What if photon has a mass

Zero mass vs Ultralight darkmatter?

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PLAN

1. *Schrodinger and mass of photon*
   - Proca& Stueckelberg theory
2. *Dark matter question*
   - Candidates for dark matter
3. *Bose Einstein Condensate*
   - Dark matter and BEC
   - Dark matter halos
   - Half radius and Density of Halo
4. *Superfluid Aether*
5. *CMB and photon mass*
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7. *Stuckeberg extn of Standard model*
8. *Summary & Future work?*

Schrodinger and DIAS

- I gave a talk on 'Must photon be massless?' at Dublin Inst of Advanced Studies in January 2018. To my surprise, I found Erwin Schrodinger talked about same question..in 1955.
- *Must the Photon Mass be Zero?*. **L. Bass and E. Schrodinger, Proc. Roy. Soc. London. A, Mathematical and Physical Sciences,232,1188 (1955)**.
- He was posing the question since massive photon will have an extra degree of freedom. While calculating in blackbody radiation energy density as a function of frequencies we multiply by a factor of 2 to account for the transverse degrees of freedom. Density of modes $\rho(\nu) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}} - 1}$: Stefan's constant $E = \sigma T^4$


ERWIN
SCHRÖDINGER
1887 1961
$i\hbar\dot{\psi} = H\psi$

TWO OR THREE?

Bose vs Einstein

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Ans: No. If vector potential A_μ is coupled to conserved j_μ

$$\partial_\mu j^\mu = 0$$

only a small change in the s-matrix contributions if m_γ is small.

Massive photon and conserved current

- Moral is we will still get the factor of 2 instead of 3
- But we can estimate the bound on the mass of the 'photon'
- Schrodinger himself estimated mass of the photon. The massive photon equation would be

$$(\nabla^2 - m_\gamma^2)\vec{A} = -\mu_0 \vec{J} \quad (1)$$

- The solution is

$$\vec{A} \approx \frac{\mu_0}{4\pi}\nabla \times \left(\vec{m}\frac{e^{-m_\gamma r}}{r}\right) \quad (2)$$

where $\vec{m}$ is magnetic dipole moment.

- We compare $B_{ext}(m_\gamma)$ to $B(0)$ at the equator (B_{ext} is the 'extra' magnetic field):

$$\frac{B_{ext}(m_\gamma)}{B(0)} = \frac{\frac{2}{3}(m_\gamma R)^2}{1 + m_\gamma R + \frac{1}{3}(m_\gamma R)^2} \quad (3)$$

- Schrodinger used only earth based measurement data. But now satellite data is also available..

Estimates of mass of the photon

- Considering the earth as a magnet we can estimate the photon mass using the above formula and comparing with data.
- The magnetic field has extra 'non-potential' contribution. which depends on the mass of the photon m_γ^2.
- It has a negative contribution and reduces the magnetic field.
- This can be compared with experimental estimates to obtain the mass limits..
- This gave Bass and Schrodinger $m_\gamma \ \leq \ 10^{-47} g$. By careful analysis of the geomagnetic field this was improved by Goldabher and Nieto to $\leq \ 10^{-48} g$.
- Schrodinger's estimates based on geo-magnetic surveys are still very good. Based on dissipation of large scale magnetic fields in the galaxy there are now estimates $\leq \ 10^{-56} g$ which are in the list with question marks.

Estimates of mass of the photon-contd

- Compton wavelength $\lambda = h/m\,c$ of these correspond to radius of solar sytem and galaxy or $\leq\ 10^{-16}eV$ and $10^{-24}eV$. Size of the universe provides a cutoff which can be maximum we can provide!!
- There are laboratory experiments too. Wavelength independence of velocity of light is one of the direct consequences of photon mass being zero.
- Using radio wave interferometer over a large frequency range veleocity difference has been measured. it was found to be $\dfrac{\Delta c}{c} \leq 10^{-5}$. This corresponds to $m_\gamma\ \leq\ 10^{-42}g$. Astronomical estimates are better.
- There are several other estimates with lot of uncertainties and the best so far can take is either Erwin Schrodinger estimate of $m_\gamma \leq 10^{-18}eV$ or galactic magnetic field estimate $m_\gamma\ \leq 10^{-24}eV$.

Proca and Stueckelberg theory

- If we introduce mass term of the photon to the conventional Maxwell action, it breaks local gauge invariance. But global invariance is still there, and current is conserved and charge is still superselected.
- But the massive Proca theory describing spin 1 has 3 degrees of freedom unlike Maxwell theory which has only 2. Hence there is discontinuity in the degrees of freedom when we take $m_\gamma \to 0$.
- But the Massive QED is renormalisable (ultraviolet). If we make the mass to be tiny but nonzero we will find the contribution of the longitudinal photon to several processes are extremely small as used by the text book of Banks! There is no infrared divergence either because of 'mass'!
- Also what happens in the $m \longrightarrow 0$ limit, for the infrared divergence and the question of lack of local gauge invariance which has been our guiding principle. Stueckelberg theory avoids discontinuity and gauge invariance question nicely.

Massive gauge theory

- We can preserve local gauge invariance and still give mass to the photon in three ways. (1) Stueckelberg theory (2) Higgs mechanism (3)topological massive $B \wedge F$ theory.
- The Lagrangian for Stueckelberg theory is:

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}m^2\left(A_\mu - \frac{1}{m}\partial_\mu\phi\right)^2 + \bar{\psi}[\gamma^\mu(i\partial_\mu + eA_\mu) - M]\psi \tag{4}$$

- The gauge fixing: $-\frac{1}{2}(\partial_\mu A^\mu + m\,\phi)^2$. The gauge transformations are:

$$\psi \to e^{i\lambda(x)}\psi, \;\; A_\mu \to A_\mu - \partial_\mu\lambda(x), \;\; \phi \to \phi + m\lambda(x) \tag{5}$$

where ϕ is Stueckelberg scalar field.

Higgs mechanism

- Here introduce a complex scalar field Φ. This will have nonzero vacuum expectation value giving mass to the the photon.
- We can write in the symmetry broken phase $\Phi = R\, e^{i\phi}$.
- Phase of this field will be like Stueckelberg field and this mechanism in a specific limit of freezing the fluctuations of R goes to Stueckelberg theory.
- For topological massive theory we use two form $B = B_{\mu\nu} dx^\mu \wedge dx^\nu$ and $H = dB$.
- We can take as Lagrangian $\mathcal{L} = \frac{1}{2} H \wedge^* H + \frac{1}{2} m^2\, B^2$
- Massive $B_{\mu\nu}$ will describe a spin -1 particle after elimination of constraints. But in the massless limit it describes a spin -0 particle!! Again a discontinuity of the degrees of freedom.
- In all these mechanisms extra degrees of freedom are introduced, but local gauge invariance gives the correct massive spin-1 theory.

Topologically massive gauge theory

- The Lagrangian is:

$$\mathcal{L} = -\frac{1}{2}F\wedge^{*}F + \frac{1}{2}H\wedge^{*}H + m\,B\wedge F + \bar{\psi}[\gamma^{\mu}(i\partial_{\mu} - eA_{\mu}) + M]\psi \quad (6)$$

- Again the combined gauge transformations leave the Lagrangian upto total divergence invariant.
- In $2+1$ D we also have Maxwell Chern Simon theory given by the Lagrangian:

$$\mathcal{L} = -\frac{1}{2}F\wedge^{*}F + m\,A\wedge F + \bar{\psi}[i\gamma D - M]\psi \quad (7)$$

- Stueckleberg theory in $2+1$ which is equivalent to Proca theory gives 2 degrees of freedom unlike 1 degree of freedom through Maxwell-Chern Simon theory.
- Both Maxwell and Maxwell CS theory describes a scalar field. but with different helicity. But we will focus on the 3+1 Stueckelberg QED.

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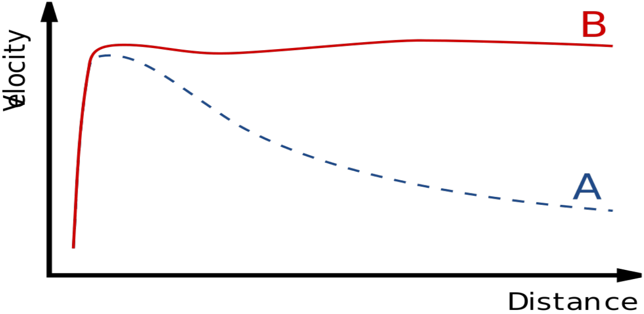
WHAT THE HELL IS DARK MATTER?.

AMUSING TO CALL LIGHT AS DARK!

Matter, Dark matter, Dark energy

- Our universe consists of matter, dark matter and dark energy. (5%, 20% and 75%).
- From observations of rotation curve in galaxies, gravitational lensing and acceleration of the universe.
- Spiral galaxies rotate around the center. The visible mass density decreases from the center to the edge. If they are the only mass, Kepler's Second Law will give the rotation velocities will decrease with distance from the center. But this remains flat.



Dark matter

- Galaxy clusters mass distributions can be measured in couple of independent ways.
- First by Radial velocities within the clusters. X-rays from the cluster spectrum and flux give detailed estimate of temperature of pressure leading to the mass profile of the clusters. Gravitational lensing of distant galaxies measures cluster without the need for velocities.
- All these point to existence of dark matter overwhelmingly at 5:1.
- Though Dark matter and usual matter are matter they do not evolve in the same way. Dark matter does not interact with matter and radiation and affects CMB by gravitational effects and their differential evolution.
- Dark matter is crucial for structure formation itself..
- Several candidates have been proposed for the dark matter.

Dark matter

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- Fuzzy dark matter is a model which suggests that dark matter is composed of ultralight particles having Compton wavelength on cosmological scales, with mass comparable in order of magnitude to $10^{-20}eV$.
- We saw the Stueckelberg field ϕ decouples at from physical processes and only exhibits gravitational interactions and has mass in this range.

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AND FOLLOW UP BY EINSTEIN

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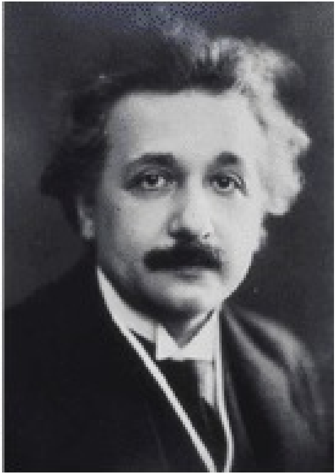
BOSE EINSTEIN CONDENSATE-BEC



Stueckelberg field and dark matter

- In the FDM picture, we may treat the constituent particles as these Stueckelberg particles which do not interact.
- Furthermore, this particle will be such a candidate only if they form a Bose-Einstein condensate.
- The formation of a Bose-Einstein condensate needs (i) a conservation law for particle number (ii) the system should be at a temperature below the critical temperature T_c.
- For massless scalar fields, the conserved quantity is by shift invariance of the field, which is broken by a mass term and self interactions. It is an approximate symmetry.
- The $T_c$is given by (where ρ is the number density of a gas of particles),

$$T_c = \frac{\hbar c}{k_B} \left(\frac{\rho \pi^2}{m_\gamma \zeta(3)} \right)^{1/3} . \tag{8}$$

What is BEC?

- Ideal Bose gas is quantum state of matter similar to ideal gas in classical statistical mechanics. They obey Bose statistics and have integer spin. This was given by Bose for photon gas and extended to massive particles by Einstein.
- While at large temperature they behave similarly, but at very low temperatures form a condensate under certain conditions due to Bose Statistics. (After seeing Bose's letter, Einstein realised immediately such a state can exist!)
- BEC was obtained for Rubidium 87 gas whose mass is 86 amu (O(86 Gev)).
- Critical temperature is $\approx$ nano kelvins. Very low value because of high mass of the atoms..making up the gas.
- Stueckelberg particles have mass $\approx 10^{-20} eV$. This makes the critical temperature very high (close to temperature of Big bang)

Dark matter and BEC

- We need to give density and the change in density effected as a consequence of the Friedman expansion. If dark matter is given by an initial density of ρ_0 at the time of decoupling, before the radiation dominated era begins. The epochs are given in table:

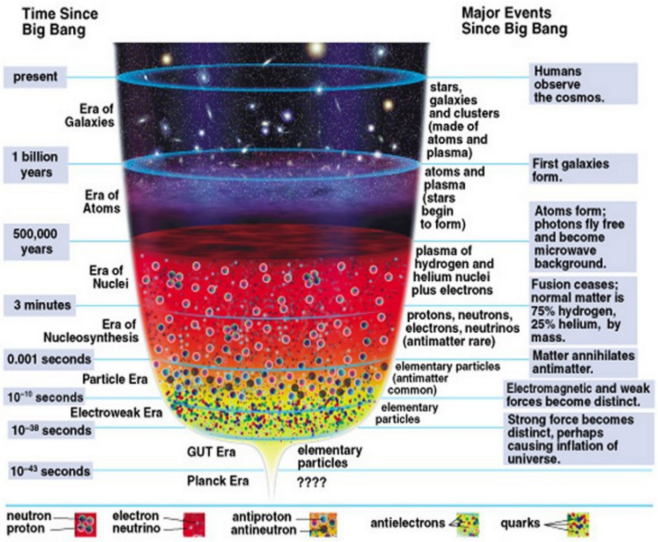
Table: Epochs

Epoch and Time	Scale Factor	Temp.
Radiation Era $1s$ to $1.2 \times 10^{12}s$	$\propto t^{1/2}$	$10^{12}K$-$10^{4}K$
Matter Era $4.7 \times 10^{4}y$ to $9.8 \times 10^{9}y$	$\propto t^{2/3}$	$10^{4}K$ - $4K$
Dark Energy $9.8 - 13.8$ billion y	$\propto e^{Ht}$	$< 4K$

- Using these, the current density ρ_{final} is:

$$\rho_{final}^{\frac{1}{3}} = \rho_0^{\frac{1}{3}} \frac{1}{(1.2 \times 10^{12})^{\frac{1}{2}}} \left(\frac{47000}{9.8 \times 10^{9}} \right)^{\frac{2}{3}} \frac{1}{1.377}. \tag{9}$$

Big Bang and afterwords



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Dark matter and BEC

- Employing this we get the relation between the observed dark matter density ρ and the critical temperature required to achieve Bose-Einstein condensation,

$$\rho \sim 10^{-22} m_\gamma \, T_c^3 \tag{10}$$

- In SI units, the observed dark matter density (PDG) $\sim 10^{-22} kg/m^3$ or 1 proton/cc is recovered if we take $m_\gamma \sim 10^{-19} eV$ and $T_c \sim 10^{17} K$. The corresponding estimate for $m_\gamma = 10^{-22}$ eV would be $T_c \sim 10^{19} K$.
- Further the condensate once it is formed at the earlier times will remain so for all epochs.

Dark matter and BEC

- We have assumed that the current dark matter density can be entirely explained in terms of Stueckelberg particles. We can compute half radii of condensates and their masses.
- We can consider the dark matter as fluid with shift invariance broken by small mass term.
- A BEC condensate is there if the temperature is less than T_c. This description is by the ansatz $\phi \sim e^{-imc^2 t}\psi$ in a perturbed FRLW universe.
- $$i\left(\partial_t + \frac{3}{2}\frac{\dot{a}}{a}\right)\psi = \left(\frac{-\nabla^2}{2m} + mV\right)\psi \qquad (11)$$

  where V is a gravitational potential, in the linear approximation.
- Following Witten et al., the half radius and mass can be worked out.

Dark matter Halo

- DM halo is a theoretical model of galaxy that bounds the galactic disc and extends beyond luminous (visible) part.
- For our galaxy Milkyway luminous matter is $\approx\ 10^{10}$ solar masses. But dark matter is estimated to be $\approx\ 10^{12}$ solar masses.
- Halo's mass dominates the galactic mass (nearly 95% !) and only postulated through observation of rotation curves.
- DM halos are crucial for galaxy formation and evolution.
- During galaxy formation temperature of matter is too high to form gravitationally bound objects. Prior formation DM is needed to add additional structure.
- After the galaxy formation it extends far beyond observable part and required for understanding velocities and lensing.
- Several density profiles are in the literature and crucial parameters for modelling are half radius and central density.

Half radius and density of the halo

- Following Hui, Ostriker,Tremaine, Witten (astro-ph/1610.08297) the formulae (Eqs.29,30) for the half radius and mass are given by:

$$r_{\frac{1}{2}} = 3.925 \frac{\hbar^2}{GMm_\gamma^2}, \tag{12}$$

$$\rho_c = 4.4 \times 10^{-3} \left(\frac{Gm_\gamma^2}{\hbar^2} \right)^3 M^4, \tag{13}$$

  where ρ_c is the central density of the halo and M is the mass of the soliton.

- We parametrize mass as $10^{-17-x} eV$ and $T_c = 10^{\frac{50}{3}-y} K$ and obtain the sample space for these parameters.
- In principle there could be self interaction parameter too which we have ignored..For future work...

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Dark matter and BEC

- It is desirable we refine our hypothesis and study the theory space of the model. There are three parameters to be taken into account.
- Mass of the photon, T_c and decoupling temperature and the self interactions. For the moment we ignore the last one. We can compare with the central density and half radius parameters of the dwarf galaxies.
- The same arguments are used for ultralight axions or any other fuzzy dark matter candidate. But one should find such candidates. But for 'massive photon' it is already there.. and can serve as candidate?
- Changing gear: Einstein's special theory of relativity and Michelson Morley experiment established the irrlevance of aether medium for propagation of light.

reverse gear: Aether

- Einstein again made revolutionary suggestion about gravity and geometry in 1915: general theory of relativity.
- In 1922: Sidelights on Relativity:To deny the ether is ultimately to assume that empty space has no physical qualities whatever. The fundamental facts of mechanics do not harmonize with this view.
- Recapitulating, we may say that according to the general theory of relativity space is endowed with physical qualities; in this sense, therefore, there exists an ether. According to the general theory of relativity space without ether is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time.
- But this ether may not be thought of as endowed with the quality characteristic of ponderable media, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it.

reverse gear 2 and 3

- In 1951, Dirac in 'Nature' asked 'Is there an Aether?' He answered: We can now see that we may very well have an aether subject to quantum mechanics and conforming to relativity, provided we are willing to consider vacuum as idealized state not attainable in practice. From experimental point of view there does not seem to be any objection to this. We must make profound alterations to the idea of vacuum.
- John Bell suggests the aether was wrongly rejected on purely philosophical grounds: "what is unobservable does not exist"
- R B Laughlin: The word 'ether' has extremely negative connotations in theoretical physics because of its association with opposition to relativity. Unfortunate because, stripped of these connotations, it nicely captures the way most physicists think about the vacuum. The modern concept of the vacuum of space, confirmed every day by experiment, is a relativistic ether. But we do not call it this because it is taboo.

DIRAC

It is no longer a trivial state but needs elaborate mathematics for its description

- The viscous drag effect of aether would still pose objections. But will disappear if it is taken superfluid.
- Going further: In 1976, KPS, CS, ECG Sudarshan in Foundations of Physics proposed: 'A new model for aether as superfluid of fermion, anti fermion pairs described by a macroscopic wavefunction, pervading entire universe and may account for missing matter..'

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In 'light'er vein..

Qn: How many Theoretical Physicists specializing in general relativity does it take to change a 'light' bulb?

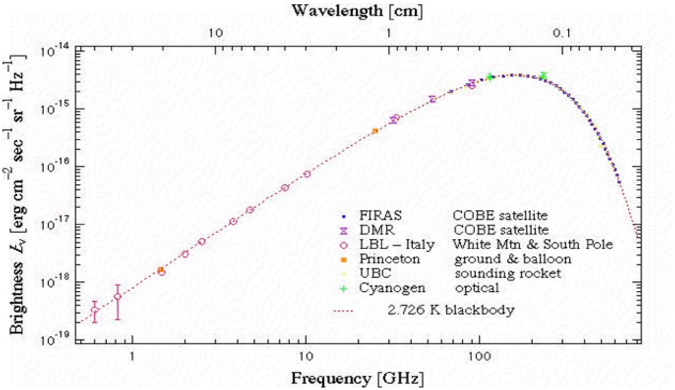
In 'light'er vein..

Qn: How many Theoretical Physicists specializing in general relativity does it take to change a 'light' bulb?

Ans: Two. One to hold the bulb... and another to rotate the Universe!

CMB and Photon mass limit

- One can go back to the issue of effect of on photon mass on black body radiation. Cosmic microwave background being one of most perfect 'blackbody' we can check its effect on the CMB distribution.



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- One can put limits on the mass of the photon from the deviation from black body spectrum for CMB.

CMB and Photon mass limit

- Density $\propto \frac{E^3}{e^{\frac{E}{kT}} - 1}\sqrt{1 - \frac{m^2}{E^2}}$, due to the modified dispersion relation $p^2 = E^2 - m^2$
- Such an anlaysis was done by Julian Heeck (Phy Rev. Letts 111, 2013) and gives poor estimate! $m_\gamma < 10^{-6}eV$. Geo and Solar Magnetic field analysis give much tighter bound as of now.
- He also estimated the lifetime of photon assuming it can decay to couple of neutrinos with likely much less mass. (There is no justification for this).
- It turns out lifetime in the rest frame of photon (assuming $m_\gamma = 10^{-18}eV$) is 3 years.
- Since the photon travels with almost the universal speed time dilation will enhance this extensively. This translates in nearly 'light speed' photon to be 10^{18} years, about 10^4 times that of the age of Universe.

Stueckelberg field in Weinberg Salam

- Stuckelberg idea can be incorporated in Weinberg Salam mdoel too.
-

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_f + \mathcal{L}_S, \tag{14}$$

$$\mathcal{L}_g = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}Tr(f_{\mu\nu}f^{\mu\nu}) + \frac{1}{2}m_{\gamma^2}\left(B_\mu - \frac{1}{m_\gamma}\partial_\mu\phi\right)^2, \tag{15}$$

$$\mathcal{L}_S = |D_\mu\Phi|^2 - \lambda\left(|\Phi|^2 - \frac{f^2}{2}\right)^2 \tag{16}$$

- where B_μ is used to denote the weak hypercharge field and D_μ is the covariant derivative acting on the Higgs. $B_{\mu\nu}$ is the hypercharge field strength and $f_{\mu\nu}$ is the $SU(2)$ field strength.

Holography and Stueckelberg theory

- Net effect is mass of the photon is m_γ and small corrections to M_Z and Weinberg angle. Details are given the review by Henri Ruegg, Marti Ruiz-Altaba, "The Stueckelberg Field", Int.J.Mod.Phys.**A19** (2004) 3265.
- Dvali etal: propose holography can be formulated in terms of information capacity of Stueckelberg degrees of freedom.
- These degrees of freedom act as qubits to encode quantum information.
- The capacity is controlled by the inverse Stueckelberg energy gap to the size of the system.
- They relate the scaling of the gap of the boundary Stueckelberg edge modes Bogoliubov modes..
- ideas are not clear but needs further work...

Summary and Conclusions

- Revival of Infrared question through asymptotic symmetries is interesting. When we regulate theory through mass maintaining local gauge invariance gives the Stueckelberg scalar a new role. It regulates the divergence, and breaks the asymptotic symmetry. Can the charges due to the new symmetries be observed? Since they are tied up with masslessness of the photon in a limiting process probably they can be observed only to the extent we can measure the mass of the photon.
- What about QCD? Unfortunately there is no Stueckelberg theory for non abelian gauge theory...Speculations about $\frac{1}{N}$?.
- What about gravity? Probably massive gravity theories?
- Last speculation? Can it help in dark matter? Stueckelberg field is not coupled to matter but can only gravitate...
- Remarks of Bernhard Riemann:..infinitesimal and infinite.. Axioms underlying the basis of geometry...!!

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