Sifting through the SM for the hints of an ALP

by

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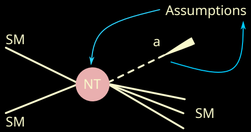
Indirect Detection

- ALPs in the final state



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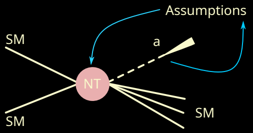

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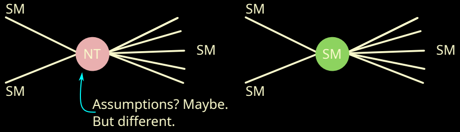

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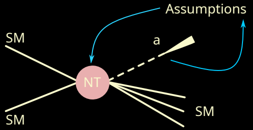

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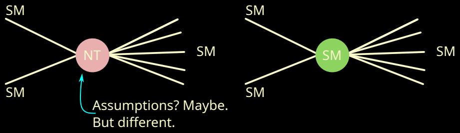


- $O_{\text{expt}} = O_{\text{SM}} + O_{NP}$
  - Better measurements, Better computations
  - Worst case: Better SM

Overview

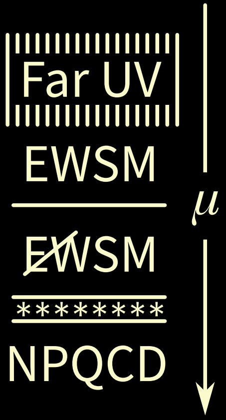
Theory of light mesons and an ALP: AχPT

- $\mathcal{L}_{A\chi PT} = \mathcal{L}^{SM}_{A\chi PT} + \mathcal{L}^{a}_{A\chi PT}$
  - $\mathcal{L}^{SM}_{A\chi PT} \to$ *Only* SM-like fields
  - Two qualitatively different modifications:

    Fields redefined due to mixing with *a*

    Interactions of *a* source interactions of $\pi^0,\ \eta$

Scales

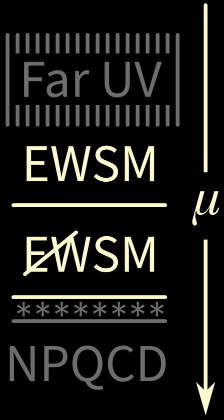


Scales

- Here be demons $\rightarrow$

Scales

- Here be demons →
- This we know →
  SM particles, SM symmetries
  ALP (a), symmetries of a



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- Chiral Lagrangian + ALP, AχPT →

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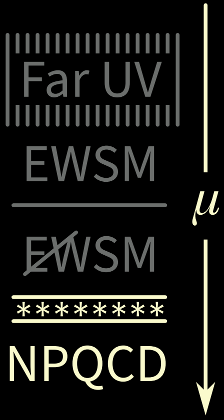
  SM particles, SM symmetries
  ALP (a), symmetries of a

! Match currents (tree level)

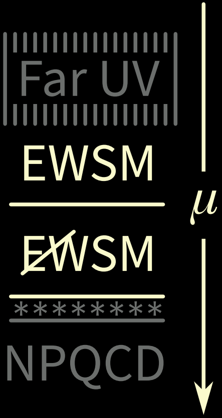
- Chiral Lagrangian + ALP, AχPT →



Symmetries & Lagrangians

Overview of operators

- Shift symmetric $(a \to a + x)$
  - $aG\tilde{G}$ $aW\tilde{W}$ $aB\tilde{B}$
  - $\frac{1}{f_a}\partial_\mu a\,[\,\bar{q}^i_L T^a_{ij}\, q^j_L \;+\; L \leftrightarrow R\,]$



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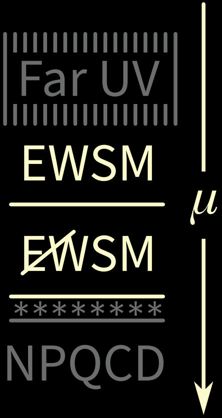
- The mass term
  - $\frac{1}{2}m_a^2 a^2$



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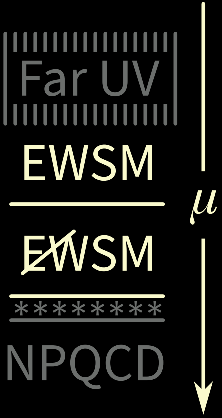
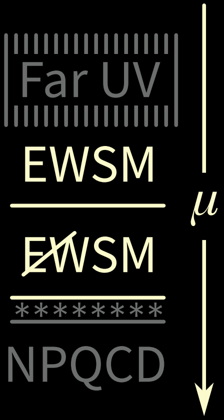
- The mass term
  - $\frac{1}{2}m_a^2 a^2$
- Periodic symmetry ? $(a \to a + \frac{2\pi}{n})$
  - aGG aWW aBB, $\frac{1}{f_a}a\bar{q}\gamma^\mu q j_\mu$

! e.g., Leading terms of $\sin(a)$, $\cos(a)$



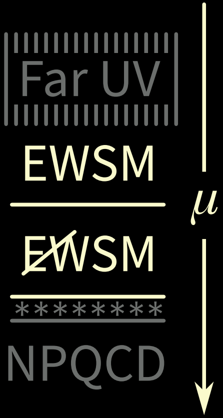
Basis and ground rules

- $SU(3)_L \times SU(3)_R \to SU(3)_V$ (flavor)



Basis and ground rules

- $SU(3)_L \times SU(3)_R \to SU(3)_V$ (flavor)
- No $aG\tilde{G}$
  - $q_R \to e^{ia/f_a} q_R$ (any axial)



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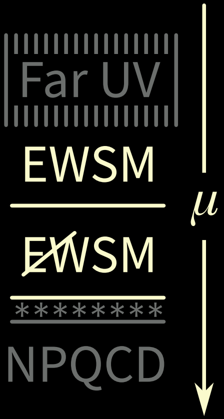
- EW symmetric basis
  - u_L, d_L same footing
  - Only t_8^L allowed for $SU(3)_L$



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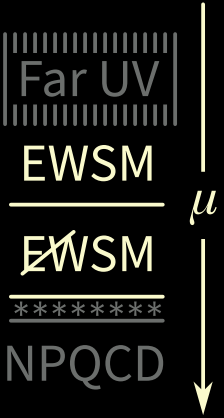
  - Only t_8^L allowed for $SU(3)_L$
- No FCNC
  - No $T_{6,7}^{L,R}$



The operators, finally.

$$\mathcal{L} \supset \sum_a C_a \mathcal{O}^a$$

$$\mathcal{O}_L^i : \quad \frac{1}{f_a}\partial_\mu a \cdot \bar{q}_L t^i \gamma^\mu q_L$$

$$\mathcal{O}_R^i : \quad \frac{1}{f_a}\partial_\mu a \cdot \bar{q}_R t^i \gamma^\mu q_R$$

$$\mathcal{O}_{LR}^i : \quad \frac{a}{f_a} \cdot \bar{q}_L t^i M q_R$$

$$\mathcal{O}_W : \quad -\frac{a}{f_a} \cdot \bar{q}_L Q^W \gamma_\mu q_L j_\pm^\mu$$

$$\mathcal{O}_Z : \quad -\frac{a}{f_a} \cdot \left(\bar{q}_L Q_L^Z \gamma_\mu q_L + \bar{q}_R Q_R^Z \gamma_\mu q_R\right) j_Z^\mu$$

G_F modification

- $V(a) \;=\; -\mu^2(a)\, H^\dagger H + \lambda(a)\,(H^\dagger H)^2$

$\Longrightarrow \quad v \to v\,(a)$

$\Longrightarrow \quad G_F \to G_F(a) \;=\; G_F(1 + C_W\, a)$

$\Longrightarrow \quad G_F j^+_\mu J^{-\mu} \to G_F(a)\, j^+_\mu J^{-\mu} \;=\; G_F(1 + C_W\, a)\, j^+_\mu J^{-\mu}$

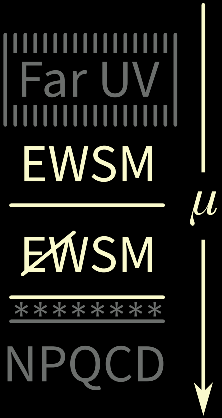
ALP-Quark Lagrangian

$$\mathcal{L} \supset \bar{q}_L \gamma^\mu \left(i\partial_\mu + L_\mu\right) q_L + L \to R$$
$$+ \bar{q}_L \overline{M} q_R + \cdots$$

$$L^\mu(a) = \left(1 + C_W \frac{a}{f_a}\right) Q^W j^\mu_\pm + \frac{\partial^\mu a}{f_a} C_L^8 t_8$$

$$R^\mu(a) = \frac{\partial_\mu a}{f_a} \sum_{i=3,8} C_R^i t^i$$

$$\overline{M} = \sum_i^{0,3,8} \left(1 + i C^i_{LR} \frac{a}{f_a} t^i + \cdots\right) M$$



Current Matching

$$U_\pi \equiv \exp\left(\frac{2i\pi^a t^a}{f_\pi}\right) \xrightarrow[L\times R]{} L\, U_\pi\, R^\dagger$$

$$J^a_{L_\mu} = -i\frac{f_\pi^2}{2}\mathrm{Tr}\left[U_\pi^\dagger t^a \partial^\mu U_\pi\right] + \cdots$$

$$\frac{\partial_\mu a}{f_a}\bar{q}_L t^3\gamma^\mu q_L \to -\frac{if_\pi^2}{2f_a}\,\partial_\mu a\,\mathrm{Tr}\left[U_\pi^\dagger t^3\partial^\mu U_\pi\right]$$

Chiral Lagrangian

$$U_\pi \equiv \exp\left(\frac{2i\pi^a t^a}{f_\pi}\right) \quad \xrightarrow[L\times R]{} \quad L_3\, U_\pi\, R_3^\dagger,$$

$$\mathcal{L} \supset \frac{f_\pi^2}{4}\,\mathrm{Tr}\left[\,\left|\partial_\mu U_\pi - i(L_\mu U_\pi - U_\pi R_\mu)\right|^2\,\right] + \frac{\Lambda f_\pi^2}{2}\,\mathrm{Tr}\left[\overline{M} U_\pi^\dagger\right] + \text{h.c.} + \cdots$$

Power Counting

- p_μ/Λ (Chiral Lagrangian)
- m_q/Λ (Breaking operators)
- α_{EM} (EM)
- G_F (Electroweak)
- $\xi \equiv f_\pi/f_a$ (ALP)

- We work at ξ^2

Observables

Field Redefinition

- $$\mathcal{L}_{2p} = \partial_\mu \begin{pmatrix}\pi^0 & \eta & a\end{pmatrix} \mathbf{K}\, \partial_\mu \begin{pmatrix}\pi^0 \\ \eta \\ a\end{pmatrix} + \begin{pmatrix}\pi^0 & \eta & a\end{pmatrix} \mathbf{M} \begin{pmatrix}\pi^0 \\ \eta \\ a\end{pmatrix}$$
- $$\begin{pmatrix}\pi^0 \\ \eta \\ a\end{pmatrix} \to R_{\text{Mass}} \times R_{\text{Kinetic}} \begin{pmatrix}\pi^0 \\ \eta \\ a\end{pmatrix}$$
- $$R_{\text{Mass}} \times R_{\text{Kinetic}} \equiv 1 + \epsilon \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \xi \begin{pmatrix} 0 & 0 & \checkmark \\ 0 & 0 & \checkmark \\ \checkmark & \checkmark & 0 \end{pmatrix} + \xi^2 \begin{pmatrix} \checkmark & \checkmark & 0 \\ \checkmark & \checkmark & 0 \\ 0 & 0 & \checkmark \end{pmatrix}$$
  - $\checkmark \equiv$ Functions of Wilson coefficients, C_i

Meson Masses

$$
\begin{aligned}
M_{\pi^\pm}^2 &= 2B_0\hat{m} + \Delta_e\,,\\
M_{\pi^0}^2 &= 2B_0\hat{m}\left[1 + \frac{\xi^2}{6}\left(3C_3^2 - 2\sqrt{3}C_{LR}^8 C_R^3 \frac{m_\Delta}{\hat{m}}\right)\right],\\
M_{K^\pm}^2 &= B_0\left(m_s + m_u\right) + \Delta_e\,,\\
M_{K^0}^2 &= M_{\bar{K}^0}^2 = B_0\left(m_s + m_d\right),\\
M_\eta^2 &= \frac{4}{3}B_0\left(m_s + \frac{1}{2}\hat{m}\right)\left[1 + \frac{\xi^2}{4}C_8^2\right].
\end{aligned}
$$

$$
\Delta_{\mathrm{GMO}} \equiv \frac{4M_K^2 - M_\pi^2 - 3M_\eta^2}{M_\eta^2 - M_\pi^2} = 0 - \frac{3}{4}\xi^2 C_8^2 + \cdots
$$

Form Factors: Field Redefinitions

- $\langle f|\mathcal{O}_{\mu\nu}|i\rangle = F_1(p\cdot p)g_{\mu\nu} + F_2(p\cdot p)p_\mu q_\nu + \cdots$

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- $\langle \pi^0(p_\pi)|\bar{s}\gamma_\mu u|K^+(p_K)\rangle \equiv \frac{1}{\sqrt{2}}\left[f_{+,\,\mathrm{SM}}^{K^+\pi^0}(q^2)\;Q_\mu + f_{-,\,\mathrm{SM}}^{K^+\pi^0}(q^2)\;q_\mu\right]$
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- $\dfrac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} = 1 - \sqrt{3}\,\epsilon - \xi^2\frac{C_3}{8}\left[C_A^3 + C_{LR}^3 + 2\sqrt{3}C_{LR}^8\right]$
- $f_-^{K^+\pi^0}(0) = 0$ at LO in the SM

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- $f_-^{K^+\pi^0}(0) = 0$ at LO in the SM

$\rightarrow$ Modified differential width spectra $\leftarrow$

Semileptonic $K^{\pm}$ decay: SM

- The decay channel: $K^+ \to \pi^0 \ell \nu$

- Lagrangian:

$$\mathcal{L} = iG_F V_{\bar{s}u} \left[K^+ \partial_\mu (\pi_0 + \sqrt{3}\eta) - \partial_\mu K^+ (\pi_0 + \sqrt{3}\eta) \right] j^\mu_{-,\ell}$$

- Amp squared:

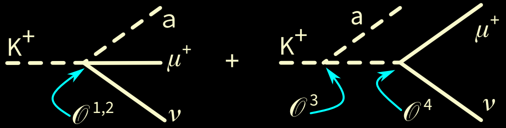
$$\overline{|\mathcal{A}|}^2_{K_{l3}} = 2G_F^2 |V_{\bar{s}u}|^2 C_{\rm cor} (2H \cdot p_\ell \, H \cdot p_{\nu_\ell} - H^2 p_\ell \cdot p_{\nu_\ell}),$$

$$H_\mu \equiv f_{+,\mathrm{SM}}^{K^+\pi^0}(t)\, Q_\mu + f_{-,\mathrm{SM}}^{K^+\pi^0}(t)\, q_\mu$$

$K^+ \to \pi^0 \ell \nu$: AχPT

- $\mathcal{O}^1_{K^+_{\ell 3}}$: $(K^+\partial_\mu a - \partial_\mu K^+ a)\, j^\mu_{-,\ell}$ $\quad iG_F V_{\bar{s}u} \frac{\xi}{2}(\mathcal{C}_R - 2iC_W)$
- $\mathcal{O}^2_{K^+_{\ell 3}}$: $(K^+\partial_\mu a + \partial_\mu K^+ a)\, j^\mu_{-,\ell}$ $\quad iG_F V_{\bar{s}u} \frac{\xi}{2}(\mathcal{C}_R + 2iC_W)$
- $\mathcal{O}^3_{K^+_{\ell 3}}$: $\partial^\mu a\, (\partial_\mu K^+ K^- - K^+ \partial_\mu K^-)$ $\quad \frac{i}{4}\frac{1}{f_\pi}\xi\, (\mathcal{C}_R + \sqrt{3}C^8_L)$
- $\mathcal{O}^4_{K^+_{\ell 3}}$: $\partial_\mu K^+ j^\mu_-$ $\quad -2f_\pi\, G_F V_{\bar{s}u}$

! $\mathcal{C}_\mathcal{R} = C^3_R + \sqrt{3}C^8_R$



Form Factors: New operators

$$\mathrm{Re}\left(\tilde{f}_{+}^{K^+\pi^0}(q^2)\right) = \left(\alpha_{K^+\pi^0}^{(0)} + \xi^2\alpha_{K^+\pi^0}^{(2)}\right)\left[1 + \lambda_{K^+\pi^0}^{+,\,(0)}\frac{q^2}{M_\pi^2} + \lambda_{K^+\pi^0}^{\prime +,\,(0)}\frac{q^4}{2M_\pi^4}\right]$$

$$\simeq \left[1 + \xi^2\frac{\alpha_{K^+\pi^0}^{(2)}}{\alpha_{K^+\pi^0}^{(0)}}\right] f_{+,\,\mathrm{SM}}^{K^+\pi^0}(q^2)\ ,$$

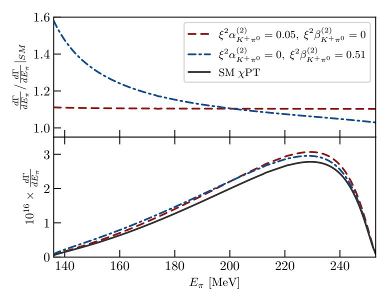
$$\mathrm{Re}\left(\tilde{f}_{-}^{K^+\pi^0}(q^2)\right) = \left(\delta\beta_{K^+\pi^0}^{(0)} + \xi^2\beta_{K^+\pi^0}^{(2)}\right)\left[1 + \lambda_{K^+\pi^0}^{-,\,(0)}\frac{q^2}{M_\pi^2} + \lambda_{K^+\pi^0}^{\prime -,\,(0)}\frac{q^4}{2M_\pi^4}\right]$$

$$\simeq \left[1 + \xi^2\frac{\beta_{K^+\pi^0}^{(2)}}{\delta\beta_{K^+\pi^0}^{(0)}}\right] f_{-,\,\mathrm{SM}}^{K^+\pi^0}(q^2)\ ,$$

Distortion of differential spectrum

$$\overline{|\mathcal{A}|}^2_{K_{l3}} = 2G_F^2 |V_{\bar{s}u}|^2 C_{\text{cor}} \left[1 + 2\,\xi^2 \frac{\alpha^{(2)}_{K^+\pi^0}}{\alpha^{(0)}_{K^+\pi^0}} \right] \left(2H \cdot p_\ell \, H \cdot p_{\nu_\ell} - H^2 p_\ell \cdot p_{\nu_\ell} \right),$$

$$H_\mu \equiv f^{K^+\pi^0}_{+,\text{SM}}(t)\, Q_\mu + \left[1 + \xi^2 \left(\frac{\beta^{(2)}_{K^+\pi^0}}{\delta\beta^{(0)}_{K^+\pi^0}} - \frac{\alpha^{(2)}_{K^+\pi^0}}{\alpha^{(0)}_{K^+\pi^0}} \right) \right] f^{K^+\pi^0}_{-,\text{SM}}(t)\, q_\mu .$$

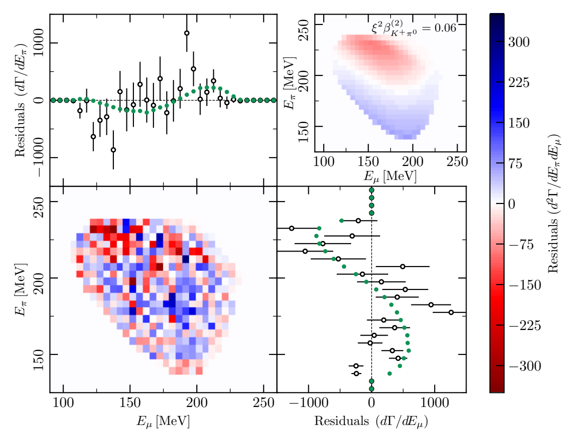


- $d\Gamma/dE$
- LO: SM – BSM ξ^2
- $\beta \to \ell$ mass suppressed
- $q_\mu \ell \gamma^\mu \nu \sim m_\ell$

Strategy

- Get lattice computations for FF parameters
  - ETM collaboration
- Get differential data from experiments
  - $K/\pi \to \pi\ell\nu$ distribution from NA48/2
  - Total rate from PDG
- Get G_F and $V_{\bar{s}u}$ from *other* places
  - $V_{\bar{s}u}$ from $K^+ \to \ell^+\nu$

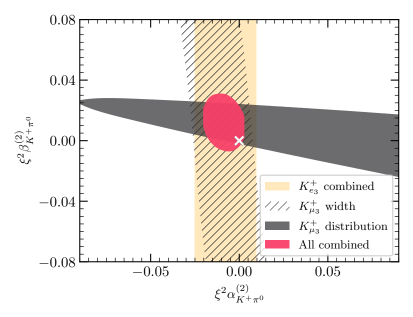
Experimental Data



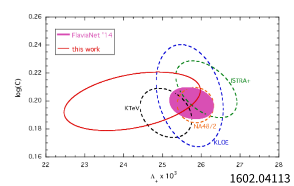
- NA48/2
- $K^+_{e_3}$, $K^+_{\mu_3}$
- $\sim 10^6$ reconstructed

Bounds

$$\xi^2\beta^{(2)}_{K^+\pi^0} \approx \frac{M_K^2-M_\pi^2}{M_\pi^2}\left[\left(\lambda^{+,\,(0),\mathrm{Fit}}_{K^+\pi^0}-\lambda^{0,\,(0),\mathrm{Fit}}_{K^+\pi^0}\right)-f^{K^+\pi^0}_{+,\,\mathrm{SM}}(0)\left(\lambda^{+,\,(0),\mathrm{SM}}_{K^+\pi^0}-\lambda^{0,\,(0),\mathrm{SM}}_{K^+\pi^0}\right)\right]$$
$$\sim 0.01 \pm 0.04$$



- Total width $\rightarrow \alpha$
- Diff. width $\rightarrow \beta$
- No β from e (m_e)



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Flat Directions

- $$\mathcal{L}_{a\ell^+\nu} \supset iG_F V_{\bar{s}u}\, \xi \Big[\left(\alpha^{(1)}_{K^+a} + i\tilde{\alpha}^{(1)}_{K^+a}\right)\left(K^+\partial_\mu \hat{a} - \partial_\mu K^+ \hat{a}\right) + \left(\beta^{(1)}_{K^+a} + i\tilde{\beta}^{(1)}_{K^+a}\right)\partial_\mu\left(K^+\hat{a}\right)\Big] j^\mu_{-,\ell}$$

$$\alpha^{(1)}_{K^+a} \equiv f(C_i);$$

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i.e. for the pion case

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$$\alpha^{(2)}_{K^+a} \equiv g(C_i);$$

i.e. for the pion case

- Careful analysis gives different condition for pion phobia

Sum Rules

- Add up amplitudes to get an idea of underlying theory
- In the SM:

$$\frac{1}{4}\left|f_{+,\,\mathrm{SM}}^{K^+\pi^0}(0)\right|^2 + \frac{3}{4}\left|f_{+,\,\mathrm{SM}}^{K^+\eta}(0)\right|^2 = 1$$

  - Completeness of basis (Also think of Cabbibo angle etc)

- The same sum in the AχPT:
  - $\frac{1}{4}\left|\tilde{f}_{+}^{K^+\pi^0}(0)\right|^2 + \frac{3}{4}\left|\tilde{f}_{+}^{K^+\eta}(0)\right|^2 =$

$$1 - \frac{\xi^2}{16}\left(C_3 + \sqrt{3}C_8\right)^2 + \xi^2\frac{3}{16}(C_L^8)^2$$

- The first term is expected
- These sums can tell us about the *structure* of the theory

Summary

- Low lying ALPs modify χPT in non-trivial ways
- These modifications can be observed and categorized
  - Meson mass spectrum
  - Differential widths
  - Sum rules
- The search for these modifications will complement direct searches
- More precise computations needed
  - Derivation of the modified Lagrangian
  - Precise computations of the SM parameters

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Thank You

Questions/Inputs/Critique?