\phi^{x_4}} - \frac{\delta^3(-\Gamma)}{\delta\phi^{x_1}\delta\phi^{x_2}\delta\phi^{y}}(iD^{yz})\frac{\delta^3(-\Gamma)}{\delta\phi^{z}\delta\phi^{x_3}\delta\phi^{x_4}} - \ldots$$

$$\mathcal{A} = \overline{V}_{;(\alpha_1\alpha_2\alpha_3\alpha_4)} + \frac{2}{3} s_{12} \overline{R}_{\alpha_1(\alpha_3\alpha_4)\alpha_2} + \ldots.$$

Do they construct and transform covariantly in an analogous way?

Recursive construction by covariant derivative

Main result: adding another leg to a correlator amounts to covariant differentiation

$$\mathcal{M}_{x_1 \cdots x_n x} = \nabla_x \mathcal{M}_{x_1 \cdots x_n} = \frac{\delta}{\delta \phi^x} \mathcal{M}_{x_1 \cdots x_n} - \sum_{i=1}^{n} G^y_{x x_i} \mathcal{M}_{x_1 \cdots \hat{x}_i y \cdots x_n}$$

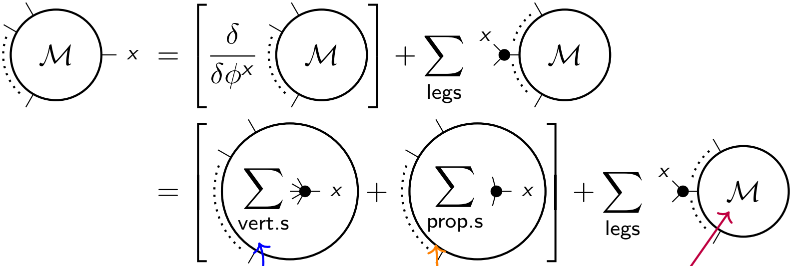
The $n+1$-index $\mathcal{M}$ is the n-index $\mathcal{M}$, with the new leg x added in all possible ways to its graphs.

The 'partial derivative' piece adds it to all the vertices and propagators.

The 'connection' piece adds it by splitting an existing leg.

Covariant derivative adds new leg in all possible ways

$$\mathcal{M}_{x_1\cdots x_n x} = \nabla_x \mathcal{M}_{x_1\cdots x_n} = \frac{\delta}{\delta\phi^x}\mathcal{M}_{x_1\cdots x_n} - \sum_{i=1}^{n} G^y_{x x_i}\mathcal{M}_{x_1\cdots \hat{x}_i y\cdots x_n}$$



$$\frac{\delta}{\delta\phi^x}\left[-i\frac{\delta^k(-\Gamma)}{\delta\phi^{y_1}\cdots\delta\phi^{y_k}}\right] = -i\frac{\delta^{k+1}(-\Gamma)}{\delta\phi^{y_1}\cdots\delta\phi^{y_k}\delta\phi^x}$$

$$\frac{\delta}{\delta\phi^x}D^{y_1 y_2} = D^{y_1 z_1}\left[-i\frac{\delta^3(-\Gamma)}{\delta\phi^{z_1}\delta\phi^x\delta\phi^{z_2}}\right]D^{z_2 y_2}$$

$$-G^y_{x x_i}\mathcal{M}_{x_1\cdots\hat{x}_i y\cdots x_n} = -i\frac{\delta^3(-\Gamma)}{\delta\phi^x\delta\phi^{x_i}\delta\phi^z}D^{zy}\mathcal{M}_{x_1\cdots\hat{x}_i y\cdots x_n}$$

Off-shell recursion, but more covariant, and more off-shell

The Taylor expansion of ϕ w.r.t. J contains amplitudes

$$\frac{\delta^{n+1} W[J]}{\delta J \cdots \delta J \delta J_z}\Big|_{J=0} \equiv \frac{\delta^n \phi^z[J]}{\delta J \cdots \delta J}\Big|_{J=0}.$$

Berends-Giele recursion makes $\phi[J]$ by iteratively solving ϕ's equation of motion about $\phi = J = 0$. (**Berends:1987me**)

We can write this expansion as

$$\phi^y = \hat{J}^y - \sum_{n=2}^{\infty} \frac{1}{n!} \left(G^y_{x_1 \cdots x_n}|_{J=0} \right) \hat{J}^{x_1} \cdots \hat{J}^{x_n} \text{ where } \begin{cases} \hat{J}^x & \equiv (iD^{xy}|_{J=0})\, J_y \\ G^y_{x_1 \cdots x_n} & \equiv iD^{yz} \mathcal{M}_{z x_1 \cdots x_n} \end{cases}$$

Components constructable by covariant differentiation: $G^y_{x_1 \cdots x_n x} = \nabla_x G^y_{x_1 \cdots x_n}$, also when $J \neq 0$ (hence 'more off-shell').

All quantities transform covariantly, up to off-shell terms

At tree-level, the (effective) action transforms as a scalar under field redefinitions $\phi[\tilde{\phi}]$: $\tilde{\Gamma}[\tilde{\phi}] = \tilde{S}[\tilde{\phi}] = S\big[\phi[\tilde{\phi}]\big] = \Gamma\big[\phi[\tilde{\phi}]\big]$

For some a, b,

Tensorial transformation

$$\widetilde{\mathcal{M}}_{x_1\cdots x_n} = \left(\frac{\delta\phi^{y_1}}{\delta\tilde{\phi}^{x_1}}\cdots\frac{\delta\phi^{y_n}}{\delta\tilde{\phi}^{x_n}}\right)\mathcal{M}_{y_1\cdots y_n} + a_{x_1\cdots x_n y_1}\frac{\delta(-\Gamma)}{\delta\phi^{y_1}}$$

$$+\sum_{i=1}^{n} b_{x_1\cdots\hat{x}_i\cdots x_n y_1}\frac{\delta\phi^{y_2}}{\delta\tilde{\phi}^{x_i}}\frac{\delta^2(-\Gamma)}{\delta\phi^{y_1}\delta\phi^{y_2}}.$$

$$\frac{\delta(-\Gamma)}{\delta\phi^{y_1}} = J_{y_1}$$

$$\left.\frac{\delta^2(-\Gamma)}{\delta\tilde{\phi}_{x_1}\delta\tilde{\phi}_{x_2}}\right|_{J=0} = \frac{\delta\phi_{y_1}}{\delta\tilde{\phi}_{x_1}}\frac{\delta\phi_{y_2}}{\delta\tilde{\phi}_{x_2}}\left.\frac{\delta^2(-\Gamma)}{\delta\phi_{y_1}\delta\phi_{y_2}}\right|_{J=0}$$

Covariance allows us to isolate the physical pieces.

(Similar transformations for ‘connection’ $G^{y}_{x_1 x_2}$ and ‘metric’ $-iD^{-1}_{xy}$.)

SMEFT (Standard Model Effective Field Theory)

see also (Alonso, Jenkins, and Manohar 2016b) for details

Comprising four equivalent real scalars

$$\vec{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}, \qquad \vec{\phi} \to O\vec{\phi}, \qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_4 + i\phi_3 \end{pmatrix}$$

where $O \in O(4) \supset SU(2) \times U(1)$. Electroweak symmetry is *linearly realised* on the $\vec{\phi}$.

Then the terms in the Lagrangian are

$$\mathcal{L}_{\text{SM}} = \frac{1}{2}(\partial\vec{\phi} \cdot \partial\vec{\phi}) - \frac{1}{4}\lambda(\vec{\phi} \cdot \vec{\phi} - v^2)^2$$

$$\begin{aligned} \mathcal{L}_{\text{SMEFT}} &= \frac{1}{2}\tilde{A}(\vec{\phi} \cdot \vec{\phi})(\partial\vec{\phi} \cdot \partial\vec{\phi}) + \frac{1}{2}\tilde{B}\left(\vec{\phi} \cdot \vec{\phi}\right)(\vec{\phi} \cdot \partial\vec{\phi})^2 - \tilde{V}\left(\vec{\phi} \cdot \vec{\phi}\right) + \mathcal{O}\left(\partial^4\right), \\ &\to \frac{1}{2}\partial\vec{\phi} \cdot \partial\vec{\phi} + \frac{1}{2}B\left(\vec{\phi} \cdot \vec{\phi}\right)(\vec{\phi} \cdot \partial\vec{\phi})^2 - V\left(\vec{\phi} \cdot \vec{\phi}\right) + \mathcal{O}\left(\partial^4\right), \end{aligned}$$

HEFT (Higgs Effective Field Theory)

see also (Alonso, Jenkins, and Manohar 2016b) for details

Built from a real h and a unit vector $\vec{n}$ comprising 3 Goldstones π^i

$$h\,, \qquad \vec{n} = \begin{pmatrix} n_1 = \pi_1/v \\ n_2 = \pi_2/v \\ n_3 = \pi_3/v \\ n_4 = \sqrt{1 - n_1^2 - n_2^2 - n_3^2} \end{pmatrix},$$

upon which the electroweak symmetry is *non-linearly realised*

$$h \to h \quad , \quad \vec{n} \to O\vec{n}\,, O \in O(4)\,.$$

The lagrangian is

$$\mathcal{L}_{\rm SM} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}(v+h)^2(\partial\vec{n})^2 - \frac{1}{4}\lambda(h^2+2vh)^2$$

$$\mathcal{L}_{\rm HEFT} = \frac{1}{2}\left[\tilde{K}(h)\right]^2(\partial h)^2 + \frac{1}{2}\left[v\tilde{F}(h)\right]^2(\partial\vec{n})^2 - \tilde{V}(h) + \mathcal{O}\left(\partial^4\right)$$

$$\to \frac{1}{2}(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h) + \mathcal{O}\left(\partial^4\right)\,.$$

[Canonically $F(0) = 1,\ V'(0) = 0$]

A review of unitarity violation in $W^+W^- \to W^+W^-$ (3)

(Alonso, Jenkins, and Manohar 2016a)

Put the amplitude in a correctly normalized s-wave state[2]

$$|\hat{M}| = \frac{\left|\int \mathrm{d}\Pi_i \mathrm{d}\Pi_f \mathcal{A}\right|}{\left(\int \mathrm{d}\Pi_i\right)^{\frac{1}{2}} \left(\int \mathrm{d}\Pi_f\right)^{\frac{1}{2}}} \overset{E \gg m_W}{=} \frac{1}{8\pi} \frac{E^2}{2} \left| \overline{F'}^2 - \frac{1}{v^2} \right| + \mathrm{O}(E^0)$$

Unitarity circle arguments say $|\hat{M}| \lesssim 1$, so there is a unitarity bound on the CoM energy

$$E \lesssim \sqrt{16\pi} \left| \overline{F'}^2 - \frac{1}{v^2} \right|^{-\frac{1}{2}} = \sqrt{16\pi} |\overline{\mathcal{K}_\pi}|^{-\frac{1}{2}}$$

$E \to \infty$ if the hWW coupling is SM like: $\overline{F'} = \frac{1}{v}$. I.e., target space locally flat: $\overline{\mathcal{K}_\pi} = 0$.

Four legs good, more legs better: Better unitarity bounds from higher point amplitudes when UV weakly coupled.

[2] E is the center of mass energy.

$\mathcal{A}\left(\pi_i \pi_j h^{n-2}\right)$

Following (Falkowski and Rattazzi 2019)

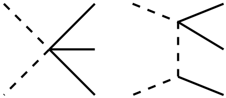
With the application of geometric/kinematic identities:

$$\begin{aligned}\mathcal{A}\left(\pi_i \pi_j h^{n-2}\right) =& \overline{V}_{;(\pi_i \pi_j h \dots h)} + \overline{R}_{\pi_i h h \pi_j ; h \dots h}\left(s_{12} - \frac{2m_h^2}{n-1}\right) \\ & + \mathrm{O}\left(\overline{R}^2\right) + \text{factorizable pieces} \\ =& \frac{1}{3}\,\delta_{ij}\,\overline{\partial_h^{n-2}\left(\nabla^2 V - \partial_h^2 V\right)} - \delta_{ij}\,\overline{\partial_h^{n-4}\mathcal{K}_h}\left(s_{12} - \frac{2m_h^2}{n-1}\right) \\ & + \mathrm{O}\left(\overline{R}^2\right) + \text{factorizable pieces}\,.\end{aligned}$$

The parts of the $n > 4$ amplitudes that grow with energy are *derivatives* of sectional curvatures.

$$\mathcal{A}\left(\pi_i \pi_j \to h^{n-2}\right) = -E^2\,\delta_{ij}\,\overline{\partial_h^{n-4}\mathcal{K}_h} + \mathrm{O}(E^0)$$

Unpack this result



Take the $\mathrm{O}(E^2)$ part of $\mathcal{A}\left(\pi_1\pi_1 \to h^3\right)$

$$\mathcal{A}\left(\pi_1\pi_1 \to h^3\right) = -E^2\overline{\partial_h \mathcal{K}_h} = E^2\overline{\partial_h\left(\frac{F''}{F}\right)} = E^2\left(\overline{F}''' - \overline{F}''\overline{F}'\right)$$

Sub in particular UV examples

$$\textbf{SM}:\ vF = (v+h) \implies \mathcal{A} = 0$$

$$\textbf{unSMEFTy}:\ vF = (v+h) + \frac{\epsilon}{v^2}h^3 \implies \mathcal{A} = \frac{6\epsilon}{v^3}E^2$$

$$\textbf{SMEFTy}:\ vF = (v+h) + \frac{\epsilon}{v^2}(v+h)^3 \implies \mathcal{A} = 0 + \mathrm{O}(\epsilon^2)$$

Parametrically faster growth in cases where the deviations are non-SMEFT like.

SM kinetic term $\implies$ 4 point amplitude unitary

SMEFT term $\implies$ higher point amplitude unitary(ish)

Correlations present in many other amplitudes

(Abu-Ajamieh, Chang, Chen, and Luty 2020)

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \delta_3 \frac{m_h^2}{2v} h^3 - \delta_4 \frac{m_h^2}{8v^2} h^4 - \sum_{n=5}^{\infty} \frac{c_n}{n!} \frac{m_h^2}{v^{n-2}} h^n + \cdots$$

$$+ \delta_{Z1} \frac{m_Z^2}{v} h Z^\mu Z_\mu + \delta_{W1} \frac{2m_W^2}{v} h W^{\mu +} W^-_\mu + \delta_{Z2} \frac{m_Z^2}{2v^2} h^2 Z^\mu Z_\mu + \delta_{W2} \frac{m_W^2}{v} h^2 W^{\mu +} W^-_\mu$$

$$+ \sum_{n=3}^{\infty} \left[\frac{c_{Zn}}{n!} \frac{m_Z^2}{v^n} h^n Z^\mu Z_\mu + \frac{c_{Wn}}{n!} \frac{2m_W^2}{v^n} h^n W^{\mu +} W^-_\mu \right] + \cdots$$

$$- \delta_{t1} \frac{m_t}{v} h \bar{t} t - \sum_{n=2}^{\infty} \frac{c_{tn}}{n!} \frac{m_t}{v^n} h^n \bar{t} t + \cdots$$

Process	$\times \frac{E^4}{1152\pi^3 v^4}$
$hZ^2 \to hZ^2$	$[4\delta_{V1} - 2\delta_{V2} + \frac{1}{2} c_{V3}]$
$h^2 Z \to Z^3$	$-\frac{\sqrt{3}}{2}[4\delta_{V1} - 2\delta_{V2} + \frac{1}{2} c_{V3}]$
$h^2 W^+ \to Z^2 W^+$	$-\frac{1}{2}[4\delta_{V1} - 2\delta_{V2} + \frac{1}{2} c_{V3}]$
$h^2 Z \to Z W^+ W^-$	$-\frac{1}{\sqrt{2}}[4\delta_{V1} - 2\delta_{V2} + \frac{1}{2} c_{V3}]$
$h^2 W^+ \to W^+ W^- W^+$	$-[4\delta_{V1} - 2\delta_{V2} + \frac{1}{2} c_{V3}]$
$hZW^+ \to hZW^+$	$[36\delta_{V1} - 13\delta_{V2} + 2c_{Vc}]$
$hW^+W^+ \to hW^+W^+$	$[36\delta_{V1} - 13\delta_{V2} + 2c_{V3}]$
$hW^+W^- \to hW^+W^-$	$-[28\delta_{V1} - 9\delta_{V2} + c_{V3}]$
$hZ^2 \to hW^+W^-$	$-\sqrt{2}[32\delta_{V1} - 11\delta_{V2} + \frac{3}{2} c_{V3}]$

Process	$\times \frac{\left(\frac{1}{2}c_{t2} - \delta_{t1}\right) m_t E^2}{32\pi^2 v^3}$
$\bar{t}_R t_R \to Zh^2$	$i\sqrt{N_c}$
$h^2 \to Z\bar{t}_L t_L$	$i\sqrt{\frac{N_c}{3}}$
$Zh \to h\bar{t}_L t_L$	$i\sqrt{\frac{2N_c}{3}}$
$t_R Z \to t_L h^2$	$\frac{i}{\sqrt{6}}$
$t_R h \to t_L Zh$	$\frac{i}{\sqrt{3}}$
$\bar{t}_R t_R \to Z^2 h$	$-\sqrt{N_c}$
$Z^2 \to \bar{t}_L t_L h$	$-\sqrt{\frac{N_c}{3}}$
$Zh \to \bar{t}_L t_L Z$	$-\sqrt{\frac{2N_c}{3}}$
$t_R h \to t_L Z^2$	$-\frac{1}{\sqrt{6}}$
$t_R Z \to t_L Zh$	$-\frac{1}{\sqrt{3}}$

Unitarity bound for $\mathcal{A}(\pi_i\pi_j \to h^n)$

For $2 \to n$, the s-wave state $|\hat{M}|^2 \sim \frac{1}{8\pi}\left(\frac{1}{(n-2)!}\right)^2 \left(\frac{E}{4\pi}\right)^{2(n-2)} |\mathcal{A}|^2$, see e.g. (Abu-Ajamieh, Chang, Chen, and Luty 2020)[3]

Unitarity bound

$$E < 4\pi \times \left|\frac{\overline{\partial_h^{n-2}\mathcal{K}_h}}{n!}\right|^{-\frac{1}{n}} \times b_n \times (n!)^{\frac{1}{n}}$$

$$= \begin{cases} 8^{\frac{1}{4}}\sqrt{16\pi} \times \left|\overline{\mathcal{K}_h}\right|^{-\frac{1}{2}} & n = 2 \\ 4\pi v_* \times (n!)^{\frac{1}{n}} & n = \text{`a few'} \end{cases}$$

v_* is the scale of '∂_h' $\approx$ the radius of convergence of $\mathcal{K}_h$.

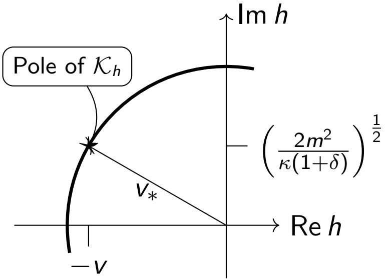
$4\pi v_*$ is a general bound. If the EFT is poorly described by SMEFT, $v_* \sim v$.

---

[3] b_n is an $\mathrm{O}(1)$ fudge factor

$$\left(\frac{1}{b_n}\right)^{2n} = \frac{(4\pi)^2}{8(n-1)}\left(1 - \frac{2m_h^2}{(n+1)E^2}\right)^2 \times \frac{\text{Vol. } n \text{ body Higgs PS}}{\text{Vol. } n \text{ body massless PS}}.$$

Example: loop-level scalar singlet scales ($\delta = \frac{\kappa}{96\pi^2}$)



Sectional curvatures

$$\mathcal{K}_h = \delta \frac{\kappa}{2} \frac{m^2}{\left(m^2 + \frac{1}{2}\kappa\left(1+\delta\right)\left(v+h\right)^2\right)^2},$$

$$\mathcal{K}_\pi = \delta \frac{\kappa}{2} \frac{1}{\left(m^2 + \frac{1}{2}\kappa\left(1+\delta\right)\left(v+h\right)^2\right)}.$$

Radius of convergence, $v_* \approx \sqrt{v^2 + \frac{2m^2}{\kappa(1+\delta)}}$.

Mass of scalar at vacuum, $m_S^2 = m^2 + \frac{1}{2}\kappa v^2 \approx \frac{1}{2}\kappa v_*^2$.

If $\kappa \sim (4\pi)^2$ and $m^2 \sim (4\pi v)^2$, $\overline{\mathcal{K}_h}^{-1/2} \sim v_*$.

If S gets majority of its mass from EWSB, $v_* \sim v$.

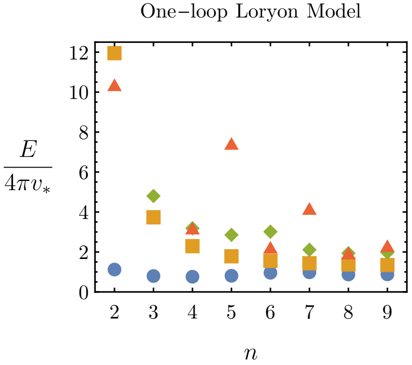
Example: loop-level scalar singlet unitarity cutoff

Unitarity bound $\pi^2 \to h^n$

$$E < 4\pi \times \left| \frac{\overline{\partial_h^{n-2}\mathcal{K}_h}}{n!} \right|^{-\frac{1}{n}} \times b_n \times (n!)^{\frac{1}{n}}$$

Strongly coupled: $v_*^2\overline{\mathcal{K}_h} \sim 1$

not SMEFT: $v_* \sim v$



	m^2/v^2	$\kappa/2$	$v_\star/v$	$v_\star^2\,\overline{\mathcal{K}_h}$	$v_\star^2\,\overline{\mathcal{K}_\pi}$
A	$(4\pi)^2$	$(4\pi)^2$	2	0.2	0.4
B	1	$(4\pi)^2$	1	1×10^{-3}	0.3
C	1	1	1	1×10^{-3}	2×10^{-3}
D	10^6	1	1000	2×10^{-3}	2×10^{-3}

● A ■ B ◆ C ▲ D

Amplitudes summary

Unitarity bound $\pi^2 \to h^n$

$$E < 4\pi \times \left| \frac{\overline{\partial_h^{n-2}\mathcal{K}_h}}{n!} \right|^{-\frac{1}{n}} \times b_n \times (n!)^{\frac{1}{n}}$$

$$\approx \begin{cases} 8^{\frac{1}{4}}\sqrt{16\pi} \times \left|\overline{\mathcal{K}_h}\right|^{-\frac{1}{2}} & n = 2 \\ 4\pi v_* \times (n!)^{\frac{1}{n}} & n = \text{`a few'} \end{cases}$$

Scalar amplitudes are geometric! We identified parts of HEFT amplitudes $\propto$ derivatives of sectional curvatures at our vacuum.

This allows us to probe locally: how curved our manifold is, and how rapidly this is changing. We derived unitarity bounds sensitive to these two scales.

Manifolds poorly described by SMEFT can't be flat over a large region, leading inexorably to TeV scale unitarity cutoffs.

$\alpha, \beta, \dots$ don't have to be flavour indices

...they can refer to momentum, spin, etc. (Cheung, Helset, and Parra-Martinez 2022)

Take the Dirac-Born-Infeld lagrangian[4]

$$\mathcal{L} = -\frac{1}{2}(\partial^\mu \phi)(\partial_\mu \phi)\left[1 - \frac{1}{4}(\partial^\nu \phi)(\partial_\nu \phi) + \mathrm{O}(\phi^4)\right]$$

The 3- and 4- point vertices are

$$g(p_1 p_2, p_3) = 0$$
$$g(p_1 p_2, p_3 p_4) = \frac{1}{2}(p_3 \cdot p_4)$$

whence

$$R(p_1 p_2 p_3 p_4) = \frac{1}{2}\left[g(p_1 p_4, p_2 p_3) + g(p_2 p_3, p_1 p_4) - g(p_1 p_3, p_2 p_4) - g(p_2 p_4, p_1 p_3)\right]$$
$$= -\frac{1}{4}(t - u)$$

Substitute into the NLSM amplitude on the previous slide to get

$$\mathcal{A}(p_1 p_2 p_3 p_4) = \frac{1}{4}(s^2 + t^2 + u^2)$$

[4] Using mostly plus metric convention.

Off-shell recursion, but more covariant, and more off-shell

The Taylor expansion of ϕ w.r.t. J contains amplitudes

$$\frac{\delta^{n+1} W[J]}{\delta J_{y_1} \cdots \delta J_{y_n} \delta J_z}\bigg|_{J=0} \equiv \frac{\delta^n \phi^z[J]}{\delta J_{y_1} \cdots \delta J_{y_n}}\bigg|_{J=0}.$$

In Berends-Giele recursion, get $\phi[J]$ by iteratively solving the equation of motion about $\phi = J = 0$

$$\Gamma[\phi]_{,x} + J_x = \sum_{n=0}^{\infty} \frac{1}{n!} \Gamma[0]_{,xx_1 \ldots x_n} \phi^{x_1} \cdots \phi^{x_n} + J_x = 0$$

We can write the result as

$$\phi^y = \hat{J}^y - \sum_{n=2}^{\infty} \frac{1}{n!} \left(G^y_{x_1 \cdots x_n}|_{J=0} \right) \hat{J}^{x_1} \cdots \hat{J}^{x_n} \text{ where } \begin{cases} \hat{J}^x & \equiv \left(iD^{xy}|_{J=0} \right) J_y \\ G^y_{x_1 \cdots x_n} & \equiv iD^{yz} \mathcal{M}_{zx_1 \cdots x_n} \end{cases}$$

which is valid when $J \neq 0$. This is like a normal coordinate map $\phi \to \hat{J}$.